A ROBUST SOURCE LOCALIZATION ALGORITHM APPLIED TO ACOUSTIC SENSOR NETWORK

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ABSTRACT

The problem of source localization using distributed acoustic sensor networks is considered in this paper. The acoustic sensors detect and estimate the time of arrival (TOA) of the acoustic transient emitted from a source of interest. The estimated TOA estimates are then transmitted to a base station for collaborative source localization. In realistic scenarios, a subset of the TOA's could typically be erroneous owing to various detrimental factors. The standard least squares-based methods totally fail in presence of even only one outlier or erroneous TOA. By modeling the TOA estimation error as Cauchy-Lorentz distribution, a robust source localization algorithm is derived. Numerical experiments are provided to show the robustness and accuracy of localization of the proposed algorithm.

Index Terms— Robust estimate, source localization, acoustic sensor network

1. INTRODUCTION

Source localization using sensor arrays has been a research topic of great interest for decades. For instance, the problem of direction of arrival (DOA) estimation of a far-field signal based on the sampled data received by an array of sensors has been extensively investigated (see [1]). More recently, near-field source localization using distributed sensor nodes has drawn considerable attention [2]. In the context of audio communication and speech processing, source (speaker) localization is usually done based on the estimate of time different of arrival (TDOA) between the elements of a microphone array [3]. Source localization is subsequently carried out based on the least square estimation (LSE) or its variants.

In this paper, we consider near-field acoustic transient source localization using time of arrivals (TOA's) in distributed sensor networks. Compared to the applications to the speech processing systems [3], we consider a more difficult and realistic scenario where some of the distributed unattended acoustic sensors may generate totally erroneous TOA estimates (or outliers) due to various detrimental factors, such as propagation delay, multipath interference, sensor location error, false alarm, or simply device failure. The solution to this problem is particularly of interest to numerous realistic battlefield surveillance systems e.g. in sniper detection and localization. In [4], the authors employed a large number of nodes in order to provide redundancy to combat the sensor failure problem. In particular, they defined a so-called *consistency function* over a four-dimensional (three spatial dimension plus on temporal dimension) space-time cubic. A bisection algorithm is then applied to find the space-time area where the consistency function is maximized. The experimental results in [4] showed the effectiveness of the proposed method at the expense of relatively high computational burden.

Clearly, the standard LSE-based methods fail even if only one outlier is present. To overcome this problem, we model the TOA estimation error as a Cauchy-Lorentz distribution. This yields a simple and yet very robust (to outliers) maximum likelihood estimate of the source localization, which can easily be solved numerically using the iterative search routine. The proposed algorithm automatically eliminates the effects of the outliers and generates very accurate localization as long as a few TOA estimates are reasonably accurate. When all the TOA estimation errors are small, the proposed method performs similarly to the LSE. Comparing to the method in [4], our proposed algorithm is simpler and computationally more efficient. Simulation results are provided to show the robustness and accuracy properties of the proposed method.

2. PROBLEM FORMULATION

A distributed acoustic sensor network for acoustic transient source localization is illustrated in Figure 1, where the K passive acoustic sensors are illustrated by circles "$\circ$". The sensor locations, $\mathbf{p}_k = [x_k, y_k, z_k]^T$, $k = 1, 2, \dots, K$, are assumed to be known. The location of the source, $\mathbf{p}^{(s)} = [x^{(s)}, y^{(s)}, z^{(s)}]^T$, is shown by an asterisk "$*$" in this figure. Here the superscript (s) stands for "source".

Suppose the source emits an acoustic transient at time T (to be determined) propagating spherically in the air at a constant and known speed of c. The TOA of the signal received at the kth line-of-sight sensor is

$$T_k = T + \frac{\|\mathbf{p}_k - \mathbf{p}^{(s)}\|}{c} \qquad (1)$$

where $\|\cdot\|$ is Euclidean norm. The signal received by the kth

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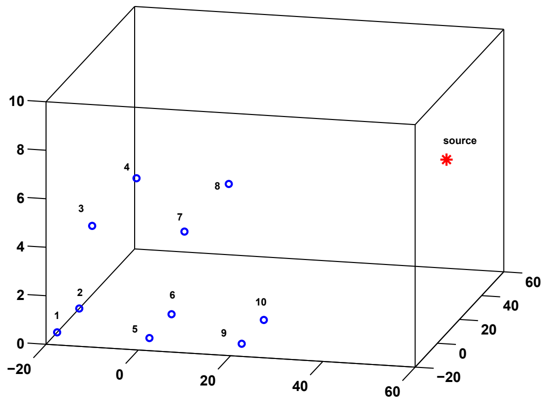

Fig. 1. The layout of the 3-D sensor networks (illustrated by "o"). The transient acoustic source is illustrated by "$*$".

distributed sensor can then be represented by

$$x_k(t) = \begin{cases} \alpha_k s(t - T_k) + I_k(t) + n_k(t) & t \geq T_k \\ n_k(t) & t < T_k \end{cases} \quad (2)$$

where $s(t)$ is the transient signal waveform, $\alpha_k = \frac{1}{d_{k,s}}$ represents the effect of signal attenuation which is proportional to its travel distance, $d_{k,s} = \|\mathbf{p}_k - \mathbf{p}^{(s)}\|$, $I(t)$ represents the interference and the multipath signal component, and $n(t)$ is the background noise. The sensors use either the energy detector or zero crossing encoding-based method to estimate TOA [4]. Clearly, due to many factors such as ambient noise, multipath effects, time synchronization error, and sensor location error, the estimated TOA at each sensor differs from the true TOA by some error, i.e.

$$\hat{T}_k = T_k + e_k, \quad (3)$$

where e_k is the estimation error. We assume that the sensors are synchronized. A method for time synchronization is discussed in [4]. The estimated TOA's $\hat{T}_k$'s are then sent to the base station (or a gateway node), where the source is localized in both spatial and temporal domain, i.e., $\mathbf{p}^{(s)}$ and T are estimated based on $\hat{T}_k$'s. In theory we need only four TOA estimates to estimate the unknowns. However, the redundancy of having more than four TOA estimates leads to much better robustness in source localization.

In what follows, we devise a robust source localization algorithm that can still yield accurate source localization assuming that a few of the sensors have sufficiently accurate TOA estimates while the others provide erroneous TOA estimates.

3. ROBUST SOURCE LOCALIZATION

3.1. Least Squares-Based Estimation

First, let us examine the LSE-based source localization in order to provide insights into the reasons why this type of method is not robust against the outliers, hence leading us to the robust solution.

To jointly estimate the source location $\mathbf{p}^{(s)}$ and the time T, the LSE gives

$$\hat{\mathbf{p}}^{(s)}, \hat{T} = \arg \min_{\mathbf{p}^{(s)}, T} \sum_{k=1}^{K} \left[\|\mathbf{p}_k - \mathbf{p}^{(s)}\| - c(\hat{T}_k - T) \right]^2. \quad (4)$$

Indeed, if the TOA estimation error, $e_k \sim N(0, \sigma^2)$ is i.i.d. Gaussian, then it is straightforward to show that the LSE is also a maximal likelihood estimate (MLE). In practice, the distribution of the estimation error may be far from the Gaussian assumption. For example, it is typical in the practical scenario that some sensors yield reasonably accurate TOA estimates, while the others are totally erroneous. Even a single failed TOA estimate (outlier) can lead to completely wrong LSE, which is qualitatively explained as follows. Notice that the cost function in (4) is the sum of the K squared fitting errors. If one of estimates $\hat{T}_k$ is an outlier, then the corresponding squared error

$$\mathcal{E}_k(\mathbf{p}^{(s)}, T) \triangleq [\|\mathbf{p}_k - \mathbf{p}^{(s)}\| - c(\hat{T}_k - T)]^2 \quad (5)$$

will "blow up", and so is the total squared error. This consequently fails the whole estimation process. In other words, the LSE is very sensitive to the presence of outliers. One remedy to make the estimate robust against the outliers is to deemphasize the cost function for the very large $\mathcal{E}_k(\mathbf{p}^{(s)}, T)$, which is the underlying idea of the proposed robust algorithm.

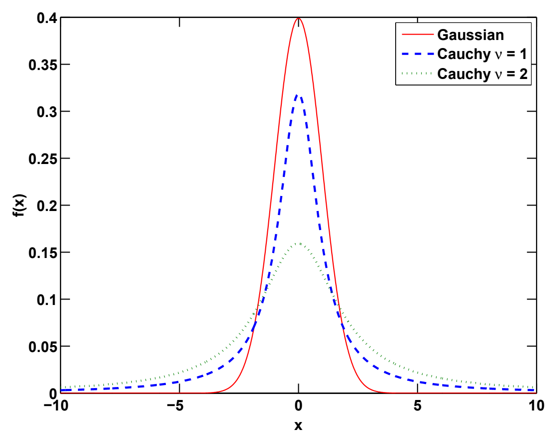

Fig. 2. The PDFs of Gaussian distribution and Cauchy-Lorentz Distribution.

3.2. Proposed Robust Algorithm

Here, we model the scaled TOA estimation error

$$c \cdot e_k = c(\hat{T}_k - T_k) = c(\hat{T}_k - T) - \|\mathbf{p}_k - \mathbf{p}^{(s)}\|$$

as a random variable of Cauchy-Lorentz distribution [5]:

$$f(x) = \frac{1}{\pi\nu \left(1 + \frac{x^2}{\nu^2}\right)}, \quad (6)$$

where the parameter ν determines the half-width at half maximum. The probability of functions (PDF's) of the Gaussian distribution $N(0,1)$ and the Cauchy distributions of (6) with parameter $\nu = 1$ and $\nu = 2$ are plotted in Figure 2. We can see that the Cauchy-Lorentz distribution has much heavier "tails" compared to the Gaussian PDF. This heavy tail allows for better ability to represent the effects of erroneous measurements or the outliers.

Based on the Cauchy-Lorentz assumption of the TOA estimation errors, we derive the MLE of $\mathbf{p}^{(s)}$ and T:

$$\hat{\mathbf{p}}^{(s)}, \hat{T} = \arg\max_{\mathbf{p}^{(s)},T} \prod_{k=1}^{K} f\left(c(\hat{T}_k - T) - \|\mathbf{p}_k - \mathbf{p}^{(s)}\|\right), \quad (7)$$

where $f(\cdot)$ is given in (6). After some straightforward manipulations, (7) can be simplified to be

$$\begin{aligned} & \hat{\mathbf{p}}^{(s)}, \hat{T} \\ = \quad & \arg\min_{\mathbf{p}^{(s)},T} \sum_{k=1}^{K} \log\left[1 + \frac{(\|\mathbf{p}_k - \mathbf{p}^{(s)}\| - c(\hat{T}_k - T))^2}{\nu}\right] & (8) \\ = \quad & \arg\min_{\mathbf{p}^{(s)},T} \sum_{k=1}^{K} \log\left[1 + \frac{\mathcal{E}_k(\mathbf{p}^{(s)}, T)}{\nu}\right] & (9) \end{aligned}$$

where from (8) to (9) we have used the definition in (5).

Here we see that even if the kth sensor has an erroneous TOA estimation (i.e. a very large $\mathcal{E}_k\left(\mathbf{p}^{(s)}, T\right)$), the detrimental influence is deemphasized by the function $\log\left[1 + \frac{\mathcal{E}_k(\mathbf{p}^{(s)},T)}{\nu}\right]$. Therefore, compared to the standard LSE the proposed method effectively puts less weight on the outliers. On the other hand, if all the TOA estimates are reasonably accurate, then

$$\mathbf{p}^{(s)}, \hat{T} \approx \arg\min_{\mathbf{p}^{(s)},T} \sum_{k=1}^{K} \mathcal{E}_k\left(\mathbf{p}^{(s)}, T\right), \quad (10)$$

since $\log(1+x) \approx x$ for x around zero. In other words, the robust source localization algorithm performs similarly to the LSE in this case. Now, we see the role of parameter ν; as ν increases, the robust algorithm performs more like the standard LSE, but for small ν, the proposed algorithm puts relatively less weights to the erroneous TOA's.

3.3. Iterative Algorithm

The nonlinear estimation in (9) (as well as in (4)) can be performed using the standard Newton's method. Let $l(\boldsymbol{\theta})$ be the log-likelihood cost function in (9), where

$$\boldsymbol{\theta} \triangleq [x^{(s)}, y^{(s)}, z^{(s)}, T]^T,$$

the Newton's method can be written as

$$\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i - \mathbf{J}^{-1}(\boldsymbol{\theta}_i)\mathbf{d}(\boldsymbol{\theta}_i) \quad (11)$$

where

$$\mathbf{J}(\boldsymbol{\theta}_i) = \left[\frac{\partial^2 l(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^T}\right]\Bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_i} \quad (12)$$

and

$$\mathbf{d}(\boldsymbol{\theta}_i) = \left[\frac{\partial l(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right]\Bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_i}. \quad (13)$$

$$\frac{\partial l(\boldsymbol{\theta})}{\partial x_k} = \sum_{k=1}^{K} \frac{2\left(x_k - x^{(s)}\right)\left(\|\mathbf{p}^{(s)} - \mathbf{p}_k\| - c\,(\hat{T}_k - T)\right)}{\|\mathbf{p}^{(s)} - \mathbf{p}_k\|\left(\nu + \left(\|\mathbf{p}^{(s)} - \mathbf{p}_k\| - c\,(\hat{T}_k - T)\right)^2\right)}. \quad (14)$$

For $\frac{\partial l(\boldsymbol{\theta})}{\partial y_k}$ and $\frac{\partial l(\boldsymbol{\theta})}{\partial z_k}$, they can be obtained from (14) by replacing x_k by y_k and z_k, respectively.

$$\frac{\partial l(\boldsymbol{\theta})}{\partial T} = \sum_{k=1}^{K} \frac{2\,c\left(\|\mathbf{p}^{(s)} - \mathbf{p}_k\| - c\,(\hat{T}_k - T)\right)}{\nu + \left(\|\mathbf{p}^{(s)} - \mathbf{p}_k\| - c\,(\hat{T}_k - T)\right)^2}. \quad (15)$$

The second order derivatives are more complicated which we omit here. As an alternative approach we can use the build-in Matlab™ function FMINUNC to solve this unconstraint optimization problem. In both approaches, we need to set an initial value $\boldsymbol{\theta}_0$ for the iterative search. Although global optimum cannot be guaranteed as the cost function is not a convex function of the estimated parameters, the extensive experiments suggest that the global optimal can be reached most of the times even if the initial value is quite far away from the true value.

4. SIMULATION RESULTS

We present the results of three simulations to demonstrate the effectiveness of the proposed algorithm. For all these experiments, we consider the distributed sensor network with layout shown in Figure 1 where the 10 sensors are indexed. The 3-D locations of the 10 sensors are

$$(-20,-10,0),\ (-20,10,0),\ (-10,-20,5),\ (-10,20,5),$$
$$(0,-10,0),\ (0,10,0),\ (10,-20,5),\ (10,20,5),$$
$$(20,-10,0),\ (20,10,0).$$

The source location is $\mathbf{p}^{(s)} = (50,50,5)$ and it gives off an acoustic transient signal at time $T = 0$, which propagates spherically with speed $c = 344$ m/s.

In the first study, we assume that all the 10 sensors estimate the TOA's quite accurately and the estimation errors are iid Gaussian, i.e., $e_k \sim N(0,\sigma^2)$. In this case, LSE is indeed an ML estimate and hence is optimal. Figure 3 compares the mean squared errors (MSE) of the estimated spatial location using standard LSE and the proposed robust estimate with the parameter $\nu = 1$. We see that the performance of the robust method is very close to the LSE, which agrees with the analysis in Section 3.2. In this simulation, the initial value of the source location is chosen to be $\hat{\mathbf{p}}_0^{(s)} = (0,0,0)$ and $\hat{T}_0 = 200$ (ms) that far from actual values.

In the second study, we check the robustness of the proposed method against the erroneous TOA estimates. We let the first N ($N < K$) sensors to have TOA estimation error with large variance $\sigma_\tau^2 = 1$ (sec^2), and the rest to have good estimate with $\sigma_\tau = 10^{-4}$ sec, or 0.1 ms. Such cases occur in scenarios where building or other objects are in the line-of-sight of

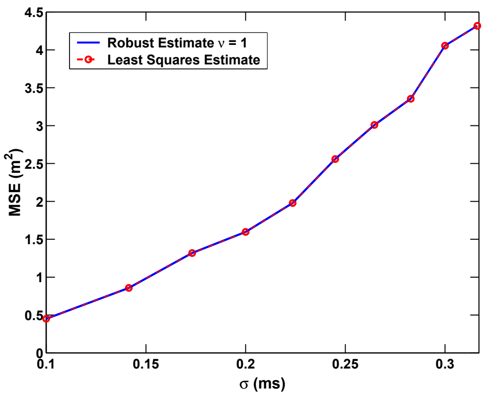

Fig. 3. Comparison of the MSEs of the estimated location using the LSE of (4) and the robust estimate (9). The result is based on 500 Monte Carlo Trials.

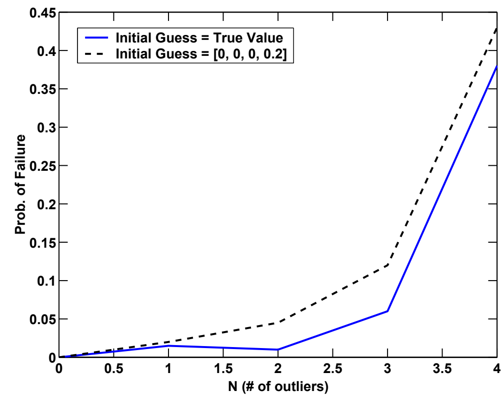

Fig. 5. The influence of the initial guess of $\mathbf{p}^{(s)}$ and T over and Probabilities of failure of robust estimate (9). The result is based on 200 Monte Carlo Trials.

the first N sensors. Suppose the required 3D localization precision is 2 meters, i.e., we regard the estimated source location more than 2 meters away from the true value as a failed estimate. The results in Figure 4 indicate that the LSE produces failed estimate with probability one even if there is only one outlier. In contrast, our proposed method is very robust against the outliers. It yields accurate estimate (3D estimation error of less than 2 meters) with probability close to 60% even when five out of ten sensors produce erroneous TOA estimates. The performance of the proposed method is clearly dependent on choice of the parameter ν. As we have pointed out in Section 3.2, when ν increases, the propose method performs more like the LSE, this observation is verified in the results in Figure 4. On the other hand, small ν, e.g. $\nu = 0.1$, does not yield better performance than $\nu = 0.25$. The theoretical analysis on the influence of ν remains an open problem.

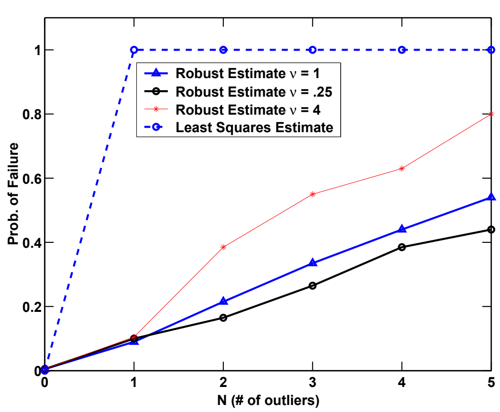

Fig. 4. Probabilities of failure of robust estimate (9) with respect of the number of unreliable sensors. The total number of sensors $K = 10$. The result is based on 200 Monte Carlo Trials.

In the final study, we compare the performances of the proposed method with different initial value $\boldsymbol{\theta}_0$ for the source location. We include, as a benchmark, the case where we set $\boldsymbol{\theta}_0$ as the true value. Figure 5 shows that compared to this benchmark the relative performance degradation is very moderate in spite of the fact that the initial values $\hat{\mathbf{p}}_0^{(s)} = (0, 0, 0)$ and $\hat{T}_0 = 200$ ms are far from the true ones $\mathbf{p}^{(s)} = (50, 50, 5)$ and $T = 0$. In practice, some *a priori* information can be exploited to choose a reasonably accurate initial guess of the source location.

5. CONCLUSIONS

In this paper, we have considered the problem of acoustic source localization based on the sensor-level TOA estimates in distributed sensor networks. By modeling the TOA estimation errors as Cauchy-Lorentz distribution, we derived a simple algorithm which is shown to be very robust against the TOA outliers. Although the proposed algorithm must be implemented iteratively using an initial guess of the unknown parameters, we have shown via simulations that it can converge with high probability to the global optimal even if the initial value is quite far away from the true value. Simulation results are presented to validate the robust performance of the proposed algorithm.

6. REFERENCES

[1] H. L. Van Trees, *Optimal Array Signal Processing*. New York, NY: John Wiley and Sons, Inc., 2002.

[2] J. C. Chen, K. Yao, and R. E. Hudson, "Source localization and beamforming," *IEEE Signal Processing Magazine*, pp. 30–39, March 2002.

[3] Y. Huang, *Real-time acoustic source localization using passive microphone array*. PhD Dissertation, Georgia Institute Technology 2001.

[4] S. H. S. *et. al*, "Sensor network-based countersniper system," *Proceedings of the 2nd international conference on Embedded networked sensor systems, Baltimore MD*, pp. 1–12, 2004.

[5] G. Casella and R. L. Berger, *Statistical Inference*. Duxbury Press, 2001.