

TIME-VARYING CHANNEL ESTIMATION FOR OFDM SYSTEMS

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ABSTRACT

In the new generation wireless communication systems where high data rates are desired, Orthogonal Frequency Division Multiplexing (OFDM) has become the standard method because of its advantages over single carrier modulation schemes on multi-path, frequency selective fading channels. However, inter-carrier interference due to Doppler frequency shifts, and multi-path fading severely degrades the performance of OFDM systems. Estimation of channel parameters is required at the receiver. In this paper, we present a time-varying channel modeling and estimation method based on the Discrete Evolutionary Transform that provides a time-frequency procedure to obtain a complete characterization of a multi-path, fading and frequency selective channel. Performance of the proposed method is tested on different levels of channel noise, and Doppler frequency shifts.

Index Terms— Time-varying channel modeling, Time-frequency analysis, OFDM systems

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is considered an effective technique for broadband wireless communications because of its great immunity to fast fading channels and inter-symbol interference (ISI). It has been adopted in several wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), the wireless local area network (W-LAN) standard; IEEE 802.11a, and the metropolitan area network (W-MAN) standard; IEEE 802.16a [1, 2]. OFDM partitions the entire bandwidth into parallel subchannels by dividing the transmit data bitstream into parallel, low bit rate data streams to modulate the subcarriers of those subchannels. However, inter-carrier interference (ICI) due to Doppler shifts, phase offset, local oscillator frequency shifts, and multi-path fading severely degrades the performance of multi-carrier communication systems [1, 3]. For fast-varying channels, especially in mobile systems, large fluctuations of the channel parameters are expected between consecutive transmit symbols. Estimation of the channel parameters is required to employ coherent receivers. Most of the channel estimation methods assume a linear time-invariant model for the channel, which is not valid for fast varying environments [4, 5]. A complete time-varying model of the channel can be obtained by employing time-frequency representation methods. We present a time-varying channel modeling and estimation method based on the discrete evolutionary representation of channel output. The Discrete Evolutionary Transform (DET) provides a time-frequency representation of the received signal by means of which

the spreading function of the multi-path, fading and frequency-selective channel can be modeled and estimated.

2. WIRELESS CHANNEL MODEL

In wireless communications, the multi-path, fading channel with Doppler frequency shifts may be modeled as a linear time-varying system with the following discrete-time impulse response [6, 7]

$$h(m, \ell) = \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} \delta(\ell - N_i) \quad (1)$$

where L is the total number of transmission paths, ψ_i represents the Doppler frequency, α_i is the relative attenuation, and N_i is the time delay caused by path i . The Doppler frequency shift ψ_i , on the carrier frequency ω_c , is caused by an object with radial velocity v and can be approximated by $\psi_i \cong \frac{v}{c} \omega_c$ where c is the speed of light in the transmission medium [7]. In wireless mobile communication systems, with high carrier frequencies, Doppler shifts become significant and have to be taken into consideration. Time-varying channel parameters cannot be easily estimated in the time-domain, however the estimation problem can be solved in the time-frequency domain by means of the so called spreading function which is related to the time-varying frequency response and the bi-frequency function of the channel. Time-varying transfer function of this linear channel is calculated by taking the discrete Fourier transform (DFT) of the impulse response with respect to ℓ , i.e.,

$$H(m, \omega_k) = \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} e^{-j\omega_k N_i} \quad (2)$$

where $\omega_k = \frac{2\pi}{K} k$, $k = 0, 1, \dots, K-1$. Now, the bi-frequency function is found by computing the discrete Fourier transform of $H(m, \omega_k)$ with respect to time variable, m :

$$B(\Omega_s, \omega_k) = \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_k N_i} \delta(\Omega_s - \psi_i). \quad (3)$$

and $\Omega_s = \frac{2\pi}{K} s$, $k = 0, 1, \dots, K-1$. Furthermore, the spreading function of the channel is obtained by calculating the DFT of $h(m, \ell)$ with respect to m , or by taking the inverse DFT of $B(\Omega_s, \omega_k)$ with respect to ω_k :

$$S(\Omega_s, \ell) = \sum_{i=0}^{L-1} \alpha_i \delta(\Omega_s - \psi_i) \delta(\ell - N_i) \quad (4)$$

which displays peaks located at the time-frequency positions determined by the delays and the corresponding Doppler frequencies, and

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with α_i as their amplitudes [7]. If we extract this information from the received signal, we will be able to eliminate the effects of the time-varying channel and estimate the transmitted data symbol.

3. OFDM SYSTEM MODEL

In an OFDM communication system, the available bandwidth B_d is divided into K subchannels. The input data is also divided into K -bit parallel bit streams, and then mapped onto some transmit symbols $X_{n,k}$ drawn from an arbitrary constellation points where n is the time index, and $k = 0, 1, \dots, K-1$, denotes the frequency or sub-carrier index. We then insert some pilot symbols, $p_{n,k} \in \{-1, 1\}$ at some pilot positions (n', k') , known to the receiver: $(n', k') \in \mathcal{P} = \{(n', k') | n' \in \mathcal{Z}, k' = iS + (n' \bmod(S)), i \in [0, P-1]\}$ where P is the number of pilots, and the integer $S = K/P$ is the distance between adjacent pilots in an OFDM symbol [8].

The n^{th} OFDM symbol $s_n(m)$ is obtained by taking the inverse DFT and then adding a cyclic prefix of length L_{CP} where L_{CP} is chosen such that $L \leq L_{CP} + 1$, and L is the time-support of the channel impulse response. This is done to mitigate the effects of intersymbol interference (ISI) caused by the channel time spread [1, 2].

$$s_n(m) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} e^{j\omega_k m} \quad (5)$$

$m = -L_{CP}, -L_{CP}+1, \dots, 0, \dots, K-1$ where again $\omega_k = \frac{2\pi}{K}k$, and each OFDM symbol has $N = K + L_{CP}$ length. The channel output suffers from multi-path propagation, fading and Doppler frequency shifts:

$$\begin{aligned} y_n(m) &= \sum_{\ell=0}^{L-1} h(m, \ell) s_n(m - \ell) \\ &= \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} s_n(m - N_i) \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} e^{j\omega_k(m - N_i)} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} H_n(m, \omega_k) e^{j\omega_k m} X_{n,k} \end{aligned} \quad (6)$$

The transmit signal is also corrupted by additive white Gaussian noise $\eta(m)$ over the channel. The received signal for the n^{th} frame can then be written as $r_n(m) = y_n(m) + \eta_n(m)$. The receiver discards the Cyclic Prefix and demodulates the signal using a K-point DFT as

$$\begin{aligned} R_{n,k} &= \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} [y_n(m) + \eta_n(m)] e^{-j\omega_k m} \\ &= \frac{1}{K} \sum_{s=0}^{K-1} X_{n,s} \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_s N_i} \\ &\quad \times \sum_{m=0}^{K-1} e^{j\psi_i m} e^{j(\omega_s - \omega_k)m} + Z_{n,k}. \end{aligned} \quad (7)$$

If the Doppler effects in all the channel paths are negligible, $\psi_i = 0$, $\forall i$, then the channel is almost time-invariant within one OFDM

symbol. In that case, above equation becomes

$$\begin{aligned} R_{n,k} &= X_{n,k} \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_k N_i} + Z_{n,k} \\ &= X_{n,k} H_{n,k} + Z_{n,k} \end{aligned} \quad (8)$$

where $H_{n,k}$ is the channel frequency response, and $Z_{n,k}$ is the Fourier transform of the channel noise. By estimating the channel frequency response coefficients $H_{n,k}$, data symbols, $X_{n,k}$, can be recovered by a simple equalizer, $\hat{X}_{n,k} = R_{n,k}/H_{n,k}$. However, if there are large Doppler frequency shifts in the channel, then the time-invariance assumption above is no longer valid. Here we consider time-varying channel modeling and estimation and approach the problem from a time-frequency point of view [7, 9].

4. TIME-VARYING CHANNEL ESTIMATION FOR OFDM SYSTEMS

In the following we briefly explain the Discrete Evolutionary Transform as a tool for the time-frequency representation of wireless channel output.

4.1. Time-Frequency Analysis by DET

A non-stationary signal, $x(n)$, $0 \leq n \leq N-1$, may be represented in terms of a time-varying kernel $X(n, \omega_k)$ or its corresponding bi-frequency kernel $X(\Omega_s, \omega_k)$. The time-frequency discrete evolutionary representation of $x(n)$ is given by [10],

$$x(n) = \sum_{k=0}^{K-1} X(n, \omega_k) e^{j\omega_k n}, \quad (9)$$

where $\omega_k = 2\pi k/K$, K is the number of frequency samples, and $X(n, \omega_k)$ is the evolutionary kernel. The discrete evolutionary transformation (DET) is obtained by expressing the kernel $X(n, \omega_k)$ in terms of the signal. This is done by using conventional signal representations [10]. Thus, for the representation in (9), the DET that provides the evolutionary kernel $X(n, \omega_k)$, $0 \leq k \leq K-1$, is given by

$$X(n, \omega_k) = \sum_{\ell=0}^{N-1} x(\ell) w_k(n, \ell) e^{-j\omega_k \ell}, \quad (10)$$

where $w_k(n, \ell)$ is, in general, a time and frequency dependent window. Details of how the windows can be obtained are given in [10]. However, for the representation of multipath wireless channel outputs, we need to consider signal-dependent windows that are adapted to the Doppler frequencies of the channel.

4.2. OFDM Channel Estimation

We will now consider the computation of the spreading function by means of the evolutionary transformation of the received signal. The output of the channel, after discarding the cyclic prefix, for the n^{th} OFDM symbol can be written as,

$$y_n(m) = \frac{1}{\sqrt{K}} \sum_{i=0}^{L-1} \sum_{k=0}^{K-1} \alpha_i e^{j\psi_i m} e^{j\omega_k(m - N_i)} X_{n,k} \quad (11)$$

Now calculating the discrete evolutionary representation of $y_n(m)$, we get

$$\begin{aligned} y_n(m) &= \sum_{k=0}^{K-1} Y_n(m, \omega_k) e^{j\omega_k m} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} H_n(m, \omega_k) X_{n,k} e^{j\omega_k m} \end{aligned} \quad (12)$$

The above equation can also be given in a matrix form as,

$$\mathbf{y} = \mathbf{A} \mathbf{x} \quad (13)$$

where

$$\begin{aligned} \mathbf{y} &= [y_n(0), y_n(1), \dots, y_n(K-1)]^T, \\ \mathbf{x} &= [X_{n,0}, X_{n,1}, \dots, X_{n,K-1}]^T, \\ \mathbf{A} &= [a_{m,k}]_{K \times K} = H_n(m, \omega_k) e^{j\omega_k m}. \end{aligned}$$

If the time-varying frequency response of the channel $H_n(m, \omega_k)$ is known, then $X_{n,k}$ may be estimated by

$$\hat{\mathbf{x}} = \mathbf{A}^{-1} \mathbf{y}. \quad (14)$$

A time-frequency procedure to estimate $H_n(m, \omega_k)$ is explained in the following. Comparing the representations of $y_n(m)$ in (11) and (12), we get the kernel as

$$Y_n(m, \omega_k) = \frac{1}{\sqrt{K}} \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} e^{-j\omega_k N_i} X_{n,k} \quad (15)$$

Finally, the channel frequency response is

$$H_n(m, \omega_k) = \frac{\sqrt{K} Y_n(m, \omega_k)}{X_{n,k}} \quad (16)$$

The evolutionary kernel $Y_n(m, \omega_k)$ can be calculated directly from $y_n(m)$ [10] and channel parameters α_ℓ , ψ_ℓ , and N_ℓ can be obtained from the spreading function $S(\Omega_s, \ell)$. According to (16), we need the input data symbols $X_{n,k}$ to estimate the channel frequency response. This can be achieved by estimating the frequency response $H_{n,k}$ by using any of the pilot aided channel estimation methods [2, 5], and using it to get a rough estimate of $X_{n,k}$. Then the detected data can be used for the estimation of the spreading function via (16). The DFT of $H(m, \omega_k)$ with respect to m , gives the bi-frequency function $B(\Omega_s, \omega_k)$, and the inverse DFT with respect to ω_k , gives us the spreading function $S(\Omega_s, \ell)$ from which all the parameters of the channel will be obtained and the transmitted data symbol will be detected.

The time-frequency evolutionary kernel of the channel output is obtained by replacing $y_n(m)$ in equation (10), or

$$\begin{aligned} Y_n(m, \omega_k) &= \sum_{\ell=0}^{K-1} y_n(\ell) w_k(m, \ell) e^{-j\omega_k \ell} \\ &= \frac{1}{\sqrt{K}} \sum_{s=0}^{K-1} X_{n,s} \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_s N_i} \\ &\quad \times \sum_{\ell=0}^{N-1} w_k(m, \ell) e^{j(\psi_i + \omega_s - \omega_k) \ell} \end{aligned} \quad (17)$$

We consider windows of the form $w_p(m, \ell) = e^{j\psi_p(m-\ell)}$, for $0 \leq \psi_p \leq \pi$ presented in [9] that depends on the Doppler frequency ψ_p .

This window will give us the correct representation of $Y_n(m, \omega_k)$ only when $\psi_p = \psi_i$, in fact, using the window $w_i(m, \ell) = e^{j\psi_i(m-\ell)}$, above representation of $Y_n(m, \omega_k)$ becomes,

$$Y_n(m, \omega_k) = \sqrt{K} \sum_{i=0}^{L-1} \alpha_i e^{j(\psi_i m - \omega_k N_i)} X_{n,k}$$

which is the expected result multiplied by K .

4.3. Time-Frequency Receiver

After estimating the spreading function and the corresponding frequency response $H_n(m, \omega_k)$ of the channel, data symbols $X_{n,k}$ can be detected using a time-frequency receiver given in (14). In fact, the channel output in equation (7) can be rewritten as

$$\begin{aligned} R_{n,k} &= \frac{1}{K} \sum_{s=0}^{K-1} \left\{ \sum_{m=0}^{K-1} H_n(m, \omega_k) e^{j(\omega_s - \omega_k) m} \right\} X_{n,s} + Z_{n,k} \\ &= \frac{1}{K} \sum_{s=0}^{K-1} B_n(\omega_s - \omega_k, \omega_k) X_{n,s} + Z_{n,k}. \end{aligned} \quad (18)$$

where $B_n(\Omega_s, \omega_k)$ is the bi-frequency function of the channel during n^{th} OFDM symbol, and above equation indicates a circular convolution with the data symbols. It is possible to write the above equation in a matrix form as

$$\mathbf{r} = \mathbf{B} \mathbf{x} + \mathbf{z} \quad (19)$$

where $\mathbf{B} = [b_{s,k}]_{K \times K} = B_n(\omega_s - \omega_k, \omega_k)$ is a $K \times K$ matrix and, $\mathbf{r}$, $\mathbf{x}$ and $\mathbf{z}$ are $K \times 1$ vectors defined by $\mathbf{r} = [R_{n,1}, R_{n,2}, \dots, R_{n,K}]^T$, $\mathbf{x} = [X_{n,1}, X_{n,2}, \dots, X_{n,K}]^T$, and $\mathbf{z} = [Z_{n,1}, Z_{n,2}, \dots, Z_{n,K}]^T$ respectively. Finally, data symbols $X_{n,k}$ can be estimated by using a simple time-frequency equalizer

$$\hat{\mathbf{x}} = \mathbf{B}^{-1} \mathbf{r}$$

which is an extension of the LTI channel equalizer to the time-varying channel model given in (1).

5. SIMULATIONS

In the experiments, the wireless channel is simulated randomly, i.e., the number of paths, $1 \leq L \leq 5$, the delays, $0 \leq N_i \leq L_{CP} - 1$ and the doppler frequency shift $0 \leq \psi_i \leq \psi_{\max}$, $i = 0, 1, \dots, L - 1$ of each path are picked randomly. Input data is BPSK coded and modulated onto $K = 128$ sub-carriers, 12 % of which is assigned to the pilot symbols. The OFDM symbol duration is chosen to be $T = 200 \mu s$, and $T_{CP} = 50 \mu s$. Frequency spacing between the sub-carriers is $F = 5 \text{ kHz}$. First, the Signal-to-Noise Ratio (SNR) of the channel noise is changed between 0 and 15 dB, for fixed values of the maximum doppler $\psi_{\max}$ on each path, and the bit error rate (BER) is calculated by four different approaches: 1) No Channel Estimation, 2) Pilot Symbol Assisted (PSA) Channel Equalization 3) Proposed Approach, and 4) Known Channel parameters. The spreading function, hence all the parameters of the channel are estimated by the proposed method. Fig. 1 shows the BER versus SNR for maximum Doppler frequency $\psi_{\max} = 500 \text{ Hz}$. Notice that our proposed method outperforms the PSA channel estimation even with low SNR values. Finally, the SNR is fixed to 15 dB while the normalized Doppler frequency is first changed between 50 Hz and 500 Hz, then between 500 Hz and 3500 Hz. BER is calculated for each of the above methods and given in Fig. 2 and Fig. 3 respectively. We see that the performance degrades for large Doppler frequency shifts for all methods.

6. CONCLUSIONS

In this work, we present a time-varying modeling of the multi-path, fading OFDM channels with Doppler frequency shifts by means of discrete evolutionary transform of the channel output. This approach allows us to obtain a representation of the time-dependent channel transfer function from the noisy channel output. At the same time, using the estimated channel parameters, a better detection of the input data can be achieved. Examples show that, our method has a considerably better BER performance than PSA channel estimation.

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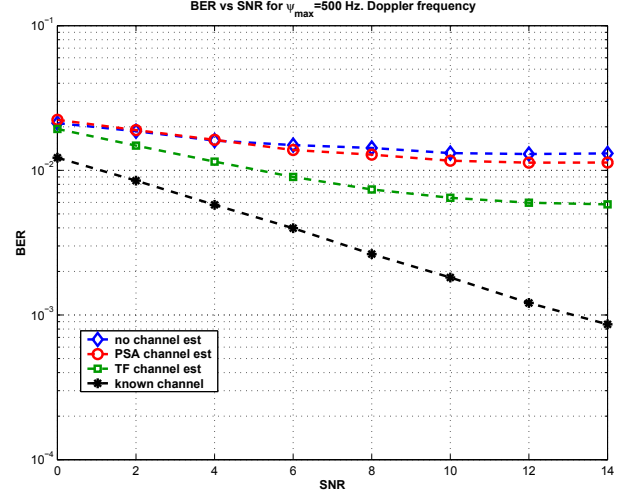


Fig. 1. BER versus SNR for $\psi_{\max} = 500 \text{ Hz}$.

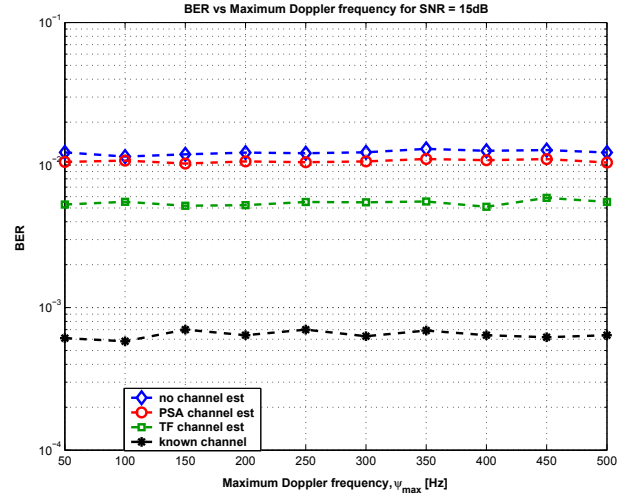


Fig. 2. BER versus $\psi_{\max} = [50 - 500] \text{ Hz}$.

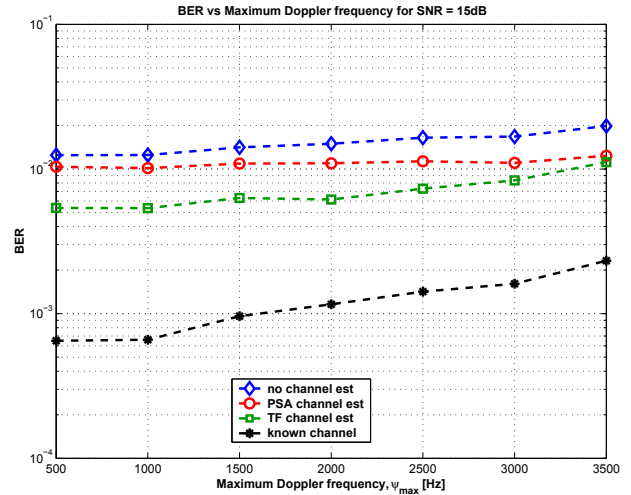


Fig. 3. BER versus $\psi_{\max} = [500 - 3500] \text{ Hz}$.