

# MULTI-MODEL RAO-BLACKWELLISED PARTICLE FILTER FOR MANEUVERING TARGET TRACKING IN DISTRIBUTED ACOUSTIC SENSOR NETWORKS

*Yu Zhi-jun, You Guang-xin, Wei Jian-ming, Liu Hai-tao*

Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Science,  
Shanghai, 200050, China  
Email: seawave.yu@yahoo.com

## ABSTRACT

In this paper, a multi-model Rao-Blackwellised Particle Filter algorithm is presented for tracking high maneuvering target in distributed acoustic sensor networks. It is more efficient for high-dimension nonlinear and non-Gaussian estimation problems than generic particle filter, and by stratified particles sampling from a set of system models, it can tackle the target's maneuver perfectly. In the simulation comparison, a high maneuvering target moves through an acoustic sensor network field. The target is tracked using both the RBPF and the multi-model RBPF algorithms, and a location-central protocol is applied for energy conservation. The results show that our approach has great performance improvements, especially when the target is making maneuver.

**Index Terms**— Particle filter, RBPF, Maneuvering target, Multi-model, sensor networks

## 1. INTRODUCTION

Target tracking is emerging as one of the new attractive applications in large-scale wireless sensor networks (WSN) such as wild animal habit monitoring and intruder surveillance in military regions. For many practical target tracking problems, the target motion models are uncertain and the observations are incomplete, and the state equation or the measurement equation is nonlinearly modeled. Additionally, the system noises are also maybe non-Gaussian. There have been many suboptimal methods for these nonlinear problems. The classical method is extending the standard Kalman filter (KF) to nonlinear system by local linearizing all nonlinear models around certain points, which is so called extended Kalman filter (EKF) [1]. In 1995, Julier and Uhlmann proposed a new algorithm called Unscented Kalman filter (UKF) [2]. Particle filter (PF) algorithm is now a popular and useful method for nonlinear and non-Gaussian estimation problems [3]. The PF uses a set of random samples with associated weights to represent the required posterior density function (PDF) and computes state estimates based on these samples and weights. However, when applied to a high dimensional state space, the computation burden may explode and the estimation accuracy may deteriorate rapidly. An effective method to solving this problem is using the Rao-Blackwell theorem [4] to reduce the dimension of the state vector that needs to be estimated by nonlinear method. Based on this theorem, Arnaud Doucet and Simon Godsill proposed the Rao-Blackwellised Particle Filter (RBPF) algorithm which is also called marginalized particle filter in other papers [5][6].

The main predominance of the PF and RBPF algorithms is being able to handle any functional nonlinearity and any distribution of system noise or measurement noise. But the highly uncertainty and incompleteness of the measurement information in maneuvering target tracking application largely weaken this predominance. The difficulty mainly lies on the fact that the observation at a single step is always of highly uncertainties and incomplete to some extent and the valid information in single step is not enough for calculating an effective estimate of the target state. The predefined target model will miss match with the true target motion mode when the target is maneuvering, so it is most difficult to detect the maneuver efficiently and adjust the target model quickly. In order to tackle these kind of problems, here we propose a multi-model RBPF algorithm (we will call it MRBPF for simplification below) in the scenario of target tracking in acoustic sensor networks and the sensory information used for tracking are the acoustic energies. The proposed RBPF algorithm draws valid information from successive observations. First, we should choose a set of possible target motion models and associated initial model probabilities. Then we use stratified sampling theory [7] and successive observations to adjust the model probabilities and the particles distribution among these models. When modifying the PDF of the system modes with the observations of next time step, the maneuverability is settled and better performance is achieved.

The remainder of this paper is organized as follows. In the next section we will introduce the maneuvering target tracking problem in acoustic WSN. In section 3, the new multi-model RBPF algorithm is presented. Section 4 is the simulation experiment to evaluate the performance of our algorithm. Finally, a conclusion is given in section 5.

## 2. PROBLEM FORMULATION

### 2.1. Measurement Model

In this paper, we focus on the target tracking task using acoustic sensors (microphones) in wireless ad hoc sensor networks. Existing acoustic source localization or tracking methods make use of three types of physical measurements: time delay of arrival (TDOA), direction of arrival (DOA) and source signal strength or energy. In practice, energy based acoustic features is an appropriate choice for WSN passive target tracking applications because of the shortages of sensing, communication, energy and computation ability in sensor networks [8]. It is known that most of the sensor energy is consumed at the wireless communication module. The acoustic energy is computed as the moving average of the squared

magnitude of the acoustic time series, so the acoustic energy time series can be sampled at a much lower rate compared to the raw acoustic time series and the communication module awakening frequency can be reduced which means the retrenchment of sensor energy.

Assuming there are  $K$  targets and  $N$  acoustic sensors in a sensor field. The energy received by the  $i^{th}$  sensor is the sum of the decayed energy emitted from each of these  $K$  targets. Ideally, the acoustic energy measured on the  $i^{th}$  sensor at time step  $t$  can be expressed as follows:

$$y_i(t) = g_i \sum_{k=1}^K \frac{s_k(t-t_{ki})}{\|r_k(t-t_{ki}) - r_i\|^\alpha} + v_i(t) \quad (1)$$

Where  $s_k(t)$  is a scalar denoting the energy emitted by the  $k^{th}$  target during the energy sampling period,  $s_k(t)$  can be assumed varying very slowly during the run of the  $k^{th}$  target.  $t_{ki}$  is the time delay for the sound signal propagates from the  $k^{th}$  target to the  $i^{th}$  sensor.  $r_k(t)$  is a vector denoting the coordinates of the  $k^{th}$  target at the time step  $t$ .  $r_i$  is a vector denoting the coordinates of the  $i^{th}$  sensor.  $g_i$  is the energy gain factor of the  $i^{th}$  sensor.  $\alpha$  is an energy decay factor whose value can be measured during sensor calibration.  $v_i(t)$  is the cumulative effects of the modeling error of above-mentioned parameters and addition observation noise of  $y_i(t)$ ,  $v_i(t)$  can be approximated very well with a normal distribution.

## 2.2. Target Motion Model

For the ground maneuvering target tracking applications in a 2D sensor field, the target's state vector can be expressed as follows:

$$x_t = (\xi_t, \eta_t, \dot{\xi}_t, \dot{\eta}_t)^T \quad (2)$$

Where  $\xi_t, \eta_t$  denote the target positions and  $\dot{\xi}_t, \dot{\eta}_t$  denote the velocities in x and y axes respectively. In section 2.1, we have mentioned that the emitted acoustic power  $s_k(t)$  of  $k^{th}$  target varies very slowly, but it is not suitable to consider it as a constant during a long time. So we introduce  $s_k(t)$  into the target state vector, and equation (2) can be modified as:

$$x_t = (\xi_t, \eta_t, \dot{\xi}_t, \dot{\eta}_t, s_t)^T \quad (3)$$

For linear case, the maneuvering target dynamic can be expressed by:

$$x_{t+1} = F_t x_t + G_t w_t \quad (4)$$

Where  $F_t$  is state transition matrix, and  $G_t$  is noise matrix.  $F_t$  and  $G_t$  are defined according to available priori knowledge such as the target type or terrain information. More models may tackle the true target motion mode better, but also will result in more complicated computation. In this paper, we assume that the target is making a double-turn maneuver. The state transition matrix is given by:

1) Constant velocity(CV) model,

$$F_t = \begin{bmatrix} 1 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

2) First coordinated turn model,

$$F_t = \begin{bmatrix} 1 & 0 & \sin(\omega T)/\omega & (\cos(\omega T)-1)/\omega & 0 \\ 0 & 1 & (1-\cos(\omega T))/\omega & \sin(\omega T) & 0 \\ 0 & 0 & \cos(\omega T) & -\sin(\omega T)/\omega & 0 \\ 0 & 0 & \sin(\omega T) & \cos(\omega T) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

3) Second coordinated turn model,

$$F_t = \begin{bmatrix} 1 & 0 & \sin(\omega T)/\omega & (\cos(\omega T)-1)/\omega & 0 \\ 0 & 1 & (1-\cos(\omega T))/\omega & \sin(\omega T) & 0 \\ 0 & 0 & \cos(\omega T) & -\sin(\omega T)/\omega & 0 \\ 0 & 0 & -\sin(\omega T) & \cos(\omega T) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Here  $\omega$  denotes the turn rate in radians per second.

## 3. MULTI-MODEL RAO-BLACKWELLISED PARTICLE FILTER

The RBPF is a clever combination of generic PF and standard Kalman filter, which can be used when the system model contains a linear substructure, subject to Gaussian noise. The RBPF is implicitly suitable for most of target tracking problems which have linear dynamic with a nonlinear measurement relation.

Using the notation  $x_t^p$  for the states that are estimated using the PF and  $x_t^k$  for the states that are estimated using the KF, the system dynamic described in section 2 can be partitioned as:

$$\begin{cases} x_{t+1}^p = A_t^p x_t^p + A_t^k x_t^k + w_t^p, & w_t^p \in N(0, Q_t^p) \\ x_{t+1}^k = F_t^p x_t^p + F_t^k x_t^k + w_t^k, & w_t^k \in N(0, Q_t^k) \\ y_t = h_t(x_t^p) + v_t, & v_t \in N(0, R_t) \end{cases} \quad (8)$$

The total target state vector is  $x_t = (x_t^p, x_t^k)^T$ . All noise signals are considered white and independent. The generic RBPF algorithm is summarized in [7]. From equation (1) we know that the measurement equation is only the function of targets's positions and the acoustic power, so we select the state-vector partition as  $x_t^p = (\xi_t, \eta_t, s_t)^T$  and  $x_t^k = (\dot{\xi}_t, \dot{\eta}_t)^T$ . The dimension of state vector estimated by PF is reduced to 3, this is considerable when there are several targets or the number of particles used in algorithm is very large.

The target may do many types of maneuvers which are unobservable when passing the sensor field. Here we propose a multi-model Rao-Blackwellised particle filter(MRBPF), which samples particles from several models belonging to a system

model set and mixes the result of each model. The model probabilities are also updated simultaneously.

Now, we redefine the system dynamic of this form:

$$x_{t+1} = f(x_t, \mu_t^j, w_t) \quad (9a)$$

$$y_t = h(x_t, \mu_t^j, v_t) \quad (9b)$$

Where  $\mu_t^j \in M_s$  denotes that model  $\mu_t^j$  is in effect at time step  $t$ ,  $M_s$  is the mode space.  $N_s$  is the number of models in  $M_s$ . The mode transition is defined by a first homogeneous Markov chain[9],

$$P_{ij} \{ \mu_{t+1}^j | \mu_t^i \} = p_{ij}, \quad \forall i, j \in N_s \quad (10)$$

Where  $p_{ij}$  is the Markov transition probability from model  $i$  to model  $j$ .

For maneuvering target tracking, a crucial problem is to estimate the system model  $\mu_t^j$ , when applying the PF for system state estimation, it is natural to combine the model transition probabilities with distribution of particles which represents the PDF of the system state. The proposed MDRBPF algorithm is summarized in Table 1.

**Table 1.** The proposed MDRBPF algorithm

1. Initialization:
 

Let time step  $t=0$ , For  $i=1, \dots, N$  ( $N$  is the number of particles used),  $x_{0-1}^{p,(i)} \sim p_{x_0^p}(x_0^p)$ , and set  $\{x_{0-1}^{k,(i)}, P_{0-1}^i\} = \{x_0^k, \bar{P}_0\}$ ; For  $k=1, \dots, N_s$ , set the initial model probability  $p(\mu_0^k | \mathbb{Y}_0) = p(\mu_k)$ .
2. Prediction step:
  - a) For  $i=1, \dots, N$ , compute  $x_{t+1}^{*p,(i)} = f(x_t^{*p,(i)}, \mu_t^i, w_t)$ , where  $\mu_t^i$  is a sample drawn from the system model set  $M_s$  with distribution  $\{p(\mu_t^j | \mathbb{Y}_t)\}_{j=1, \dots, N_s}$  and  $w_t$  is a sample drawn from the known noise PDF. Then, for  $k=1, \dots, N_s$ , compute  $N_{\mu_t^k} = N \cdot p(\mu_t^k | \mathbb{Y}_t)$  and  $\bar{x}_{t+1}^{*p,(k)} = E\{x_{t+1}^{*p,(k)}\}$ , where  $N_{\mu_t^k}$  is the number of particles drawn from model  $\mu_t^k$ .
  - b) For  $k=1, \dots, N_s$ , compute the posterior model probabilities  $p(\mu_t^k | \mathbb{Y}_{t+1}) = \sum_{j=1}^{N_s} p(y_{t+1} | \bar{x}_{t+1}^{*p,(k)}) \gamma_t^{k,j} / C$ , where  $C$  is the normalizing factor.
  - c) Using the posterior model probabilities to predict the particles again. Namely,  $x_{t+1|t}^{p,(i)} = f(x_{t|t}^{p,(i)}, \mu_t^i, w_t)$ ,  $\mu_t^i$  is sampled with distribution  $\{p(\mu_t^j | \mathbb{Y}_{t+1})\}_{j=1, \dots, N_s}$ .
  - d) The KF measurement update step.
3. For  $i=1, \dots, N$ , Compute the importance weights  $\tilde{\gamma}_{t+1}^{(i)} = p(y_{t+1} | x_{t+1|t}^{p,(i)}, \mathbb{Y}_t)$ .  $p(y_{t+1} | x_{t+1|t}^{p,(i)}, \mathbb{Y}_t)$  is chosen as

likelihood function. Normalize  $\tilde{\gamma}_{t+1}^{(i)}$  by

$$\gamma_{t+1}^{(i)} = \tilde{\gamma}_{t+1}^{(i)} / \sum_{i=1}^N \tilde{\gamma}_{t+1}^{(i)}$$

4. For  $k=1, \dots, N_s$ , update the probability of the system model at time step  $t+1$ ,

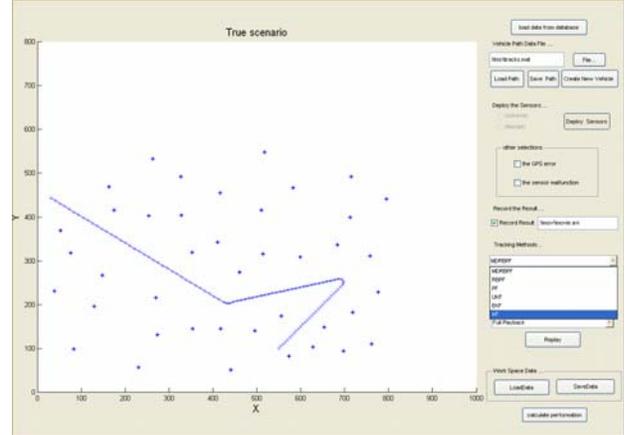
$$p(\mu_{t+1}^k | \mathbb{Y}_{t+1}) \propto \sum_{j=1}^{N_s} p(\mu_{t+1}^k | \mu_t^j) p(\mu_t^j | \mathbb{Y}_{t+1}) = \sum_{j=1}^{N_s} P_{jk} p(\mu_t^j | \mathbb{Y}_{t+1})$$

and then normalize these probabilities.

5. Resampling step: Using the importance weights in step 3 to generate  $N$  new particles, according to,  $\text{Prob}(x_{t+1|t+1}^{p,(i)} = x_{t+1|t}^{p,(j)}) = \gamma_{t+1}^{(j)}$
6. The KF time update step
7. Increase  $t$  and return to step 2.

#### 4. SIMULATION RESULTS

We have developed a simulation platform for ground target tracking in WSN which is shown in Fig. 1.



**Fig. 1.** The simulation platform for target tracking in wireless sensor networks

In the simulation, we assume that a vehicle moves across the randomly deployed sensor field as shown in Fig. 1. The detection range of each sensor is 150m. The target's initial state vector is  $(550, 100, 5, 5)^T$  and the entire target moving time is 100s. Assume the target makes a double turn maneuver, the turn rate is  $45^\circ/s$  in the first turn and  $22.5^\circ/s$  in the second turn. During other time the target moves with approximately constant velocities.

In the simulation, the target number is 1, so equation (7) can be simplified as follows:

$$y_i(t) = g_i \frac{s(t-t_i)}{\|r(t) - r_i\|^\alpha} + v_i(t) \quad (11)$$

We have carried out some preliminary experiments and the average of  $\alpha$  is calculated:  $\alpha \approx 2.0929$ . Value of  $\alpha$  may have a little difference when in different environments. The results validate the hypothesis that the acoustic energy decreases approximately as the inverse of the square of the distance between the source and the sensor.

Because the sensor energy is very short, here we adopt a location-central protocol for energy saving and prolonging the lifetime of the whole sensor network. In this protocol, the chosen sensors which are responsible for tracking are four sensors which are nearest to the predicted target position at the current time step, simultaneously a processing header is created among these four sensors by appropriate strategy. The processing header is responsible for collecting the information of other three sensors, executing the target tracking algorithm and sending the results to the sink.

The target is tracked by both RBPF algorithm using the first target model and our proposed algorithm, and we use  $N_{MC} = 100$  Monte Carlo simulations for each algorithm. The results in form of the position root of mean square error(RMSE) are given in Table 2. The unit of the quantities is meter.

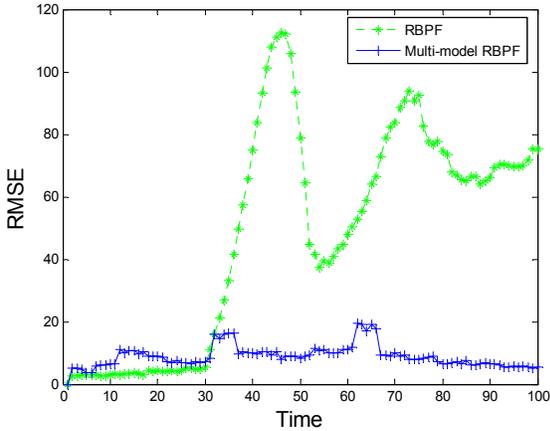
**Table 2.** RMSE for 100 Monte Carlo runs

Algorithms	RBPF	MRBPF
RMSE	59.0321	9.4187

In Fig. 2, we also give the RMSE for each time step of different algorithms, according to the following equation:

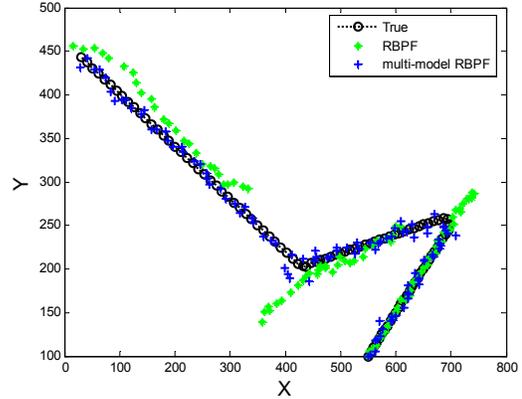
$$RMSE(t) = \sqrt{\frac{1}{N_{MC}} \sum_{j=1}^{N_{MC}} ((\hat{\zeta}_k^j - \zeta_k^{true})^2 + (\hat{\eta}_k^j - \eta_k^{true})^2)} \quad (12)$$

where  $\hat{\zeta}_k^j, \hat{\eta}_k^j$  are the filter position estimations at time step  $k$  in Monte Carlo run  $i$ .  $\zeta_k^{true}, \eta_k^{true}$  are the true position at time step  $k$ .



**Fig. 2.** The comparison of RMSE(t) for the RBPF and MDRBPF algorithms

Fig. 3 is the results extracted from one Monte Carlo run. From the simulation results we know that the performance of the RBPF is better than our algorithm when the target is moving with a constant velocity, because here all of the RBPF's particles match the true motion model very well. But when the target performs high maneuver, the RBPF could not keep up with the target and need to initialize itself to a proper state again, whereas the MDRBPF can tackle the maneuverability well.



**Fig. 3.** The true and estimated trajectory of the RBPF and MDRBPF algorithms in one Monte Carlo run

## 5. CONCLUSION

In this paper, we have briefly described the characteristics of maneuvering target tracking in acoustic sensor networks. We use acoustic energy as the feature information for tracking and a new multi-model Rao-Blackwellised particle filter is proposed. Compared with RBPF in simulation, the RMSE of the proposed method on maneuvering part of the tracking process has been shown to be markedly improved.

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