ADAPTIVE ANGULAR-DISPLACEMENT VOLD-KALMAN ORDER TRACKING

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ABSTRACT

The paper proposes and implements an adaptive Vold-Kalman filtering order tracking (VKF_OT) approach to improve the original VKF_OT scheme in an adaptive manner for the interpretation and diagnosis of rotary machinery during operation. The paper comprises theoretical derivation and numerical implementation. Comparisons of the improved VKF_OT scheme to the original are accomplished through processing a synthetic signal. Parameters such as the weighting factor and the correlation matrix of process noise, which influences tracking performance, are investigated. The adaptive OT scheme based on the Kalman filter can be computed on-line and implemented as a real-time processing application.

Keywords: Adaptive filtering, signal extraction, order tracking, acoustic and vibration signals.

1. INTRODUCTION

Mechanical systems under periodic loading due to rotary operation usually respond in measurements with a superposition of sinusoids whose frequencies are integer (or fractional integer) multiples of the reference shaft speed, which is usually the revolution speed of the loading. The fundamental frequency corresponding to the shaft speed is called a basic *order*. Most rotary machinery consists of various kinds of rotating parts, e.g. gear, chain, or shaft for power transmission. The frequencies of noise or vibration signals caused by these parts relate to the revolution speed of the shaft connected with rotating parts. Order tracking (OT) is one of effective tools for the diagnosis of rotary machinery by the analysis of the dynamic signals.

Conventional OT approaches such as the windowed Fourier transform (WFT) and the resampling methods are primarily based on Fourier analysis, and have limited resolution in some situations and suffer from a number of shortcomings. Vold and Leuridan proposed a novel algorithm, Vold-Kalman filtering order tracking (VKF_OT), for the estimation of a *single* order component, and further *simultaneous* estimation of multiple orders, respectively in 1993 and 1997 [1]-[3]. Pan and Lin [4],[5] further explored the theoretical details of VKF_OT and compared the differences of two VKF_OT schemes, the angular-velocity and angular-displacement VKF_OTs. But it is noted that these two VKF_OT schemes must be calculated off-line and implemented as post-processing techniques. They are not real-time applications yet. The computations for both the angular-velocity and angular-displacement VKF_OTs are rather time-consuming since all the data of an acquired signal to be tracked for specific order/spectral components are included in huge matrix manipulation [4]. To cope with this problem, an adaptive real-time processing VKF_OT algorithm based on the Kalman filter conducting on-line computation is derived and realized in this paper.

2. THEORETICAL BASIS

The *k*th-order component arising from the operation of a rotary machine can be shown as [4]

$$x_k(t) = a_k(t)\theta_k(t) + a_{-k}(t)\theta_{-k}(t)\,, \tag{1}$$

where $a_k(t)$ denotes the complex envelope, and $a_{-k}(t)$ is the complex conjugate of $a_k(t)$ to make $x_k(t)$ a real waveform. It is noted that $\theta_k(t)$ is a carrier wave in the formality of [3]

$$\theta_k(t) = \exp\left(ki\int_0^t \omega(u)du\right), \tag{2}$$

where $\omega(u)$ denotes the speed of the reference axle, and $\int_0^t \omega(u)du$ is the elapsed angular displacement.

The basic idea of the angular-displacement VKF_OT is to define local constraints, the so-called structural equation and data equation [3], which make the unknown complex envelopes smooth and relate the tracked orders to the measured signal. These two equations are solved by using a least-squares technique through a large number of matrix computation. It resembles the Wiener filter in mathematical operation. Furthermore, the Kalman filter is essentially a recursive algorithm whose solution converges to the optimum Wiener solution in some statistical sense. The mathematical model of the Kalman filter can be embodied in a pair of equations, i.e., the process and measurement equation [6], as below.

Process equation : $\mathbf{x}(n+1) = \mathbf{F}(n+1,n)\mathbf{x}(n) + \mathbf{v}_1(n)\,.$ (3)

Measurement equation : $\mathbf{y}(n) = \mathbf{C}(n)\mathbf{x}(n) + \mathbf{v}_2(n)\,.$ (4)

The proposed adaptive angular-displacement VKF_OT here provides us a recursive algorithm without having to solve the complex matrix problem and evaluate a huge inverse matrix with all observed time points. Thus the original (off-line) angular-displacement VKF_OT [3],[4] with its computational complexity in solving inverse matrix can be improved.

2-1. Structural equation

As the order component $x_k(t)$ to be tracked is formulated by (1), the envelope $a_k(t)$ needs to be computed. Generally, $a_k(t)$ is a relatively smooth polynomial with a low degree due to the intrinsic inertia of a rotary mechanical system. It fulfills [3]

$$\frac{d^s a_k(t)}{dt^s} = \psi_k(t), \tag{5}$$

where $\psi_k(t)$ denotes a higher-degree term in $a_k(t)$. Thus using a difference operator their discrete form can be expressed as

$$\nabla^s a_k(n) = \psi_k(n), \tag{6}$$

where ∇ denotes the difference operator, the index s is the differentiation order, and $\psi_k(t)$ physically means a combination of other spectral components and additional measurement noise. For s = 2, (6) can be further shown in matrix form like

$$\begin{bmatrix} a_k(n) \\ a_k(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a_k(n-1) \\ a_k(n) \end{bmatrix} + \begin{bmatrix} 0 \\ \psi_k(n) \end{bmatrix}. \tag{7}$$

Then Eq.(7) can be symbolized as

$$\underline{a}_k(n+1) = \mathbf{M}\,\underline{a}_k(n) + \Psi_k(n). \tag{8}$$

To simultaneously track multiple order/spectral components, (8) can be augmented to all K-order/spectral components to be extracted. For multiple-order tracking and decoupling, (7) can be extended as

$$\begin{bmatrix} \underline{a}_1(n+1) \\ \underline{a}_2(n+1) \\ \vdots \\ \underline{a}_K(n+1) \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \underline{a}_1(n) \\ \underline{a}_2(n) \\ \vdots \\ \underline{a}_K(n) \end{bmatrix} + \begin{bmatrix} \Psi_1(n) \\ \Psi_2(n) \\ \vdots \\ \Psi_K(n) \end{bmatrix}. \tag{9}$$

Likewise, the symbolized form of (9) is

$$\mathbf{A}(n+1) = \mathbf{F}(n+1,n)\mathbf{A}(n) + \mathbf{Z}(n). \tag{10}$$

The structural equation, $\mathbf{A}(n+1)$, is like an IIR lowpass filter, and characterizes the smoothness condition of unknown complex envelopes. $\mathbf{A}(n+1)$ can be regarded as the output of the lowpass filter excited by white noise $\mathbf{Z}(n)$.

2-2. Data Equation

A measured signal *y(n)* can be considered as a combination of several order/spectral components $x_k(n)$ expressed by (1), and measurement noise, i.e.,

$$y(n) = \sum_{k\in j} a_k(n)\theta_k(n) + \xi(n), \tag{11}$$

where the integral j(= ±1, ±2, ±3, …, and/or ±K) denotes the order of spectral components to be tracked, and $\xi(n)$ comprises unwanted spectral components and measurement errors. Let $\underline{B}_k(n)$ comprise a carrier signal with the form $\underline{B}_k(n) = \begin{bmatrix} 0 & \theta_k(n) \end{bmatrix}$. Therefore, (11) can be rewritten as

$$y(n) - \begin{bmatrix} \underline{B}_1(n) & \underline{B}_2(n) & \underline{B}_3(n) & \cdots & \underline{B}_K(n) \end{bmatrix} \begin{bmatrix} \underline{a}_1(n) \\ \underline{a}_2(n) \\ \vdots \\ \underline{a}_K(n) \end{bmatrix} = \xi(n), \tag{12}$$

or symbolized as

$$y(n) = \mathbf{B}(n)\mathbf{A}(n) + \xi(n). \tag{13}$$

Equation (13) is the data equation of the adaptive angular-displacement VKF_OT, and has the same form as the measurement equation (4) of a Kalman filter.

2-3. Kalman filter based on one-step prediction

As derived before, the deduced data equation, (10), and structure equation, (13), are formed identical to the process and measurement equations of Kalman filtering. Consequently, the order-tracking problem can be considered as a state estimation problem like the procedure of Kalman filtering. In the process equation of a Kalman filter, the $2K\times1$ vector $\mathbf{Z}(n)$ represents process noise, modeled as a zero-mean, white-noise process whose correlation matrix is defined by

$$E[\mathbf{Z}(n)\mathbf{Z}^H(m)] = \begin{cases} \mathbf{Q}_1(n), & n = m \\ \mathbf{0}, & n \neq m \end{cases}. \tag{14}$$

Accordingly, we define the correlation matrix of the process noise as

$$E[\psi_k(n)\psi_l^*(m)] = \begin{cases} \mathbf{Q}_1^{k,l}(n), & n = m \cap k = l \\ 0, & n \neq m \cup k \neq l \end{cases}, \tag{15}$$

where $\psi_k(n)$ or $\psi_l(m)$ is the element in the process noise vector $\mathbf{Z}(n)$, $k, l \in j(=1, 2, 3, ..., \text{and/or K})$ represents the index of the order components to be tracked, $\mathbf{Q}_1^{k,l}(n)$ is the element at the kth row and lth column of $\mathbf{Q}_1(n)$, and the asterisk denotes complex conjugation. The variable $\xi(n)$ in the measurement equation of the Kalman filter is the measurement noise, modeled as a zero-mean, white-noise process whose correlation is defined by

$$E[\xi(n)\xi^*(m)] = \begin{cases} Q_2(n), & n = m \\ 0, & n \neq m \end{cases}. \tag{16}$$

The process noise and the measurement noise are statistically independent, so

$$E[\psi_k(n)\xi^*(m)] = \mathbf{0}, \text{ for all } n \text{ and } m. \tag{17}$$

To design the parameters $\mathbf{Q}_1(n)$ and $Q_2(n)$, we apply the weighting factor introduced in the *angular-velocity* VKF_OT [3] to the Kalman filtering process. In the angular-velocity VKF_OT, the measure $S_\varepsilon(n)$ and $S_\eta(n)$ are defined as the standard deviations of the heterogeneity $\varepsilon(n)$ and nuisance component $\eta(n)$ of the structural and data equations, respectively. The ratio of the two standard deviation functions written as $r(n) = S_\eta(n) / S_\varepsilon(n)$ is the weighting factor of the angular-velocity VKF_OT [3]. Correspondingly, the introduced weighting factor $r(n)$ can also satisfy (17) to be defined as (15) for all $n \neq m$.

For a real scalar weighting factor corresponding to constant bandwidth, $\mathbf{Q}_1(n)$ is a diagonal matrix taken to be a $2K\times 2K$ identity matrix multiplied by a designated variance value. Thus the relation between $\mathbf{Q}_1(n)$ and $Q_2(n)$ for the adaptive VKF_OT scheme can be acquired as

$$\frac{S_\eta^{\,2}(n)}{S_\varepsilon^{\,2}(n)} = r^2(n) = \frac{Q_2(n)}{\mathbf{Q}_1^{k,l}(n)}, \;=> Q_2(n) = r^2(n)\mathbf{Q}_1^{k,l}(n) \cdot \qquad (18)$$

As a result, the tracked order envelopes can be recursively solved by using Kalman filtering based on one-step prediction as illustrated in [6].

3. NUMERICAL IMPLEMENTATION

A synthetic signal is designated to validate the proposed adaptive OT scheme, as well as to compare with the original angular-displacement method. Computation parameters that influence tracking performance, such as the weighting factor and the correlation matrix of process noise, are investigated.

3-1. Multi-axle Crossing Order Decoupling

Two reference speeds with order crossing are designated, where one shaft speed linearly increases from stationary to 3000 rpm in 5 sec, and the other speeds up from stationary to 9000 rpm at 2 s and 5 s. Accordingly, this synthetic signal comprises two sets of order components, one with orders 1, 4, and 9, and the other with orders 2 and 5 in accordance with respective reference speeds. The signal is designed to justify the performance of the proposed VKF_OT scheme in discriminating crossing orders arising from different rotating systems. Figure 1 shows the time-frequency spectrum of the synthetic signal via the computation of the WFT, which illustrates two sets of orders and characterizes order-crossing phenomena. The computed amplitudes of orders 1, 4, and 9 using the original and adaptive VKF_OTs are illustrated in Fig. 2. Both VKF_OT techniques are capable of tracking specific orders crossing with other spectral components. The estimated amplitudes using the adaptive scheme show indentations although they have fewer end effects than the original scheme. The computed Order 1 shows unevenness at around 2 s due to the mixing of three run-up orders crossing concurrently. Figure 3 displays the waveform estimations of specific orders extracted by the adaptive angular-displacement VKF_OT method.

As to the computation efficiency for these two OT methods, it is noted that computation consumption using the original scheme is 3.5 times as much as the adaptive scheme for tracking Order 9, which crosses once with other spectral components, as shown in Fig. 1. The CPU consumption dramatically increases up to around 17 times when extracting Orders 4 and 1, both of which cross twice with the other set of orders. In general, the computation time required for using VKF_OT methods increases as more observed time points and as the tracked order/spectral components increase. Another factor dominating computational efficiency is the number of crossing occurrences for an order to be tracked with other spectral components. When more crossing occurrences exist in an order, this increases computational complexity in solving for the inverse matrix.

3-2. Influence of the Designed Parameters

To investigate the design parameters of adaptive angular-displacement VKF_OT on tracking performance, amplitude estimation of Order 9 presented in the synthetic signals is conducted. The relation of main design parameters in the adaptive scheme formulates as shown in (21). As the weighting factor r and the correlation matrix of process noise $\mathbf{Q}_1$ are assigned, the correlation of measurement noise Q_2 can be decided. Figure 4 illustrates tracked amplitudes using different weighting factors and a fixed $\mathbf{Q}_1 = 10^{-2} \times \mathbf{I}_{2\times 2}$,

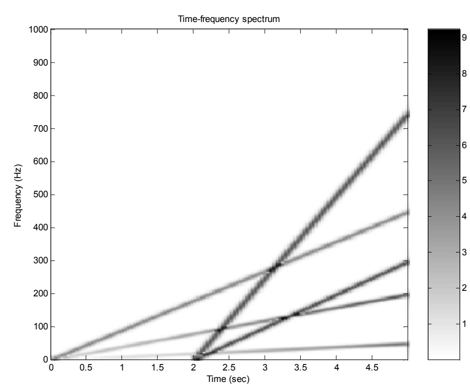

Figure 1: Spectrogram of two sets of order components.

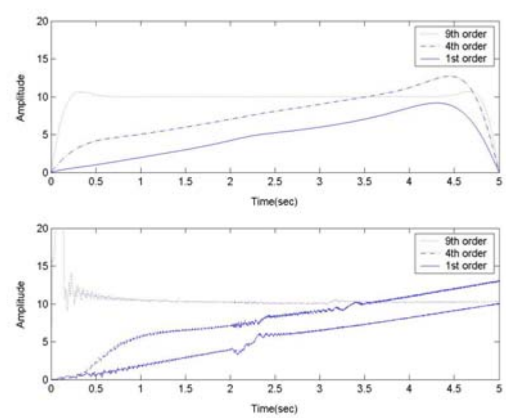

Figure 2: Amplitudes of orders 1, 4, and 9 tracked by (a) angular-displacement VKF_OT, and (b) adaptive angular-displacement VKF_OT, respectively.

where the amplitude of the synthesized Order 9 is shown at the top. The results demonstrate that a large weighting factor may slow down this convergence rate of the tracked order component, but obtain a smooth steady-state value of the tracked amplitude. This signifies that the weighting-factor tuning will affect both the convergence rate and the smoothness of the complex envelops defined by the structural equation. Moreover, this reveals that a tradeoff exists between the rate of convergence and tracking accuracy. Figure 5 illustrates a comparison of amplitude estimation with different $\mathbf{Q}_1$ and a fixed weighting factor $r = 1\times 10^6$, where $\mathbf{Q}_1$ defined in the process equation of Kalman filter is designated to be a 2×2 identity matrix multiplied by different variance values q_1, 10^{-2}, 10^{-5}, 10^{-8}, and 10^{-15}, respectively. It is noted that as the variance value q_1 is small, and the amplitude estimation converges fast to reach its designated value. Furthermore, it is observed for the value of $q_1 = 1\times 10^{-15}$ that fast convergence to the amplitude, 10, accompanies an acute initial transient. Observing both Figs. 4 and 5, it is worth mentioning that a large weighting factor r associated with a small variance matrix $\mathbf{Q_1}$ can yield both fast convergence and good tracking performance, but with a side-effect of an acute initial transient.

4. CONCLUSIONS

The paper has proposed derives, and implemented an advanced VKF_OT approach, i.e., the adaptive angular-displacement VKF_OT technique, to overcome the deficiencies of the original scheme. Comparison and discussion of the developed scheme to the original was accomplished. The proposed scheme can simultaneously extract multiple order/spectral components, and effectively decouple close and/or crossing orders. The adaptive scheme extracts order components with slight indentation, but its computation is extremely efficient. This merit benefits real-time processing, especially for the purpose of industrial applications.

5. REFERENCES

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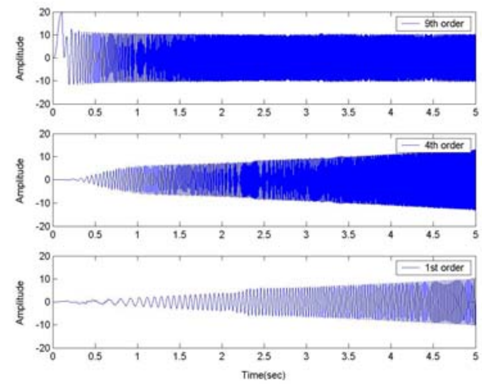

Figure 3: Order components tracked by adaptive angular-displacement VKF_OT scheme, waveform of (a) order 9; (b) order 4; and (c) order 1.

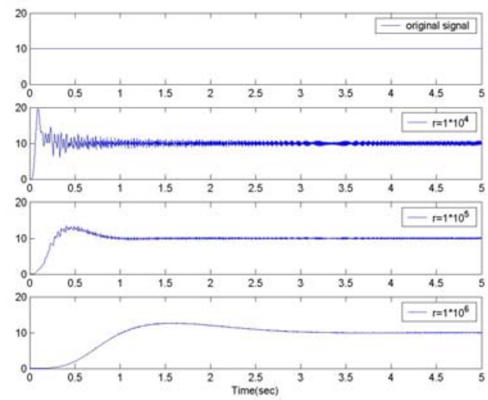

Figure 4: Amplitudes of order-9 extracted by using the adaptive angular-displacement VKF_OT with various weighting factor r.

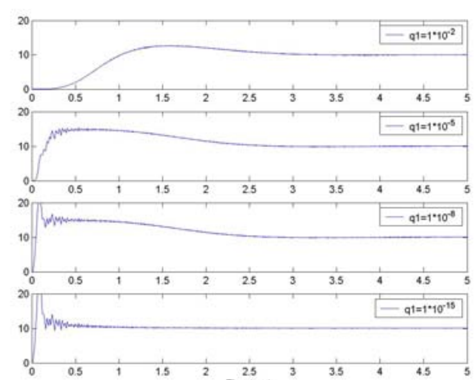

Figure 5: Amplitude estimation of order 9 by using the adaptive scheme with various correlation matrix of process noise $\mathbf{Q_1}$.