

MULTIPATH ESTIMATION IN THE GLOBAL POSITIONING SYSTEM FOR MULTICORRELATOR RECEIVERS

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ABSTRACT

In urban areas, multipath (MP) is one of the main error sources when tracking signals used in global navigation satellite systems. The received signals subjected to MP are the sum of several delayed replicas leading to biased estimations. This paper studies a Sequential Monte Carlo (SMC) algorithm which mitigates MP effects. The proposed algorithm is based on a state-space model associated to a multicorrelator GPS receiver and on a Rao Blackwellized technique which allows to achieve good performance.

Index Terms— Global Positioning System, Monte Carlo methods, Multipath, Sequential estimation

1. INTRODUCTION

The confidence in the computed position must be high for many professional applications based on satellite navigation technology. These applications include safety of life (like automated plane landing) and liability critical (like road tolling or offender tracking) services. For safety of life applications, an error in the computation of the position can lead to highly dangerous situations. For liability critical applications, an error can be responsible for financial losses or criminal prosecution. Thus, there is a need for developing algorithms ensuring an accurate positioning in real time. Mitigating MP effects at the GPS receiver is a critical issue to obtain this high accuracy, which will be used to prevent potentially damaging decisions.

On-board GPS receivers measure propagation delays of satellite signals. The problem of delay estimation is solved on-line by early-late correlators that drive tracking loops in charge of aligning the incoming signals and local replicas. If the received signal is degraded by MP, the shape of the correlation function is distorted, which results in biased delay estimates. Different approaches have been proposed in the literature to mitigate MP effects. A first class of methods consists of deriving new delay estimation techniques appropriate to GPS signals corrupted by MP. For instance, the shape of the correlation function can be adjusted to estimate the delay associated to the line-of-sight (LOS) signal. This kind of method has led to the narrow correlator [1] and the edge and strobe correlators [2]. Some alternatives aim at estimating jointly the direct and reflected signal parameters, for instance by using a maximum likelihood technique [3]. The advent of multicorrelator receivers has widened the range of possible solutions for handling GPS signals in presence of MP. Indeed, these receivers allow to fully characterize MP effects by

providing samples of the whole correlation function. These samples were recently processed with an extended Kalman filter (EKF) to solve the joint direct/indirect path estimation problem [4]. However, particle filters (PF) offer potential good solutions to this problem since they allow to handle nonlinear measurement equations. The main contribution of this paper is to study a Rao-Blackwellized PF for multicorrelator GPS signals and to compare its performance with the EKF.

The paper is organized as follows: Section 2 presents the mathematical model for the GPS signal at the output of the multicorrelator receiver. Section 3 studies a sequential Monte Carlo (SMC) technique for MP mitigation. Simulation results are presented in Section 4. Conclusions and perspectives are reported in Section 5.

2. SIGNAL MODEL

In the absence of MP, the received GPS signal corresponding to a given satellite can be written

$$s(t - \tau) = Ad(t - \tau)c(t - \tau) \cos(2\pi(f_c + f_d)t + \phi) + n(t),$$

where A is the GPS signal amplitude, τ is the time of arrival (i.e., the delay between the moment the signal was transmitted by the satellite and its arrival to the GPS receiver), f_c is the GPS carrier frequency, f_d is the Doppler frequency, ϕ is the signal phase, $d(t)$ is the data sequence, $c(t)$ is the PN sequence specific to a given satellite and $n(t)$ is the additive noise affecting the GPS signal.

In an urban medium, the received signal is composed of a line-of-sight (LOS) signal and delayed replicas due to MP. Assume that the receiver is locked at every sampling time to an estimated LOS frequency and that the received signal consists of a LOS and M reflected signals. The resultant in-phase baseband signal at the receiver can be expressed as:

$$s_I(t - \tau) = Ac(t - \tau) \cos(2\pi\tilde{f}t + \phi) + \sum_{i=1}^M A_i c(t - \tau - \alpha_i) \cos(2\pi\tilde{f}_i t + \phi_i) + n_I(t), \quad (1)$$

where \tilde{f} is the difference between the actual LOS Doppler frequency f_d and its estimate \hat{f}_d , the subscript i corresponds to the i th reflected signal, A_i is the i th MP amplitude, α_i is the relative delay between the LOS and the i th MP signal, \tilde{f}_i is the i th MP error Doppler frequency and ϕ_i is the i th MP phase. Note that the data sequence

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$d(t)$, being of low frequency, is no longer present in (1) because it has been filtered at the frequency converter stage. Note also that the quadrature baseband signals can be expressed similarly to (1) by changing $\cos(\cdot)$ to $\sin(\cdot)$.

2.1. Receiver Tracking Loop

The general GPS receiver is formed by several parallel channels, each of them tracking one of the LOS satellite signals. Assuming the cross-correlation between different satellites codes is negligible, each of the arriving signals can be independently analyzed as proposed in this paper. Moreover, we assume that a multicorrelator tracking loop is used in the receiver for the estimation of the signal parameters (see Fig. 1). After multiplying the signal by in-phase and quadrature carrier replicas, the I and Q components are formed. These components are then correlated with J different delayed replicas of the code characterized by different delays $\delta_1, \dots, \delta_J$. This results in J in-phase signals defined as:

$$I_{\delta_j} = \frac{A}{2} R(\tilde{\tau} + \delta_j) \text{sinc}(\pi \tilde{f} T) \cos(\pi \tilde{f} T + \tilde{\phi}) \quad (2)$$

$$+ \sum_{i=1}^M \frac{A_i}{2} R(\tilde{\tau} - \alpha_i + \delta_j) \text{sinc}(\pi \tilde{f}_i T) \cos(\pi \tilde{f}_i T + \tilde{\phi}_i) + n_I^j(t),$$

where $R(\tau)$ is the autocorrelation function of the satellite code, T is the integration time for the correlation computation, $\tilde{\tau} = \tau - \hat{\tau}$, $\tilde{\phi} = \phi - \hat{\phi}$. The J in-quadrature signals are obtained similarly by changing $\cos(\cdot)$ to $\sin(\cdot)$ and $n_I^j(t)$ by $n_Q^j(t)$ in (2). The noise sequences n_I^j and n_Q^j are assumed to be i.i.d. noise sequences. By concatenating eq's (2) for $j = 1, \dots, J$ and the corresponding in-quadrature equations, the following measurement equation is obtained:

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{n}_k, \quad (3)$$

where \mathbf{x}_k is the unknown parameter vector, $h(\cdot)$ is a non-linear function and \mathbf{n}_k contains all noise terms. This paper assumes that the noise vector \mathbf{n}_k is a zero-mean Gaussian vector whose covariance matrix is $\mathbf{R}_n = \sigma_n^2 \mathbb{I}_{2J}$ (where \mathbb{I}_{2J} is the $(2J) \times (2J)$ identity matrix). The GPS state vector \mathbf{x}_k is defined as follows:

$$\mathbf{x}_k = [\tau_k, v_k, A_k, \phi_k, \boldsymbol{\alpha}_k^{\text{MP}}, \mathbf{v}_k^{\text{MP}}, \mathbf{A}_k^{\text{MP}}, \boldsymbol{\phi}_k^{\text{MP}}]^T, \quad (4)$$

where the exponent MP stands for the MP parameters, τ_k, A_k, ϕ_k are the LOS parameters (time of arrival, amplitude and phase) at time instant k , v_k is the pseudo velocity at time k (related to the Doppler frequency by $f_d = -(f_c/c)v$, where c is the speed of light) and $\boldsymbol{\alpha}_k^{\text{MP}} = (\alpha_{1,k}, \dots, \alpha_{M,k})^T$, $\mathbf{A}_k^{\text{MP}} = (A_{1,k}, \dots, A_{M,k})^T$, $\tilde{\mathbf{f}}_k^{\text{MP}} = (\tilde{f}_{1,k}, \dots, \tilde{f}_{M,k})^T$ and $\boldsymbol{\phi}_k^{\text{MP}} = (\phi_{1,k}, \dots, \phi_{M,k})^T$ are the MP vectors containing delays, amplitudes, Doppler frequency errors and phases at time instant k . Note that the GPS state vector \mathbf{x}_k belongs to $\mathbb{R}^{4(M+1)}$, where M is the number of MP delayed replicas. Note also that M is assumed to be known in this paper. However, this assumption could be relaxed by implementing a model order selection rule. The interested reader is invited to consult the review paper [5] for more details.

2.2. State Space Model

The tracking problem is defined by the evolution of the following state space model

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{u}_k, \quad (5)$$

where \mathbf{x}_k is the state vector defined in (4). The state equation is assumed to be linear with a time independent state matrix $\mathbf{F}_k =$

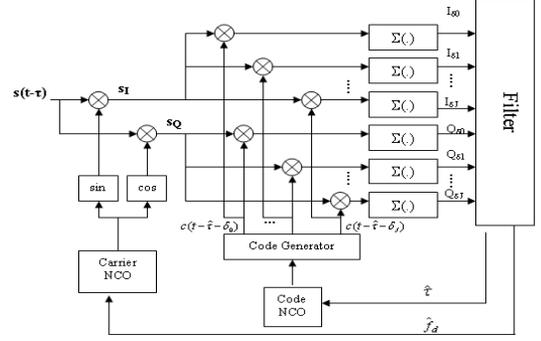


Fig. 1. Multicorrelator receiver structure.

\mathbf{F} and the noise sequence \mathbf{u}_k is a zero mean Gaussian sequence with known covariance matrix \mathbf{Q} . This paper focuses on a constant acceleration model where \mathbf{F} and \mathbf{Q} are defined as follows:

$$\mathbf{F} = \begin{pmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & \Delta T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathbb{I}_2 \end{pmatrix},$$

$$\mathbf{Q} = \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}' \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \sigma_a^2 \frac{\Delta T^3}{3} & \sigma_a^2 \frac{\Delta T^2}{2} & 0 & 0 \\ \sigma_a^2 \frac{\Delta T^2}{2} & \sigma_a^2 \Delta T & 0 & 0 \\ 0 & 0 & \sigma_A^2 & 0 \\ 0 & 0 & 0 & \sigma_\phi^2 \end{pmatrix},$$

where ΔT is the sampling period. The matrix \mathbf{C}' is obtained by replacing σ_a in \mathbf{C} by $\sigma_{a[MP]}$. This model is associated to a vehicle whose dynamics are described by a random Gaussian acceleration (with zero mean and variance σ_a^2). The dynamics of the MP parameters are also described by a random Gaussian acceleration model. However, the variance $\sigma_{a[MP]}$ associated to the MP parameters is smaller than the one proposed for the dynamics of the LOS parameters, which is a reasonable assumption.

3. PARAMETER ESTIMATION

3.1. Particle Filter

This section briefly reminds the principles of particle filtering (PF) (see [6] for more details). Particle filtering is a sequential Bayesian Monte Carlo method which approximates the posterior density function $p(\mathbf{x}_{0:k} | \mathbf{y}_{0:k})$ (with the usual notation $\mathbf{x}_{0:k} = (x_0, \dots, x_k)$) with a set of weighted particles $\mathbf{x}_{0:k}^i$ according to the following point mass approximation:

$$p(\mathbf{x}_{0:k} | \mathbf{y}_{0:k}) \approx \sum_{i=1}^N w(\mathbf{x}_{0:k}^i) \delta(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^i), \quad (6)$$

where $i = 1, \dots, N$ and N is the number of particles. The particles are generated according to an appropriate *importance density function* denoted as $\pi(\mathbf{x}_{0:k} | \mathbf{y}_{1:k})$ and the weights (referred to as *importance weights*) are computed as follows:

$$w(\mathbf{x}_{0:k}^i) = \frac{p(\mathbf{x}_{0:k} | \mathbf{y}_{0:k})}{\pi(\mathbf{x}_{0:k} | \mathbf{y}_{0:k})}. \quad (7)$$

Using Bayes rule, the evaluation of the *importance weights* can be easily obtained recursively:

$$w(\mathbf{x}_{0:k}^i) \propto w(\mathbf{x}_{0:k-1}^i) \frac{p(\mathbf{y}_k | \mathbf{x}_{0:k}^i, \mathbf{y}_{1:k-1}) p(\mathbf{x}_k^i | \mathbf{x}_{0:k-1}^i)}{\pi(\mathbf{x}_k^i | \mathbf{x}_{0:k-1}^i, \mathbf{y}_{1:k})}. \quad (8)$$

Particles with small (resp. large) importance weights correspond to low (resp. high) posterior probabilities. A resampling step is classically introduced to avoid degeneracy problems that would arise in the presence of many particles with small weights. The resampling step creates replicas of the particles having large weights and discards particles with small weights. The idea is to concentrate particles in regions of the state space that are pertinent. This paper uses the stratified resampling procedure proposed by Kitagawa [7].

3.2. Rao-Blackwellized Particle Filter

Rao-Blackwellization is a variance reduction technique which can be used efficiently for conditionally linear Gaussian state space models [6]. Assume that the state vector \mathbf{x}_k can be partitioned into two parts \mathbf{x}_k^1 and \mathbf{x}_k^2 such that the state equation is linear with respect to \mathbf{x}_k^1 and nonlinear with respect to \mathbf{x}_k^2 . Using the Bayes rule, the posterior distribution of the state vector $\mathbf{x}_{0:k}$ can be written:

$$p(\mathbf{x}_{0:k}^1, \mathbf{x}_{0:k}^2 | \mathbf{y}_{0:k}) = p(\mathbf{x}_{0:k}^1 | \mathbf{y}_{0:k}, \mathbf{x}_{0:k}^2) p(\mathbf{x}_{0:k}^2 | \mathbf{y}_{0:k}). \quad (9)$$

The Rao-Blackwellized PF estimates the posterior distribution of the *reduced state vector* \mathbf{x}_k^2 by a sequential Monte Carlo algorithm.

$$p(\mathbf{x}_k^2 | \mathbf{y}_k, \mathbf{x}_{0:k-1}^2) \simeq \sum_{i=1}^N w_k^{(i)} \delta(\mathbf{x}_k^2 - \mathbf{x}_k^{2,i}). \quad (10)$$

The posterior distribution $p(\mathbf{x}_{0:k}^1 | \mathbf{y}_{0:k}, \mathbf{x}_{0:k}^2)$ can then be approximated by a mixture of Gaussian distributions:

$$p(\mathbf{x}_k^1 | \mathbf{y}_k, \mathbf{x}_{0:k-1}^2) \simeq \sum_{i=1}^N w_k^{(i)} \mathcal{N}(\mathbf{m}_{k|k,1}^i, \mathbf{P}_{k|k,1}^i), \quad (11)$$

where $\mathbf{m}_{k|k,1}^i$ and $\mathbf{P}_{k|k,1}^i$ are computed by standard Kalman filter recursions. Drawing particles from a lower-dimension space allows a given accuracy to be obtained with a lower amount of particles. In our application, the measurement equation defined in (3) is clearly linear with respect to the LOS and MP amplitudes and nonlinear with respect to the other parameters, hence

$$\begin{aligned} \mathbf{x}_k^1 &= (A_k, \mathbf{A}_k^{\text{MP}})^T, \\ \mathbf{x}_k^2 &= (\tau_k, v_k, \phi_k, \alpha_k^{\text{MP}}, \mathbf{v}_k^{\text{MP}}, \phi_k^{\text{MP}})^T. \end{aligned}$$

3.3. Importance density function

The choice of the *importance distribution* is one of the key factors for the correct functioning of the particle filter. The idea is that the support of the importance distribution should have an overlapped region (long tailed behavior) with the true pdf in order to avoid divergence. In [6] the optimal importance distribution was shown to be:

$$\pi(\mathbf{x}_k | \mathbf{x}_{0:k-1}, \mathbf{y}_{0:k}) = p(\mathbf{x}_k | \mathbf{x}_{0:k-1}, \mathbf{y}_{0:k}), \quad (12)$$

in the sense that it minimizes the variance of the importance weights conditionally to the previous state and the measurement. This optimal importance distribution can be evaluated by using (3) and (5), up to a normalization constant. However, drawing samples according to this distribution is a difficult task. In such a situation, it is usual to approximate the optimal importance distribution by local linearization [6].

The local linearization procedure has been performed to obtain the importance density function $\pi(\mathbf{x}_k | \mathbf{x}_{0:k-1}, \mathbf{y}_{0:k})$. However, specific attention has been devoted to the MP delay parameter α . Indeed, this parameter is subjected to constraints inherent to the MP

model related to the multicorrelator receiver. First, any reflected path will arrive later than the LOS signal imposing the constraint $\alpha_i \geq 0$, for $i = 1, \dots, M$. Moreover, the particular 2-chip wide autocorrelation peak of the spreading GPS code imposes an upper limit for parameters α_i : $\alpha_i \geq 2T_{\text{chip}}$, T_{chip} being the period of each chip from the spreading code. Indeed, any value of $\alpha_i \geq 2T_{\text{chip}}$ yields a reflected signal whose resultant autocorrelation from the multicorrelator receiver is zero, i.e. not affecting the direct measurement. This paper proposes to draw state vectors \mathbf{x}_k according to the local linearization importance distribution and to reject the state vectors which do not satisfy the constraints $\alpha_i \in [0, 2T_{\text{chip}}]$. The resultant importance distribution $\pi(\mathbf{x}_k | \mathbf{x}_{0:k-1}, \mathbf{y}_{0:k})$ is a truncated Gaussian distribution. An example of marginal proposal distribution $\pi(\alpha_k | \mathbf{x}_{0:k-1}, \mathbf{y}_{0:k})$ is depicted on figure 2.

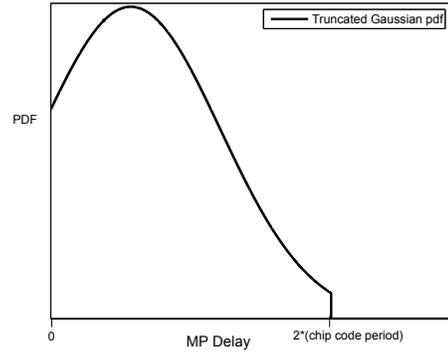


Fig. 2. Marginal importance density function for α_k .

4. SIMULATION RESULTS

Several simulations have been conducted to validate the proposed filtering solution. This paper assumes that the received GPS signal is subjected to a single MP component (i.e., $M = 1$). This assumption is realistic in many scenarios since two reflected signals very close in time can be estimated as one perturbation [3]. A realistic model has been chosen for the vehicle dynamics defined by $\sigma_a^2 = 5m/s^2$. The sampling period is $\Delta T = 1s$ and the other state parameters are $\sigma_A^2 = 0.01$ and $\sigma_\phi^2 = 0.01$. At the same time, a single MP signal described by a weaker dynamic $\sigma_{a[MP]} = \sigma_a/10$ has been introduced. The signal to noise ratio is $\text{SNR} = 10 \log(A^2/\sigma_n^2) = 20\text{dB}$ whereas the signal to MP ratio is $20 \log(A_1/A) = 6\text{dB}$. The other MP parameters used in the simulations are $\alpha_1 = 0.15T_{\text{chip}}$, $\phi_1 = 0$ and $\tilde{f}_1 = \tilde{f}$.

The number of particles for the proposed implementation is $N_p = 1000$. The results obtained with the proposed PF are compared to both the posterior Cramer Rao bound (PCRB) and the extended Kalman filter (EKF). The PCRB sets a lower limit on the variance of any unbiased estimator [8]. The EKF represents nowadays a standard navigation solution and is employed as a benchmark for performance comparison. Figures 3 and 4 compare the root mean square errors (RMSEs) of the EKF and the PF with the corresponding PCRBs, when estimating the LOS parameter τ and the MP delay α respectively. There is no clear difference between the EKF and the PF for the estimation of τ , in this particular scenario. Indeed, both RMSEs converge very quickly to the PCRBs. A small difference in the convergence time can be observed for the estimation of α_1 .

A second scenario is then investigated where the MP component

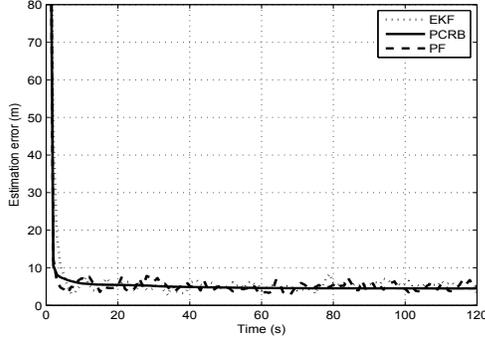


Fig. 3. RMSE for the LOS delay τ .

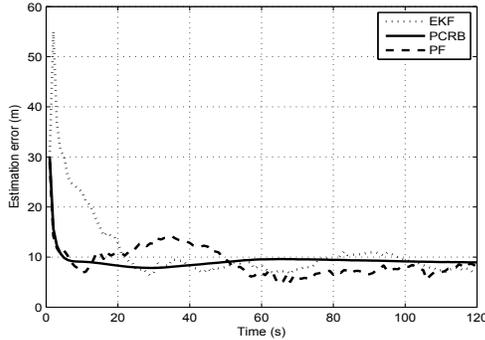


Fig. 4. RMSE for the MP delay α_1 .

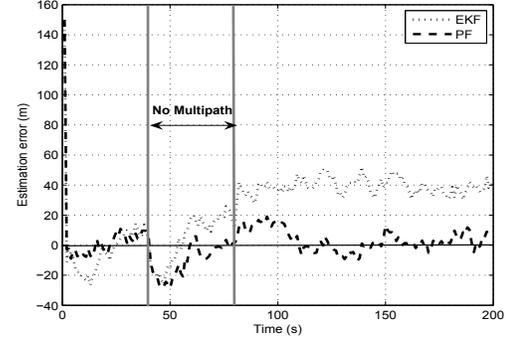


Fig. 5. RMSE for the LOS delay τ .

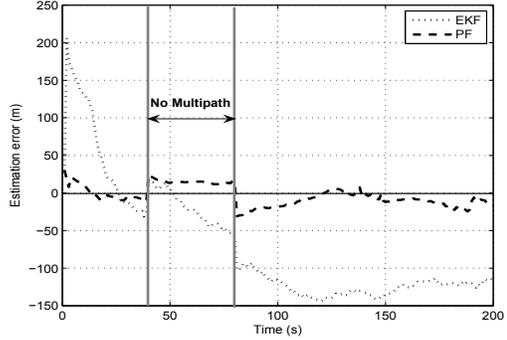


Fig. 6. RMSE for the MP delay α_1 .

is not present all along the observation window (see figures 5 and 6). More precisely, the MP component disappears at time instant $t_1 = 40$ and re-appears at time instant $t_2 = 80$. This simulation allows to analyze the algorithm behavior when the observed signal is subjected to nonstationarities. Note that the dimension of the state vector is constant for each time instant (here $M = 1$ hence $\mathbf{x}_k \in \mathbb{R}^8$): the MP parameters are estimated in the whole observation window and the amplitude estimates should agree with the actual amplitude values, i.e. $A_{1,k} = A_k/2$ for $k = 1, \dots, 39$, $A_{1,k} = 0$ for $k = 40, \dots, 80$ and $A_{1,k} = A_k/2$ for $k = 81, \dots, 200$. Figures 5 and 6 depict the RMSEs associated to the estimation of parameters τ and α_1 obtained with the EKF and the PF. Similar results would be obtained for the other parameters. They are not presented here for brevity. Fig. 6 clearly shows that the EKF is not able to track the abrupt changes in the measurements leading to a divergent estimation of the MP delay, contrary to the PF. The PF should provide reliable estimations when the GPS signal is acquired in changing environments.

5. CONCLUSIONS

Multipath (MP) is one of the main error sources when GPS signals are used for positioning and navigation. This paper studied a sequential Monte Carlo algorithm which mitigates MP effects. An extended state vector including the MP parameters was estimated by a Rao-Blackwellized particle filter (PF). The proposed PF methodology provided lower acquisition time when compared to the extended Kalman filter (EKF). Thus, the proposed algorithm seems to be a promising alternative to the EKF for applications requiring instantaneous on-line estimations. The proposed PF also showed interesting properties when the received GPS signals are contaminated by MP

components appearing and disappearing at unknown time instants. Indeed, the robustness of the PF to abrupt changes was clearly outlined when compared to the EKF. Further investigations include detecting MP presence/absence and estimating the number of replicas contained in the received GPS signals.

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