

# SOURCE LOCALIZATION IN SENSOR NETWORKS WITH RAYLEIGH FADED SIGNALS

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## ABSTRACT

Source localization is investigated for a sensor network with passive sensors. The signal emitted by the source endures Rayleigh fading during its propagation, and its average intensity is a function of the distance from the source. Maximum likelihood (ML) source location estimators that use the output, or its quantized version, of the non-coherent receiver is proposed. The ML estimators' Cramér-Rao lower bounds (CRLBs) are derived. Due to the fading effect, the proposed estimator's performance is degraded, compared to the ideal case without fading. However, it can still accurately estimate the source's position and intensity, and achieve its CRLB with relatively small amount of resources, namely small number of observations, sensors and quantization bits.

**Index Terms**— Sensor networks, localization, Rayleigh fading, quantization, Cramér-Rao lower bound

## 1. INTRODUCTION

For a sensor network with a large number of densely deployed sensors, it is possible to accurately estimate the source position based on intensity (energy) of the signal, without the need for additional sensor functionalities and measurement features, such as the direction of arrival (DOA) or time-delay of arrival (TDOA). Energy-based methods have been proposed in [1, 2], where analog measurements from sensors are required to localize the source using either a least-square or a ML estimator. For a sensor network with limited resources (energy and bandwidth), it is desired that only quantized data be transmitted from local sensors to the processing node (fusion center). In our previous work [3], we have proposed an intensity based ML target location estimation method using only quantized data.

In all the previous work on energy-based localization methods, it is assumed that the signal power is a deterministic function of the distance from the source. However, this is often not true in many practical scenarios, where the sensors are very small compared to the surrounding structures, and multipath occurs due to the reflections from the ground and surrounding structures [4]. In such realistic cases, the ML location estimator proposed in [1, 2, 3] can not function properly. Here, we introduce a new ML location estimator that takes into account

the signal fading effect due to multi-path. Experiments show that this method achieves the CRLB even with relatively small number of sensors, observations and quantization bits.

## 2. PROBLEM FORMULATION

We assume that a source is located in a sensor field with  $N$  sensors, whose locations are known. The signal emitted by the source attenuates isotropically as follows:

$$a_i^2 = P'_0 d_0^m / d_i^m \quad (1)$$

where  $a_i$  is the signal amplitude at the  $i$ th sensor,  $P'_0$  is the signal power measured at a reference distance  $d_0$ , and  $d_i$  is the  $i$ th sensor's distance from the source

$$d_i = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2} \quad (2)$$

in which  $(x_i, y_i)$  and  $(x_t, y_t)$  are the coordinates of the  $i$ th sensor and the source, respectively, and  $n$  is the power decay exponent. Here, we let  $P_0 = P'_0/4$ , and assume that  $d_0 = 1$ . As a result, (1) becomes

$$a_i^2 = 4P_0 / d_i^m. \quad (3)$$

Each sensor takes  $m$  observations of the signal, at a frequency of  $1/T_b$ . For each observation, a sensor senses the signal for a period of  $T$  ( $T < T_b$ ). It is assumed that the observation frequency  $1/T_b$  is high so that the source is static during the period of  $m$  observations. Therefore,  $d_{ij} = d_i$ , for  $j = 1, \dots, m$ , where  $d_{ij}$  is the  $i$ th sensor's distance from the source at the  $j$ th observation interval.

The source emits a narrow band signal, and during the  $j$ th observation interval at the  $i$ th sensor the signal is of the form

$$r_{ij}(t) = a_{ij}g(t)\cos(\omega_c t + \phi(t) + \varphi_{ij}) + w_i(t) \quad (4)$$

where  $a_{ij}$  is the signal amplitude,  $g(t)$  is a slowly varying envelope, which is normalized so that

$$\int_0^T g^2(t)dt = 1 \quad (5)$$

$\omega_c$  is the known carrier frequency,  $\phi(t)$  is a known phase modulation,  $\varphi_{ij}$  is the initial phase, and  $w_i(t)$  is a white Gaussian noise with a constant power spectral density  $N_0/2$ .

For the ideal case, we assume that  $\varphi_{ij}$  is known and  $a_{ij} = a_i$  is an unknown constant. Note that during each observation interval, the signal energy is

$$E_{si} = \int_0^T [a_i g(t) \cos(\omega_c t + \phi(t) + \varphi_{ij})]^2 dt = 2P_0/d_i^n \quad (6)$$

the noise energy is

$$E_{ni} = E \left[ \int_0^T w_i^2(t) dt \right] = N_0/2 \quad (7)$$

and the signal to noise ratio (SNR) at the  $i$ th sensor is therefore  $\rho_i \triangleq 4P_0/(N_0 d_i^n)$ . It is well known that the output of the optimum coherent receiver at observation interval  $j$  and sensor  $i$  is

$$u_{ij} = \int_{(j-1)T_b}^{(j-1)T_b+T} r_{ij}(t) g(t) \cos(\omega_c t + \phi(t) + \varphi_{ij}) dt$$

which follows a Gaussian  $(\sqrt{P_0/d_i^n}, N_0/4)$  distribution [5]. Under the white noise assumption, the sufficient statistic for the multi-observation case is

$$U_i = \frac{1}{m} \sum_{j=1}^m u_{ij} \quad (8)$$

which follows a Gaussian  $(\sqrt{P_0/d_i^n}, N_0/4m)$  distribution.

In more practical cases, the source signal usually propagates via multiple paths. As a result, its envelope fluctuates and its phase will be a random variable (RV) [4]. We assume that the signal endures Rayleigh fading during its propagation. The signal amplitude  $a_{ij}$  is fluctuating independently across observations and across sensors, which is modeled as

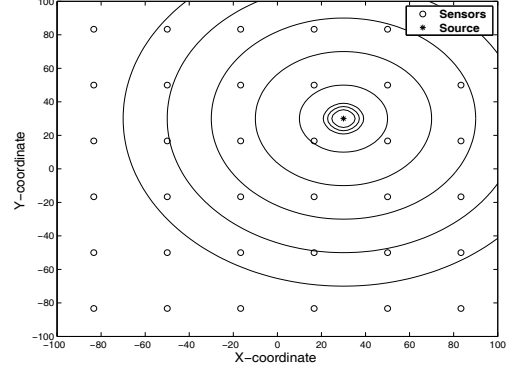
$$a_{ij} = h_j a_i \quad (9)$$

where  $h_j$  follows a Rayleigh distribution:

$$f(h_j) = 2h_j e^{-h_j^2} \quad (10)$$

The average signal energy during each observation, which is  $E[h_j^2]a_i^2/2$ , is still the same as the signal energy in the ideal case, which is  $2P_0/d_i^n$  as provided in (6). The phase  $\varphi_{ij}$  is a RV that follows a uniform distribution in  $[0, 2\pi]$ , and is also independent across observations and across sensors. It is well known that the output of the optimum non-coherent receiver to detect a signal with a random phase is

$$v_{ij} = \left[ \int_{(j-1)T_b}^{(j-1)T_b+T} r_{ij}(t) g(t) \cos(\omega_c t + \phi(t)) dt \right]^2 + \left[ \int_{(j-1)T_b}^{(j-1)T_b+T} r_{ij}(t) g(t) \sin(\omega_c t + \phi(t)) dt \right]^2$$



**Fig. 1.** The signal intensity contours of a source located in a sensor field.

With the aid of standard references for non-coherent detection, such as [5], we can show that  $v_{ij}$  follows an Exponential  $(P_0/d_i^n + N_0/2)$  distribution. For a multi-observation case, the sufficient statistic is

$$V_i = \sum_{j=1}^m v_{ij} \quad (11)$$

and it follows a Gamma  $(m, P_0/d_i^n + N_0/2)$  distribution.

### 3. DEVELOPMENT OF ML LOCATION ESTIMATOR

#### 3.1. Location Estimator using Analog Data

In this section, we assume that the processing node receives analog data from local sensors, and estimates the parameter vector:  $\theta = [P_0 \ x_t \ y_t]'$ . For the ideal case with coherent receivers, the likelihood of analog data  $\mathbf{U} = [U_1, \dots, U_N]$  is

$$f(\mathbf{U}|\theta) = \prod_{i=1}^N \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(U_i - \mu_i)^2}{2\sigma^2}} \quad (12)$$

where  $\mu_i \triangleq \sqrt{P_0/d_i^n}$  and  $\sigma^2 \triangleq N_0/4m$ . The ML estimation procedure is to find the optimal  $\theta$  that maximizes  $f(\mathbf{U}|\theta)$ . The CRLB matrix for this estimation problem is provided in the following theorem:

**Theorem 1** *The inverse of the CRLB matrix, or the Fisher information matrix (FIM), for an estimator using analog outputs from coherent receivers is*

$$\mathbf{J}_1 = \sum_{i=1}^N \alpha_{1i} \mathbf{C}_i$$

$$\mathbf{C}_i = \begin{bmatrix} \frac{1}{4P_0 d_i^n} & \frac{n(x_i - x_t)}{4d_i^{n+2}} & \frac{n(y_i - y_t)}{4d_i^{n+2}} \\ \frac{n(x_i - x_t)}{4d_i^{n+2}} & \frac{P_0 n^2 (x_i - x_t)^2}{4d_i^{n+4}} & \frac{P_0 n^2 (x_i - x_t)(y_i - y_t)}{4d_i^{n+4}} \\ \frac{n(y_i - y_t)}{4d_i^{n+2}} & \frac{P_0 n^2 (x_i - x_t)(y_i - y_t)}{4d_i^{n+4}} & \frac{P_0 n^2 (y_i - y_t)^2}{4d_i^{n+4}} \end{bmatrix}$$

where  $\alpha_{1i} \triangleq 4m/N_0$  is the Fisher information (FI) of  $\mu_i$  contained in data  $U_i$ .

We skip the details of the proof, which follows a similar procedure used in the proof of Theorem 3 in Section 3.2.

For the Rayleigh fading case with non-coherent receivers, the likelihood of data  $\mathbf{V} = [V_1, \dots, V_N]$  is

$$f(\mathbf{V}|\theta) = \prod_{i=1}^N \frac{1}{(m-1)!\beta_i^m} V_i^{m-1} e^{-\frac{V_i}{\beta_i}} \quad (13)$$

where  $\beta_i \triangleq P_0/d_i^n + N_0/2$ . The ML estimation procedure is to find the optimal  $\theta$  that maximizes  $f(\mathbf{V}|\theta)$ . The FIM for this estimation problem is provided in the following theorem:

**Theorem 2** *The FIM for an estimator using analog outputs from non-coherent receivers for Rayleigh faded signals is*

$$\mathbf{J}_2 = \alpha_{2i} \mathbf{C}_i$$

where  $\alpha_{2i} \triangleq 4mP_0/(\beta_i d_i^n)$  is the FI of  $\mu_i$  contained in  $V_i$ .

We skip the details of the proof, which follows a similar procedure used in the proof of Theorem 3 in Section 3.2.

From Theorems 1 and 2, an immediate observation is that under both the ideal and Rayleigh fading cases, the FIMs contributed by any individual sensor are proportional to each other, and are scaled by different factors ( $\alpha_{1i}$  and  $\alpha_{2i}$ ), whose ratio is

$$\alpha_{1i}/\alpha_{2i} = 1 + \rho_i/4 + 1/\rho_i \quad (14)$$

meaning that in the ideal case the output of the coherent receiver is always more informative than that of the non-coherent receiver in the fading case. This is because in the Rayleigh fading case, there is more uncertainty caused by fluctuating signal amplitude and an unknown random signal phase.

### 3.2. Location Estimator using Quantized Data

In this section, we study the location estimator using quantized output of the non-coherent receivers at sensors. We assume that each sensor sends quantized  $n_b$ -bit data to the processing node, which are denoted as  $\mathbf{D} = \{D_i : i = 1, \dots, N\}$ , where  $D_i$  can take any discrete value from 0 to  $2^{n_b} - 1$ . We assume that the set of quantization thresholds for the  $i$ th sensor is  $\vec{\eta}_i = [\eta_{i0}, \eta_{i1}, \dots, \eta_{iL}]$ , where  $L \triangleq 2^{n_b}$ ,  $\eta_{i0} = 0$  and  $\eta_{iL} = \infty$ . The probability that  $D_i$  takes a specific value  $l$  ( $0 \leq l \leq L-1$ ) is

$$p_{il}(\vec{\eta}_i, \theta) = F(\eta_{i(l+1)}, m, \beta_i) - F(\eta_{il}, m, \beta_i) \quad (15)$$

where

$$F(x, m, \beta_i) = 1 - e^{-\frac{x}{\beta_i}} \sum_{k=0}^{m-1} \frac{(x/\beta_i)^k}{k!} \quad (16)$$

is the Gamma distribution function. The likelihood of  $\mathbf{D}$  is

$$p(\mathbf{D}|\theta) = \prod_{i=1}^N \prod_{l=0}^{L-1} p_{il}(\vec{\eta}_i, \theta)^{\delta_{D_i, l}} \quad (17)$$

where  $\delta_{D_i, l}$  is Kronecker's delta function. The ML estimation procedure is to find the optimal  $\theta$  that maximizes  $f(\mathbf{D}|\theta)$ . The FIM for this estimation problem has been provided in the following theorem:

**Theorem 3** *The FIM for an estimator using quantized output of non-coherent receivers in the Rayleigh fading case is*

$$\mathbf{J}_3 = \sum_{i=1}^N \alpha_{3i} \mathbf{C}_i$$

where

$$\alpha_{3i} \triangleq \sum_{l=0}^{L-1} \frac{4P_0 \left[ \eta_{il}^m e^{-\eta_{il}/\beta_i} - \eta_{i(l+1)}^m e^{-\eta_{i(l+1)}/\beta_i} \right]^2}{d_i^n [(m-1)!]^2 \beta_i^{2m+2} p_{il}(\vec{\eta}_i, \theta)}$$

is the FI of  $\mu_i$  contained in data  $D_i$ .

*Proof:* Given the independence of data from different sensors, we have

$$\mathbf{J}_3 = \sum_{i=1}^N \mathbf{J}_{3i} \quad (18)$$

where  $\mathbf{J}_{3i}$  is the FIM of sensor  $i$ :

$$\begin{aligned} \mathbf{J}_{3i} &= E \left\{ [\nabla_{\theta} \ln p(D_i|\theta)] [\nabla_{\theta} \ln p(D_i|\theta)]^T \right\} \\ &= E \left\{ \left[ \frac{\partial \ln f(D_i|\theta)}{\partial \beta_i} \frac{\partial \beta_i}{\partial \mu_i} \right]^2 \right\} \nabla_{\theta} \mu_i [\nabla_{\theta} \mu_i]^T \end{aligned} \quad (19)$$

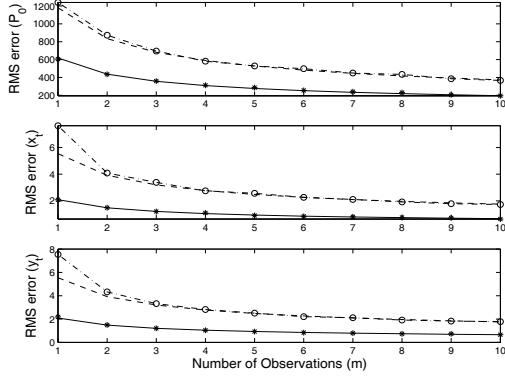
Skipping some intermediate steps, we can show that the first term in the second line of (19) is  $\alpha_{3i}$ , and in [3], we have shown that  $\nabla_{\theta} \mu_i [\nabla_{\theta} \mu_i]^T = \mathbf{C}_i$ . Q.E.D.

## 4. EXPERIMENTAL RESULTS

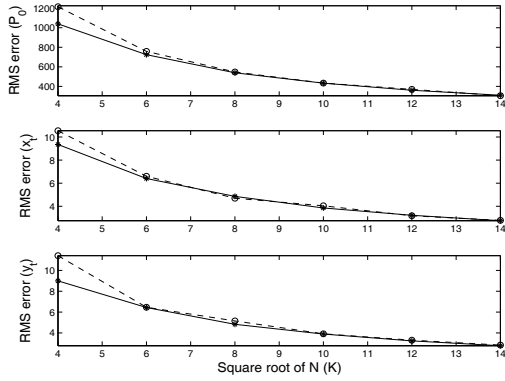
In our experiments, we assume that sensors are uniformly deployed as shown in Fig. 1,  $n = 2$ ,  $P_0 = 5000$ , and  $N_0 = 2$ .

### 4.1. Analog Data

In Fig. 2, the performance of the ML estimator using analog data is compared with the CRLB under both the ideal case and the fading case. As we can see, the estimation performance in the ideal case with coherent receivers is always better than that in the fading case with non-coherent receivers. The performance degradation in the latter case is caused by the uncertainty of the signal phase and the fluctuation of the signal amplitude. As the number of observations  $m$  increases, the ML estimator's performance improves significantly and quickly converges to the CRLB, under both cases.



**Fig. 2.** Root mean square (RMS) errors for analog data ( $x_t = y_t = 0$ ,  $N = 64$ ). Solid line: ideal case CRLB, star: ideal case ML estimation, dashed line: fading case CRLB, dashdot line + circle: fading case ML estimation.

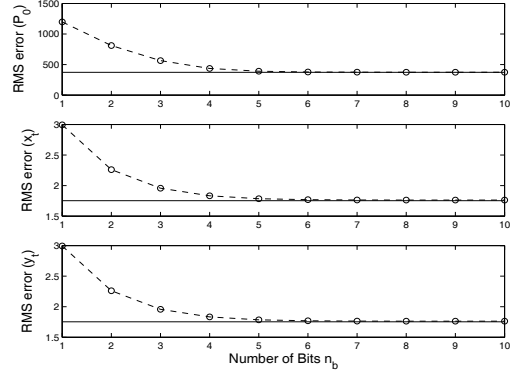


**Fig. 3.** RMS errors for quaternary data ( $x_t = 15$ ,  $y_t = 20$ ,  $m = 10$ ). Solid line + star: CRLB, dashdot line + circle: ML estimation. Quantization threshold:  $\vec{\eta} = [14.57 \ 17.95 \ 25.69]'$ .

#### 4.2. Quantized Data in Fading Case

In Fig. 3, the performance of the ML estimator using quaternary data is compared with its CRLB. For simplicity, we assume that all the sensors use the same threshold to quantize the non-coherent receiver output. Also, we define  $K \triangleq \sqrt{N}$ . As  $N$  increases, the ML estimator's performance quickly converges to the CRLB. Actually, there is not much difference between the estimator's performance and its CRLB when  $N$  is as low as 36. This means that our ML estimator is efficient, namely it reaches its CRLB, even for quantized data with a small number of bits, and a small number of sensors.

In Fig. 4, the CRLB RMS errors are shown for multi-bit quantized data. The thresholds are chosen such that they divide the interval  $[V_l, V_u]$  evenly into  $L$  intervals, where  $V_l = F^{-1}(0.05, m, \beta_{\min})$ ,  $V_u = F^{-1}(0.95, m, \beta_{\max})$ , and  $\beta_{\min}$ ,  $\beta_{\max}$  are the minimum and maximum values of  $\beta_i$ s among all the sensors, respectively. Even with this simple quantization scheme, the CRLB performance of the estimator based on quantized data converges to that using analog data quickly, as



**Fig. 4.** CRLB on RMS errors for multi-bit quantized data ( $x_t = y_t = 0$ ,  $N = 64$ ,  $m = 10$ ). Solid line: analog data, dashed line + circle: quantized data.

shown in Fig. 4. For as little as four bit data, the performance of the estimator based on quantized data is already very close to that of the estimator using analog data.

## 5. CONCLUSIONS AND DISCUSSION

We presented an intensity-based ML source location estimator that works under imperfect situations where signal suffers from Rayleigh fading. The ML estimator uses the output or its quantized version, from a non-coherent receiver. Simulation results show that the estimator based on analog data is very accurate and is efficient even with a small number of observations, and the estimator based on quantized data is efficient with a relatively small number of sensors and a very small number of quantization bits. In addition, the performance of the estimator based on quantized data converges to that of the estimator using analog data very quickly as the number of bits increases. In this paper, we adopted a very intuitive scheme to quantize the non-coherent receiver output. Our future work will include research on quantization threshold design.

## 6. REFERENCES

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