ALLIANCES WITH OPTIMAL RELAY SELECTION

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ABSTRACT

ALLIANCES is a recently proposed cooperative random access protocol for wireless networks. In this paper we modify the original model to include user location information. We also derive pair-wise error probability (PEP) under Rayleigh flat fading channel and a power-law attenuation environment. Based on the PEP analysis we propose an optimal relay selection scheme, which achieves significant throughput gains as compared to the random relay selection scheme in the original ALLIANCES.

Index Terms—Cooperation, wireless networks, collision resolution, relay selection.

I. INTRODUCTION

Traditional Medium Access Control (MAC) protocols are designed to avoid collisions. The reason behind this is that once a collision occurs, the packets involved in the collision are totally discarded thus leading to a waste of bandwidth and energy. Examples of such protocols include IEEE 802.11b/g [1], Multiple Access with Collision Avoidance Wireless (MACAW)[2], Handshake-based Channel Aware (HCA) [4], GDCF (a modified version of Distributed Coordination Function) [5], Dual-Channel Reservation (DCR) [6].

On the other hand, it has been shown [7], [3],[8] that collision resolution using signal processing techniques can achieve higher throughput. The method of [8], [12], referred to as ALLIANCES [9], is a random access scheme that by employing user co-operation resolves collisions and enables lower bit-error rate (BER) than non-cooperative protocols [12]. ALLIANCES was developed for a small scale network. User location information was not taken into account and the set of cooperating nodes was determined randomly.

However, in applications such as sensor networks, wireless LANs, and cellular networks where subscribers are equipped with a Global Positioning System (GPS), location information of network nodes is often available to base station / access point (BS/AP). In this paper we extend ALLIANCES to a more realistic system model that takes into account user location information. We provide an analytic expression for BER in a Rayleigh flat fading scenario, where users are uniformly distributed within a ring, and their transmitted power gets attenuated with distance according to a power-law expression. Based on the analysis, we propose an optimal relay selection scheme, which results in significant BER improvement at the expense of increased control overhead.

This work has been supported by NSF under grant CNS-0435052.

II. SYSTEM MODEL

The ALLIANCES protocol is a cooperative random access protocol for cellular wireless networks. In brief, once a collision occurs in time slot n, the BS declares a cooperative transmission epoch (CTE). During each CTE slot, one of the network nodes (relay), retransmits the signal that he heard during the collision slot, or simply his own packet if he was one of the collided sources. The collision signal and the retransmissions represent linear mixtures of the collided packets. Once the receiver collects enough linearly independent mixtures, it formulates a virtual MIMO problem, solution of which leads to the originally collided packets. In [8], the relays were selected using a distributed but random mechanism that involved no overhead.

Suppose now that user location information is available to the BS. Consider a network with J users. The BS/AP is located at the center of a ring area with inner radius r_{in} , and outer radius r_{out} . The distance of node i to BS/AP, denoted by d_{id} , is uniformly distributed in the range $[r_{in}, r_{out}]$. The angle of user i with respect to BS/AP is also uniformly distributed in $[0, 2\pi]$. The network nodes can be moving within the ring area, however, we assume that each node remains static during packet transmission and collision recovery. The signals between transmitter i and and receiver j, at distance d_{ij} , decays proportionally to d_{ij}^{-m} . Here m is an environment dependent exponent [10], the value of which ranges from 2 (free space) to 4 (urban environment) for omni-direction antenna. The channels between nodes and BS/AP and also inter-user channels are modeled as i.i.d. flat fading and Rayleigh distributed, and are independent between different paths.

Let us consider a K-th order collision at slot n. Let the packet transmitted by the *i*-th node during slot n consist of N symbols, i.e., $x_i(n) \stackrel{\triangle}{=} [x_{i,0}(n), \cdots, x_{i,N-1}(n)]$. Let $S(n) = \{i_1, \cdots, i_K\}$ be the set of collided sources, and $\mathcal{R}(n) = \{r_1, \cdots, r_{K-1}\}$ the set of nodes that will serve as relays during the CTE. During n-th slot, the signal heard by the BS and also by all non-source nodes is:

$$\boldsymbol{y}_{r}(n) = \sum_{i \in \mathcal{S}(n)} a_{ir}(n) d_{ir}^{-m/2} \boldsymbol{x}_{i}(n) + \boldsymbol{w}_{r}(n)$$
(1)

where $r \in \{d\} \bigcup \mathcal{R}(n), r \notin S(n)$, with $a_{ir}(n)$ denoting the channel coefficient; $w_r(n)$ representing noise; and $\{d\}$ denoting the destination node. During the (n + k)-th slot, the BS/AP receives:

$$\boldsymbol{z}_{d}(n+k) = \begin{cases} a_{rd}(n+k)d_{rd}^{-m/2}\boldsymbol{x}_{r}(n) + \boldsymbol{w}_{d}(n+k), \\ r \in \mathcal{R}(n) \bigcap \mathcal{S}(n) \\ a_{rd}(n+k)d_{rd}^{-m/2}c(n+k)\boldsymbol{y}_{r}(n) + \boldsymbol{w}_{d}(n+k) \\ r \in \mathcal{R}(n), \quad r \notin \mathcal{S}(n) \end{cases}$$
(2)

where $z_d(n+k)$ is a $1 \times N$ vector; $w_d(n+k)$ denotes the noise vector at the BS; c(n+k) represent the scaling constant, which is selected so that the transmit power is maintained within the constraints of the relay's transmitter.

Let us define matrices **X**, whose rows are the signals sent by source nodes i.e., $\mathbf{X} = [\mathbf{x}_{i_1}^T(n), \cdots, \mathbf{x}_{i_K}^T(n)]^T$, and **Z**, whose rows are the signals heard by the destination node during slots $n, n+1, \cdots, n+K-1$, i.e., $\mathbf{Z} = [\mathbf{z}_d^T(n), \mathbf{z}_d^T(n+1), \cdots, \mathbf{z}_d^T(n+K-1)]^T$ with $\mathbf{z}_d(n) = \mathbf{y}_d(n)$. The received signal at the destination can then be written in matrix form as:

$$\mathbf{Z} = \mathbf{H}\mathbf{X} + \mathbf{W} \tag{3}$$

where matrices \mathbf{H} and \mathbf{W} contain respectively channel information and noise. The precise definitions are omitted due to lack of space, but can be easily implied by (1)-(3).

Channel estimation and active user detection can be done as discussed in [8].

III. PERFORMANCE ANALYSIS

In this section we compute the Pairwise Error Probability (PEP) associated with transmitted symbols \mathbf{X} , and an arbitrary output of the Maximum Likelihood (ML) decoder, $\hat{\mathbf{X}}$.

Let σ_x^2 denote the transmitted signal power per symbol, σ_w^2 denote the power of additive Gaussian noise at non-source relays and BS/AP, and σ_a^2 denote the variance of Rayleigh fading multi-path channels. The SNR is defined as $\frac{\sigma_x^2 \sigma_a^2}{\sigma_w^2} E(d_{id}^{-m})$, where $E(d_{id}^{-m}) = \frac{r_1^{1-m} - r_2^{1-m}}{(m-1)(r_2 - r_1)}$.

Proposition 1: The PEP associated with \mathbf{X} , and $\hat{\mathbf{X}}$ given the collision order K, the number of non-source relays l, and d_{ij} , satisfies:

$$\bar{P}_{e}(\mathbf{X} \to \hat{\mathbf{X}} | K, l, d_{ij}) \leq \prod_{j=1}^{r} (1 + \frac{\sigma_{x}^{2} \sigma_{a}^{2}}{4\sigma_{w}^{2}} \tilde{\lambda}_{j})^{-1} \cdot \prod_{i=1}^{l} \prod_{j=1}^{r} \frac{1}{\gamma_{ij}} exp(\frac{1}{\gamma_{ij} \sigma_{a}^{2}}) Ei(\frac{1}{\gamma_{ij} \sigma_{a}^{2}})$$

$$(4)$$

where

$$\gamma_{ij} = \frac{\sigma_x^2 \sigma_a^2}{4\sigma_w^2} \frac{c_{r_i}^2}{1 + \sigma_a^2 c_{r_i}^2 d_{r_i d}^{-m}} \lambda_{ij}$$
(5)

$$c_{r_i}^2 = \frac{\sigma_x^2}{\sigma_x^2 \sigma_a^2 \sum_{j=1}^K d_{jr_i}^{-m} + \sigma_w^2}$$
(6)

where $\tilde{\lambda_j}$ is the *j*-th non-zero eigenvalue of $\mathbf{D}_d^2 \tilde{\mathbf{R}}_{\Delta}$; λ_{ij} is the *j*-th non-zero eigenvalue of $d_{r_i d}^{-m} \mathbf{D}_{r_i}^2 \mathbf{R}_{\Delta}$; $\mathbf{R}_{\Delta} = \frac{1}{\sigma_x^2} (\mathbf{X} - \hat{\mathbf{X}}) (\mathbf{X} - \hat{\mathbf{X}})^H$ and $\tilde{\mathbf{R}}_{\Delta} = \mathbf{R}_{\Delta} + \sum_{i=1}^{K-(l+1)} \Phi_{l+i} \mathbf{R}_{\Delta} \Phi_{l+i}$ where Φ_{l+i} is a matrix with all elements equal to zero except the element at (l + i, l + i) that equals one; $\mathbf{D}_d = diag(d_{1d}^{-m/2}, ..., d_{Kd}^{-m/2})$; $\mathbf{D}_{r_i} = diag(d_{1r_i}^{-m/2}, ..., d_{Kr_i}^{-m/2})$; *r* is the rank of \mathbf{R}_{Δ} ; \tilde{r} is the rank of $\tilde{\mathbf{R}}_{\Delta}$; Ei(x) is the exponential integral function which is defined as $\int_x^{\infty} \frac{\exp(-t)}{t} dt$.

Due to space constraints, the proof is omitted. It can be derived according to that in [12], except that here the signal model takes attenuation with distance into account. Based on PEP analysis, the BER can be obtained by averaging all error events along the lines of [12].

As an example, we apply the above PEP and BER expressions to analyze the performance of collision resolution. We evaluate the BER of collision recovery by ALLIANCES for a specific collision order K = 4 for a network with $r_{in} = 50$ meters and $r_{out} = 150$ meters. The attenuation exponent is taken to be m = 3. The additive Gaussian noise power is $\sigma_w^2 =$ -80dBm (a typical noise level of receivers). The packets contain a Binary Phase Shift Keying (BPSK) signal. Maximum likelihood decoding is used at the receiver to recover the packets. The theoretical result for K = 1 (non-cooperative, e.g., TDMA), and for K = 4 with different number of non-source relays (l), is plotted in Fig. 1. It can be seen that, for SNR > 0dB, the more the non-source relays, the better the BER performance. Also collision resolution appears to outperform non-cooperative



Fig. 1. BER vs SNR, variable K and I: analytical results.

transmissions, such as TDMA, due to its robustness to fading.

IV. OPTIMAL RELAY SELECTION

The first product in the RHS of (4) is the contribution of collided packets and retransmissions by source-relay nodes. The remainder of the RHS is the contribution of non-source relays. Let us first consider the second term. We note that as γ_{ij} increases, the terms $\frac{1}{\gamma_{ij}}$ and $exp(\frac{1}{\gamma_{ij}\sigma_a^2})$ decrease, while $Ei(\frac{1}{\gamma_{ij}\sigma_a^2})$ increases. The product of the aforementioned terms monotonically decreases as γ_{ij} increases, and is of the order of $\frac{1}{\gamma_{ij}}$.

To deal with the rank and eigenvalues of \mathbf{R}_{Δ} and $\tilde{\mathbf{R}}_{\Delta}$, let us consider the following two cases of packet length: Case I: Packet length N = 1

In this case that each packet contains only 1 information symbol, excluding the packet header. For any $\hat{\mathbf{X}} \neq \mathbf{X}$, it holds r = 1, regardless of how many symbols are in error. This means both \mathbf{R}_{Δ} and $\tilde{\mathbf{R}}_{\Delta}$ have only one non-zero eigenvalue. The only non-zero eigenvalue of $d_{r,d}^{-m} \mathbf{D}_{r_i}^2 \mathbf{R}_{\Delta}$, denoted by λ_i equals:

$$\lambda_{i} = \sum_{j=1}^{K} \lambda_{ij} = trace(d_{r_{i}d}^{-m} \mathbf{D}_{r_{i}}^{2} \mathbf{R}_{\Delta})$$
$$= d_{r_{i}d}^{-m} \sum_{j=1}^{K} d_{jr_{i}}^{-m} |\bar{\mathbf{X}}_{j}|^{2}$$
(7)

where $\bar{\mathbf{X}}_j$ represents the symbol errors for user j, i.e.,

$$\bar{\mathbf{X}}_{j} = \mathbf{X}_{j}' - \hat{\mathbf{X}}_{j}' = \begin{cases} D & \hat{\mathbf{X}}_{j}' \neq \mathbf{X}_{j}' \\ 0 & \hat{\mathbf{X}}_{j}' = \mathbf{X}_{j}' \end{cases}$$
(8)

with D being the distance of two symbols in the modulation constellation. Here \mathbf{X}' is the normalized symbols with respect to transmitted power, i.e., $\mathbf{X}' = (1/\sigma_x)\mathbf{X}$.

Inserting (7) and (6) into (5) we get

$$\gamma_{ij} = \frac{\sigma_x^2 \sigma_a^2}{4\sigma_w^2} \frac{1}{\frac{\sigma_x^2 \sigma_a^2 \sum_{j=1}^K d_{jr_i}^{-m} + \sigma_w^2}{\sigma_x^2 \sigma_a^2 d_{r_id}^{-m}} + \sigma_a^2} \sum_{j=1}^K d_{jr_i}^{-m} |\bar{\mathbf{X}}_j|^2 \qquad (9)$$

In order to minimize error probability for all possible $\hat{\mathbf{X}}$ when \mathbf{X} is transmitted, we take $\bar{\mathbf{X}}_j$ to be equal to D for j = 1, 2, ..., K. Thus (9) becomes

$$\gamma_{ij} = \frac{\sigma_x^2 \sigma_a^2}{4\sigma_w^2} \frac{D^2}{\left[1 + \frac{\sigma_w^2}{\sigma_a^2 \sigma_x^2 \sum_{j=1}^K d_{jr_i}^{-m}}\right] d_{r_i d}^m + \sigma_a^2 (\sum_{j=1}^K d_{jr_i}^{-m})^{-1}}$$
(10)

Maximizing γ_{ij} is equivalent to minimizing the denominator of (10).

Next we examine the contribution of source relays. Note that $\hat{\mathbf{R}}_{\Delta}$ is also rank 1, i.e. $\tilde{r} = 1$. Proceeding as in (7), the only non-zero eigenvalue of $\hat{\mathbf{R}}_{\Delta}$ equals:

$$\tilde{\lambda} = \sum_{j=1}^{K} d_{jd}^{-m} |\bar{\mathbf{X}}_j|^2 \Gamma_j$$
(11)

where Γ_j is an indicator function,

$$\Gamma_j = \begin{cases} 2 & j \in \mathcal{S} \cap \mathcal{R} \\ 1 & \text{else} \end{cases}$$
(12)

Assuming that $|\bar{\mathbf{X}}_j| = D$ for $j = 1, 2, \dots, K$, $\tilde{\lambda}$ is bounded according to:

$$\tilde{\lambda} = D^{2} \left[\sum_{j=1}^{K} d_{jd}^{-m} + \sum_{j \in S \cap \mathcal{R}} d_{jd}^{-m} \right]$$
(13)

$$\leq 2D^2 \sum_{j=1}^{K} d_{jd}^{-m}$$
 (14)

Thus, selecting only source nodes as relays can at most decrease BER by half of that without source relays. Source relays do not increase the diversity order. On the other hand, each non-source relay increases diversity order by 1. Therefore, the optimal relay selection scheme would be selecting K-1 non-source nodes and 0 source nodes as relays.

Minimizing PEP is equivalent to minimizing the second term of the RHS of (4), or maximizing γ_{ij} with proper selection of relay nodes r_i , i = 1, 2, ..., l and $r_i \notin S(n)$. The latter requires minimization of the denominator of (10).

Based on the analysis above, we propose the following relay selection scheme.

The following weight is associated with each network node:

$$w_i = \left[1 + \frac{\sigma_w^2}{\sigma_a^2 \sigma_x^2 \sum_{j=1}^K d_{ji}^{-m}}\right] d_{id}^m + \sigma_a^2 \left(\sum_{j=1}^K d_{ji}^{-m}\right)^{-1} (15)$$

Let the candidate non-source relays form a queue according to their weight w_i . The non-source nodes r_i , $i = 1, 2, \dots, \min(K - 1, J - K)$ with the least coefficients will serve as relays. This means that the top $\min(K - 1, J - K)$ nodes in the queue formulate the relay candidate set, which is then broadcasted by the BS/AP to all relay nodes in the beginning of collision resolution procedure.

of collision resolution procedure. At high SNR, we have $\frac{\sigma_w^2}{\sigma_a^2 \sigma_x^2 \sum_{j=1}^K d_{ji}^{-m}} \ll 1$, and the weight can be simplified to:

$$v_i = d_{id}^m + \sigma_a^2 (\sum_{j=1}^K d_{ji}^{-m})^{-1}$$
(16)

The term $\sum_{j=1}^{K} d_{ji}^{-m}$ represents power attenuation from all source nodes to the relay node *i*, while d_{id}^{-m} represents power attenuation from relay node *i* to BS/AP *d*. Thus, the optimal relay scheme chooses the nodes who are closer to base station and have low power attenuation with respect to source nodes. Note that although *m* affects the values of weighting coefficient w_i , it does not affect the order of relay candidates.

Remark - For a K-th order collision in a network with J nodes, the maximum number of non-source relays is J - K. If $K - 1 \ge J - K$, there are not enough non-source nodes to select from. In addition to all the non-source nodes, we would select as relays K - 1 - (J - K) = 2K - J - 1 source nodes that are closest to BS/AP, to maximize the contribution of source relays $\sum_{j \in S \cap \mathcal{R}} d_{jd}^{-m}$ in (13).

Case 2: Packet length N > 1

This is the most general case, where r could be larger than 1. However, we show in the sequel that the results in previous case still hold. Let us express \mathbf{R}_{Δ} as:

$$\mathbf{R}_{\Delta} = (\mathbf{X}' - \hat{\mathbf{X}}')(\mathbf{X}' - \hat{\mathbf{X}}')^{H}$$
$$= \sum_{i=1}^{N} \mathbf{X}'_{i}(\mathbf{X}'_{i})^{H} = \sum_{i=1}^{N} \mathbf{R}_{\Delta i}$$
(17)

 $\mathbf{R}_{\Delta i}$ represents the correlation matrix of the *i*-th column of \mathbf{R}_{Δ} . The rank of $\mathbf{R}_{\Delta i}$ is 1 if there are error symbols associated with column \mathbf{X}_i and $\hat{\mathbf{X}}_i$, or 0 if not. Note that we do not incorporate space time coding among all the users (that would be impossible because each node transmits his own information), thus $\mathbf{R}_{\Delta i}$ and $\mathbf{R}_{\Delta j}$ are independent for $i \neq j$. Dealing with $\mathbf{R}_{\Delta i}$ independently for i = 1, 2, ..., N, the condition (15) minimizes PEP associated with all terms in (17) simultaneously. This is also intuitively expected, as if no space time coding is used among all users, the packet length should not matter for error probability.

Summarizing, in this section we derive an optimal relay selection scheme based on information of location of network nodes. Once the relay candidate set \mathcal{R} is decided, the BS/AP broadcasts it to all nodes in the control channel. Note that more control bits would be needed, in addition to the 1 control bit required in ALLIANCES to inform users whether they are free to transmit or they are on a collision resolution mode.

V. SIMULATION RESULTS

To quantify improvement of the proposed relay selection scheme we compute BER of ALLIANCES simulations as follows. A Monte Carlo simulation was run $M = 10^6$ times for each

case. Each point on the curve is an average of the M outcomes. The network population was J = 16. The channel coefficients between users and user - base station were simulated according to the sum-of-sinusoids simulation model for Rayleigh fading channels [11], according to which, each channel multi-path was a zero mean complex Gaussian random variable, with variance $\sigma_a^2 = 1$. Note that each channel coefficient was correlated across domain. The values for r_{in}, r_{out} , noise power σ_w^2, m , and the data modulation scheme are the same as in the example in section III.

Fig. 2 shows the BER of resolving K = 4-th order collision with the proposed optimal relay selection scheme (15), and also the random relay selection scheme of [8]. The improvement is rather obvious. At SNR = 25dB, the BER for the proposed scheme is around 10^{-6} , while for the relay selection scheme of [8] it is around 10^{-4} . As the SNR becomes higher, the improvement is even larger due to the fully exploited spatial diversity.



Fig. 2. BER vs SNR for K=4: Simulation results for the proposed relay selection scheme and that of the original ALLIANCES.



Fig. 3. Simulation results: BER vs traffic Load for J=16, SNR=20 dB, and ML decoding.

Fig. 3 shows the result of Monte Carlo simulations for BER under different traffic load. Each node is fed with a Poisson traffic process with load $\frac{\lambda}{J}$. The random relay selection curve reaches minimum when the traffic load = 0.5, because at this point the expectation of the number of non-source relay nodes is maximum. The curve for the proposed optimal relay selection scheme has the

same trend, however, for moderate traffic load, it enables BER of about 10 dB lower. For low traffic load the collisions occur with low probability thus the improvement of optimal relay selection is not obvious. When the traffic load becomes larger than 0.9, high order collisions occur frequently and no enough non-source relay nodes are available. Therefore, as traffic load approaches 1, the proposed relay selection scheme becomes equivalent to the random one. Thus we conclude that the proposed relay selection scheme is more useful in moderate traffic load. A method to avoid higher collision orders has been proposed in [13].

VI. CONCLUSIONS

In this paper we extend the ALLIANCES protocol originally proposed in [8] to a more realistic system model that takes into account the location information of network nodes. An optimal location-based relay selection scheme was proposed based on the analysis of PEP and BER. Both analytical and simulation results show that the optimal relay selection scheme can significantly enhance the BER performance of ALLIANCES for moderate traffic load.

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