COMPARISON OF AVERAGE PERFORMANCE OF GPS DISCRIMINATORS IN MULTIPATH

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ABSTRACT

Signal multipath in GPS leads to undesirable tracking errors and inaccurate ranging information. The extent of the tracking error in compromising the receiver performance depends on the multipath amplitude, delay, and phase relative to the direct path. The coherent discriminator and the noncoherent early-minus-late power discriminator offer different tracking accuracy and sensitivity to multipath parameters. In this paper, we develop analytical expressions of the average performance of the two discriminators over the multipath phase distribution and other propagation channel variables.

Index Terms— GPS, Multipath, Tracking error, Coherent discriminator, Noncoherent discriminator.

1. INTRODUCTION

The multipath problem in Geolocation can cause severe degradation of the performance of GPS receivers [1-3]. Both coherent and noncoherent discriminators can be applied in the GPS receivers. . Various studies on multipath effects for the DLL discriminators have been performed [4-7]. This paper provides analytical treatment of the GPS multipath effect on the coherent discriminator and the noncoherent early-minus-late power discriminator. It compares the performance of the two discriminators under one dominant multipath for different multipath amplitude, phase, and time delay. Specifically, the role of carrier phase offset due to multipath propagation is demonstrated and its contribution to the tracking error is examined. Computer simulations of the impact of a dominant multipath on the discriminator tracking performance are provided. It is assumed that there is an infinite front-end precorrelation bandwidth, the early and late correlations are performed within the same navigation symbol, and no symbol transitions are encountered over the correlation interval. It is shown that for short multipath delay and uniform distribution of the multipath carrier offset, the noncoherent discriminator provides a superior performance over the coherent discriminator.

Closed form expressions of the tracking error as a function of multipath parameters using the coherent and the early-minus-late power discriminators, are provided respectively in Section 2 and Section 3. The performance comparison in multipath of the two discriminators based on averaged error values in multipath is provided in Section 4. The conclusion is given in Section 5.

2. PERFORMANCE ANALYSIS FOR COHERENT DISCRIMINATOR

First, we examine, in details, the effect of multipath on the coherent discriminator performance. The discriminator function in mulitpath is given by,

$$\begin{aligned} D_m &= \frac{A}{\sqrt{2}}\{\cos(\psi)[R(\tau+\frac{d}{2})-R(\tau-\frac{d}{2})] \\ &+\alpha\cos(\psi+\beta)[R(\tau+\frac{d}{2}-\Delta\tau_m)-R(\tau-\frac{d}{2}-\Delta\tau_m)]\} \\ &= D_c + D_{err} \end{aligned} \quad (1)$$

where A is the signal amplitude, α is the multipath-to-direct signal amplitude ratio, d is the early-late correlator spacing, $\Delta\tau_m$ is the multipath time delay relative to the direct path, and $R(\tau)$ is the autocorrelation function of the C/A code. β is the multipath carrier phase offset due to both the delay $\Delta\tau_m$ and the phase shift, $\Delta\theta_m$, induced by the reflector [8], with

$$\beta = -(\omega_c+\omega_d)\Delta\tau_m + \Delta\theta_m \quad (2)$$

where ω_c is the carrier frequency and ω_d is the Doppler shift. ψ is the phase error between the estimated signal carrier phase and the direct signal phase, which is given by [9],

$$\psi = \tan^{-1}[-\frac{\alpha\sin(\beta)}{1+\alpha\cos(\beta)}] \quad (3)$$

In (1), the first term, D_c, represents the multipath-free discriminator component. It is referred to as the regular

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discriminator function. Accordingly, the multipath induced error component, D_{err}, is

$$D_{err} = \frac{\alpha A}{\sqrt{2}}\cos(\psi+\beta)[R(\tau+\frac{d}{2}-\Delta\tau_m)-R(\tau-\frac{d}{2}-\Delta\tau_m)] \quad (4)$$

The error component distorts the regular discriminator function, which no longer assumes a zero value at the correct synchronous delay, leading to a tracking error. To evaluate the induced tracking error due to multipath, we approximate the autocorrelation function by

$$R(\tau) = \begin{cases} 1-|\tau|, & \text{for } |\tau| \le 1 \text{ (chip)} \\ 0, & \text{for } |\tau| > 1 \text{ (chip)} \end{cases} \quad (5)$$

We consider the discriminator is performing within its linear range, substitute (3) and (5) into (1), and then set $D_m = 0$, $d \le 1$ chip. The result is given by

$$\rho = \tau|_{D_m=0} = \begin{cases} \dfrac{\alpha[\alpha+\cos(\beta)]\Delta\tau_m}{1+2\alpha\cos(\beta)+\alpha^2}, \\ \text{for } \; 0 \le \Delta\tau_m < \dfrac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]}; \\ \dfrac{\alpha[\alpha+\cos(\beta)]d}{2[1+\alpha\cos(\beta)]}, \; \text{for } \; \dfrac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]} \\ \le \Delta\tau_m < 1-\dfrac{(1-\alpha^2)d}{2[1+\alpha\cos(\beta)]}; \\ \dfrac{\alpha[\alpha+\cos(\beta)](-\Delta\tau_m+d/2+1)}{2+\alpha\cos(\beta)-\alpha^2}, \\ \text{for } \; 1-\dfrac{(1-\alpha^2)d}{2[1+\alpha\cos(\beta)]} \le \Delta\tau_m < 1+\dfrac{d}{2}; \\ 0, \; \text{for } \; \Delta\tau_m > 1+\dfrac{d}{2} \end{cases} \quad (6)$$

3. PERFORMANCE ANALYSIS FOR NONCOHERENT DISCRIMINATOR

The early-minus-late power discriminator is capable of overcoming the effect of residual phase error. Although it provides the same tracking error envelope of the coherent discriminator, it has different multipath phase dependency within performance bounds, yielding an average behavior different than its coherent discriminator counterpart. The corresponding discriminator function in multipath, is

$$\begin{aligned} D_m = {} & \frac{A^2}{2}[R^2(\tau+\frac{d}{2})-R^2(\tau-\frac{d}{2})] \\ & +\frac{A^2}{2}\{\alpha^2[R^2(\tau+\frac{d}{2}-\Delta\tau_m)-R^2(\tau-\frac{d}{2}-\Delta\tau_m)] \\ & +2\alpha\cos(\beta)[R(\tau+\frac{d}{2})R(\tau+\frac{d}{2}-\Delta\tau_m) \\ & -R(\tau-\frac{d}{2})R(\tau-\frac{d}{2}-\Delta\tau_m)]\} = D_{nc}+D_{err} \end{aligned} \quad (7)$$

D_{nc} represents the multipath-free discriminator component. The error component is expressed as,

$$\begin{aligned} D_{err} = {} & \frac{A^2}{2}\{\alpha^2[R^2(\tau+\frac{d}{2}-\Delta\tau_m)-R^2(\tau-\frac{d}{2}-\Delta\tau_m)] \\ & +2\alpha\cos(\beta)[R(\tau+\frac{d}{2})R(\tau+\frac{d}{2}-\Delta\tau_m) \\ & -R(\tau-\frac{d}{2})R(\tau-\frac{d}{2}-\Delta\tau_m)]\} \end{aligned} \quad (8)$$

It can be readily shown that the tracking error expression for the early-minus-late power discriminator is,

$$\rho = \tau|_{D_m=0}$$

$$= \begin{cases} \dfrac{\alpha[\alpha+\cos(\beta)]\Delta\tau_m}{1+2\alpha\cos(\beta)+\alpha^2}, \\ \text{for } \; 0 \le \Delta\tau_m < \dfrac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]}; \\ \{[(\alpha\cos(\beta)(1-\Delta\tau_m)-\alpha^2 d/2+1-d/2)^2 \\ +2\alpha^2 d\cos^2(\beta)(1-d/2)+2\alpha^3 d\cos(\beta)(1-\Delta\tau_m)]^{1/2} \\ -[\alpha\cos(\beta)(1-\Delta\tau_m)-\alpha^2 d/2+1-d/2]\}/2\alpha\cos(\beta), \\ \text{for } \; \dfrac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]} \le \Delta\tau_m \\ < \dfrac{[\alpha d\cos(\beta)+1](2-d)-d(1-d/2-\alpha^2 d/2)}{\alpha d\cos(\beta)+2-d}; \\ \{[\alpha\cos(\beta)-\alpha^2]\Delta\tau_m+\alpha^2(1+d/2)-\alpha d\cos(\beta)-2+d \\ +\{\alpha^2\cos^2(\beta)\Delta\tau_m{}^2+2\alpha(2-d)[\alpha-\cos(\beta)]\Delta\tau_m \\ -4\alpha^2\cos^2(\beta)\Delta\tau_m+(2-d)[2-d+2\alpha d\cos(\beta)] \\ +4\alpha^2[d^2/4-\sin^2(\beta)]\}^{1/2}\}/[2\alpha\cos(\beta)-\alpha^2], \\ \text{for } \; \dfrac{[\alpha d\cos(\beta)+1](2-d)-d(1-d/2-\alpha^2 d/2)}{\alpha d\cos(\beta)+2-d} \le \Delta\tau_m \\ < 1+\dfrac{d}{2}; \\ 0, \; \text{for } \; \Delta\tau_m > 1+\dfrac{d}{2} \end{cases} \quad (9)$$

4. PERFORMANCE COMPARISON OF DISCRIMINATORS

Examining (6) and (9), the tracking errors for both discriminators over the multipath time delay from 0 to $\dfrac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]}$ are the same. However, over the range $\dfrac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]}$ to $1+\dfrac{d}{2}$, the two errors are different in most cases. The difference is very evident in Fig. 1, which shows the tracking error over $\Delta\tau_m$ for $\alpha = 0.56$ under different values of β. In Fig. 1, each curve of the tracking error consists of three segments. It is noted that the intersection point between the first segment and the second segment is always located on a straight line, labeled 1, whereas the intersection point between the second segment and the third segment is always located on another straight line, labeled 2. Line 1 and line 2 are, respectively, expressed as,

$$\rho = \Delta\tau_m - \frac{d}{2}\ , \qquad \rho = \Delta\tau_m - 1 + \frac{d}{2} \tag{10}$$

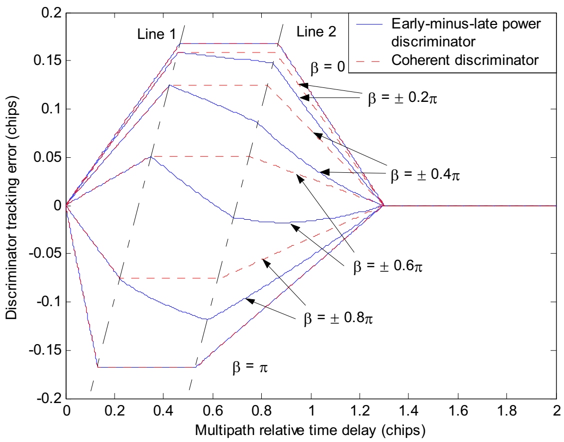

Fig. 1. Multipath tracking error of the coherent and the noncoherent discriminators

To compare the two discriminators, it is necessary to describe the overall or average receiver performance. This can be simply achieved by integrating the absolute value of tracking error, after weighting it by the proper probability density function of the multipath time delay. From (6), assuming uniform distribution, the integral of the tracking error over the multipath time delay for the coherent discriminator leads to average error,

$$\begin{aligned}
\rho_{av} &= \int_0^{1+\frac{d}{2}} |\rho| \ d\Delta\tau_m = \int_0^{\frac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]}} \left| \frac{\alpha[\alpha+\cos(\beta)]\Delta\tau_m}{1+2\alpha\cos(\beta)+\alpha^2} \right| \ d\Delta\tau_m \\
&+ \int_{\frac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]}}^{1-\frac{(1-\alpha^2)d}{2[1+\alpha\cos(\beta)]}} \left| \frac{\alpha[\alpha+\cos(\beta)]d}{2[1+\alpha\cos(\beta)]} \right| \ d\Delta\tau_m \\
&+ \int_{1-\frac{(1-\alpha^2)d}{2[1+\alpha\cos(\beta)]}}^{1+\frac{d}{2}} \left| \frac{\alpha[\alpha+\cos(\beta)](-\Delta\tau_m + d/2 + 1)}{2+\alpha\cos(\beta)-\alpha^2} \right| \ d\Delta\tau_m \\
&= \frac{\alpha|\alpha+\cos(\beta)|d^2[1+2\alpha\cos(\beta)+\alpha^2]}{8[1+\alpha\cos(\beta)]^2} \\
&+ \frac{\alpha|\alpha+\cos(\beta)|d(1-d)}{2[1+\alpha\cos(\beta)]} + \frac{\alpha|\alpha+\cos(\beta)|d^2[2+\alpha\cos(\beta)-\alpha^2]}{8[1+\alpha\cos(\beta)]^2} \\
&= \frac{\alpha d|\alpha+\cos(\beta)|(4-d)}{8[1+\alpha\cos(\beta)]}
\end{aligned} \tag{11}$$

Likewise, from (9), the corresponding integral of the tracking error for the noncoherent discriminator is

$$\begin{aligned}
\rho_{av} &= \int_0^{1+\frac{d}{2}} |\rho| d\Delta\tau_m = \frac{\alpha d|\alpha+\cos(\beta)|(4-d)}{8[1+\alpha\cos(\beta)]} + \frac{1}{2\alpha|\cos(\beta)|} \\
&\int_{\frac{[1+2\alpha\cos(\beta)+\alpha^2]d}{2[1+\alpha\cos(\beta)]}}^{\frac{[\alpha d\cos(\beta)+1](2-d)-d(1-d/2-\alpha^2 d/2)}{\alpha d\cos(\beta)+2-d}} \left| \sqrt{a_1\Delta\tau_m^{\ 2} + b_1\Delta\tau_m + c_1} + m_1\Delta\tau_m + n_1 \right| \ d\Delta\tau_m \\
&+ \frac{1}{\alpha|2\cos(\beta)-\alpha|} \\
&\int_{\frac{[\alpha d\cos(\beta)+1](2-d)-d(1-d/2-\alpha^2 d/2)}{\alpha d\cos(\beta)+2-d}}^{1+\frac{d}{2}} \left| \sqrt{a_2\Delta\tau_m^{\ 2} + b_2\Delta\tau_m + c_2} + m_2\Delta\tau_m + n_2 \right| \ d\Delta\tau_m
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&\int \left| \sqrt{a\Delta\tau_m^{\ 2} + b\Delta\tau_m + c} + m\Delta\tau_m + n \right| \ d\Delta\tau_m \\
&= \mathrm{sgn}(\sqrt{a\Delta\tau_m^{\ 2} + b\Delta\tau_m + c} + m\Delta\tau_m + n) \\
&\{(\frac{\Delta\tau_m}{2} + \frac{b}{4a})\sqrt{a\Delta\tau_m^{\ 2} + b\Delta\tau_m + c} \\
&+ (\frac{c}{2\sqrt{a}} - \frac{b^2}{8a\sqrt{a}}) \ln[2a\Delta\tau_m + b \\
&+ 2\sqrt{a(a\Delta\tau_m^{\ 2} + b\Delta\tau_m + c)}] + \frac{m}{2}\Delta\tau_m^{\ 2} + n\Delta\tau_m
\end{aligned} \tag{13}$$

$$\begin{aligned}
&a_1 = \alpha^2\cos^2(\beta)\,,\ b_1 = -2\alpha[1 - \frac{d}{2} + \frac{\alpha^2 d}{2} + \alpha\cos(\beta)]\,, \\
&c_1 = \alpha^2\cos^2(\beta) + 2\alpha\cos(\beta)(1 - \frac{d}{2} + \frac{\alpha^2 d}{2}) \\
&+ (1 - \frac{d}{2} - \frac{\alpha^2 d}{2})^2 + 2\alpha^2 d\cos^2(\beta)(1 - \frac{d}{2})\ , \\
&m_1 = \alpha\cos(\beta)\,,\ n_1 = 1 - \frac{d}{2} - \frac{\alpha^2 d}{2} - \alpha\cos(\beta)
\end{aligned} \tag{14}$$

$$\begin{aligned}
&a_2 = \alpha^2\cos^2(\beta)\,,\ b_2 = 2\alpha(2-d)[\alpha - \cos(\beta)] - 4\alpha^2\cos^2(\beta)\,, \\
&c_2 = (2-d)[2 - d + 2\alpha d\cos(\beta)] + 4\alpha^2[\frac{d^2}{4} - \sin^2(\beta)]\,, \\
&m_2 = \alpha\cos(\beta) - \alpha^2\,,\ n_2 = \alpha^2(1 + \frac{d}{2}) - \alpha d\cos(\beta) - 2 + d
\end{aligned} \tag{15}$$

The above averaged values are no longer function of $\Delta\tau_m$, but still dependent on the variables α, β, and d. The values $\alpha = 0.56$ and $d = 1$ chip are typically assumed [8,10]. Using these values, Fig. 2 compares the two discriminator average performances. It is clear that the noncoherent discriminator shows better performance with large values of β. By examining the two curves, the maximum performance differences are 0.023 chip, corresponding to 7 meters, and they occur respectively at $\beta = 97$ degrees and $\beta = 121$ degrees. In order to establish an average performance over both multipath time delay and carrier phase offset, (6) and (9) are weighted by the joint probability density function of those two parameters (random variables), followed by a double integral.

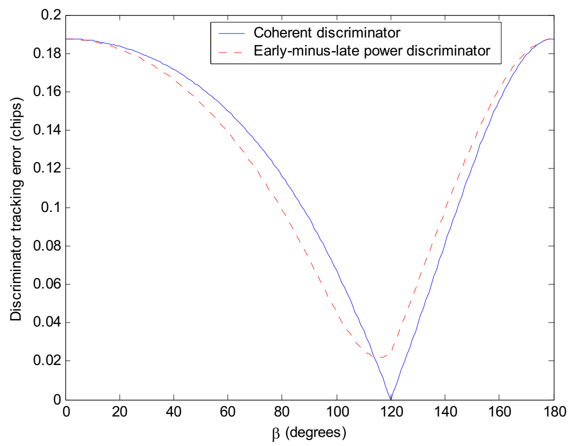

Fig. 2. Averaged multipath tracking error of cohererent discriminator and early-minus-late power discriminator

Assuming statistical independence and uniform distribution of β and $\Delta\tau_m$, Fig. 3 shows the average performance for $\alpha = 0.1$, 0.3, 0.5, 0.7, and 0.9. The positive value of the curve means the early-minus-late power discriminator works better, and the negative value indicates the coherent discriminator is preferred. It is noteworthy that the noncoherent discriminator consistently shows overall better performance for different early-late correlator spacing. This improvement, which can reach approximately 25 meters, is more pronounced at high multipath power values.

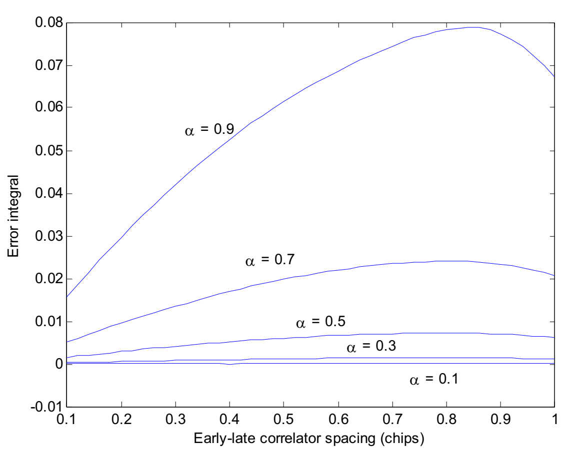

Fig. 3. Averaged multipath tracking error difference between the cohererent discriminator and the early-minus-late power discriminator

5. CONCLUSION

In this paper, we have addressed the problem of multipath for the GPS DLL. We have provided a detailed analysis of tracking performance for the DLL coherent discriminator and the DLL noncoherent early-minus-late power discriminator when the GPS receiver is subject to a single dominant multipath. This analysis has allowed examining the average tracking performance over random distribution of multipath parameters, including the carrier phase offset. It is shown that the noncoherent discriminator can reduce the tracking error by tens of meters compared to the coherent discriminator.

6. REFERENCES

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