SEQUENTIAL ESTIMATION BY COMBINED COST-REFERENCE PARTICLE AND KALMAN FILTERING

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ABSTRACT

Cost-reference particle filtering (CRPF) is a methodology for recursive estimation of hidden states of dynamic systems. It is used for tracking nonlinear states when probabilistic assumptions about the state and observations noises are not made. Recently, we have proposed a CRPF algorithm for systems with conditionally linear states that combines the use of Kalman filtering for the linear states and CRPF for the nonlinear states. We have shown that this combined method yields improved results over the standard CRPF. In this paper, we further extend that approach by relaxing some of the assumptions about the noises in the system. As a result, the only statistical assumption that remains is that the noises are stationary and zero mean. We demonstrate the performance of the proposed method by computer simulations and compare it with standard CRPF, standard particle filtering (SPF), and marginalized particle filtering (MPF).

Index Terms— Cost-reference particle filtering, Kalman filtering, Rao-Blackwellization

1. INTRODUCTION

Many problems in signal processing can be stated in the form of a dynamic system

$$\mathbf{x}_t = f(\mathbf{x}_{t-1}) + \mathbf{u}_t \tag{1}$$
$$\mathbf{y}_t = h(\mathbf{x}_t) + \mathbf{v}_t, \tag{2}$$

where the subscript t represents time index, $\mathbf{x}_t$ is the system state vector, $\mathbf{y}_t$ is a vector of measurements, $\mathbf{u}_t$ and $\mathbf{v}_t$ are state and observation noises, respectively, $f(\cdot)$ is the state transition function and $h(\cdot)$ is the observation function. The standard problem is to recursively estimate the unknown state $\mathbf{x}_t$ from the noise-corrupted observations $\mathbf{y}_t$. From a Bayesian point of view, the objective is to estimate the a posteriori density of the state $p(\mathbf{x}_t|\mathbf{y}_{1:t})$ (the notation $\mathbf{y}_{1:t}$ means the set of observations $\{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_t\}$).

Cost-reference particle filtering (CRPF) [1] is a methodology that is implemented in a similar manner as standard particle filtering (SPF). CRPF is also based on the use of particles drawn from the space of the unknown states, but instead of weights that correspond to probability masses (as in SPF), they have costs that quantify the values of the particles they are associated with. The cost function used by the filter is user-defined. A major difference between the two methods is that CRPF does *not* require any probability distribution assumptions about the state or observation noise processes. A comparison of CRPF and SPF has already been discussed and some possible simplifications of CRPF have been proposed in [2].

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There are dynamic systems where some of the state variables are conditionally linear on the remaining (nonlinear) states. It is well-known that for linear systems the Kalman filtering method is optimal [3]. This allows for improvements of the particle filtering method when it is applied on a system with conditionally linear states. The method that exploits the linear substructure is known as Rao-Blackwellized (RB) particle filter [4]. Rao-Blackwellization is a statistical procedure that is used to reduce the variance of Monte Carlo importance sampling [5]. The RB particle filtering, also known as marginalized particle filtering (MPF) [6], is a combination of the SPF and Kalman filtering.

The idea of Rao-Blackwellization has also been adopted for CRPF [7]. There one exploits the special structure of the dynamic system model by combining CRPF with Kalman filtering. Similarly to the MPF, the standard CRPF is carried out for the nonlinear states, whereas the linear states are estimated by Kalman filtering. Therefore, the conditionally linear states are estimated optimally. In [7], the combined CRPF with Kalman filtering requires knowledge of the means of the noises *and* some of the second moments of the noises in the system. In this paper, we remove the assumption of knowing the second moments of the noises. In absence of their values, we estimate them simultaneously with the rest of the states by following [8].

The rest of the paper is organized as follows. The problem is formulated in Section 2. In Section 3, we briefly review the RB scheme and the basics of the CRPF algorithm. In Section 4, the proposed combination of CRPF and Kalman filtering is discussed. Simulation results are shown in Section 5. Finally, conclusions are given in Section 6.

2. PROBLEM STATEMENT

For studying systems with conditional linearity, (1)-(2) can be presented as

$$\mathbf{x}_t^n = f_t^n(\mathbf{x}_{t-1}^n) + \mathbf{A}_t^n(\mathbf{x}_{t-1}^n)\mathbf{x}_{t-1}^l + \mathbf{u}_t^n \tag{3}$$
$$\mathbf{x}_t^l = f_t^l(\mathbf{x}_{t-1}^n) + \mathbf{A}_t^l(\mathbf{x}_{t-1}^n)\mathbf{x}_{t-1}^l + \mathbf{u}_t^l \tag{4}$$
$$\mathbf{y}_t = h_t^n(\mathbf{x}_t^n) + \mathbf{B}_t(\mathbf{x}_t^n)\mathbf{x}_t^l + \mathbf{v}_t, \tag{5}$$

where $f_t^n(\cdot)$ and $f_t^l(\cdot)$ are nonlinear state transition functions; $\mathbf{A}_t^n$ and $\mathbf{A}_t^l$ are matrices, whose entries may depend on the nonlinear states; $\mathbf{u}_t^n$ and $\mathbf{u}_t^l$ are state noise vectors; $h_t^n(\cdot)$ is the nonlinear observation function; and $\mathbf{B}_t$ is another matrix whose entries may depend on the nonlinear states. Unlike in the assumptions for

SPF, the noise distributions of $\mathbf{u}_t$ and $\mathbf{v}_t$ here are *unknown*. We only assume that the noises have zero means. Note that in [7], an additional assumption was made about some of the second moments of the noises in the dynamic system.

3. BACKGROUND

3.1. Rao-Blackwellization Algorithm

Rao-Blackwellization is a marginalization technique used for reducing the variance of estimates. The idea is to use marginalization whenever possible and thereby reduce the dimension of the space of the unknowns. Recall that the objective of particle filtering is to approximate the a posteriori density $p(\mathbf{x}_t|\mathbf{y}_{1:t})$. For the system (3)-(5), we have

$$p(\mathbf{x}_t|\mathbf{y}_{1:t}) = p(\mathbf{x}_t^l|\mathbf{x}_t^n, \mathbf{y}_{1:t})p(\mathbf{x}_t^n|\mathbf{y}_{1:t}), \tag{6}$$

where $p(\mathbf{x}_t^l|\mathbf{x}_t^n, \mathbf{y}_{1:t})$ is a Gaussian density. It can be shown that this density can be tracked by Kalman filtering. The second factor, $p(\mathbf{x}_t^n|\mathbf{y}_{1:t})$, is usually intractable, and one approximates it by particle filtering. This idea was exploited in [7], where the conditionally linear states were estimated by Kalman filtering. In the next subsection we briefly review CRPF.

3.2. Cost-Reference Particle Filtering

CRPF is a filtering method that does not rely on probabilistic information. The objective of CRPF is the online estimation of the system states from available observations. SPF uses particles and associated weights to approximate the posterior distribution of the states. CRPF also represents the unknown states by a discrete random measure that contains particles, but instead of weights it associates *costs* with the particles. The costs provide a measure of quality of the particles in that lower cost-values indicate better particles, that is, particles closer to the true value of the state and vice versa. Thus, the random measure is defined by

$$\chi_t = \left\{\mathbf{x}_{0:t}^{(i)}, \mathcal{C}_t^{(i)}\right\}_{i=1}^M \tag{7}$$

where $\mathbf{x}_{0:t}^{(i)}$ is the $i-$th particle stream, $\mathcal{C}_t^{(i)}$ is the corresponding cost, and M is the total number of particles. The cost function has a recursive additive structure, i.e.,

$$\mathcal{C}_t = \lambda\mathcal{C}_{t-1} + \Delta\mathcal{C}_t(\mathbf{x}_t|\mathbf{y}_t) \tag{8}$$

where λ is a forgetting factor ($0 \leq \lambda \leq 1$), and $\Delta\mathcal{C}(\mathbf{x}_t|\mathbf{y}_t)$ is an incremental cost. The forgetting factor λ controls the amount of contribution to the cost function by old particles. The incremental cost at time t depends only on the particles and observation at time instant t.

Another function, called *risk*, is introduced as a measure of adequacy of the state at time $t-1$ given the new observation $\mathbf{y}_t$. It is given by

$$\mathcal{R}_t = \lambda\mathcal{C}_{t-1} + \mathcal{R}_t(\mathbf{x}_{t-1}|\mathbf{y}_t) \tag{9}$$

where $\mathcal{R}_t(\mathbf{x}_{t-1}|\mathbf{y}_t) = \Delta\mathcal{C}_t(f(\mathbf{x}_{t-1})|\mathbf{y}_t)$. The risk function can be viewed as a prediction of the cost increment, which can be obtained before $\mathbf{x}_t$ is actually propagated.

One important issue of CRPF (as in SPF) is the resampling of the particles. In the original CRPF, it was proposed that a probability mass function (pmf) is generated according to

$$\pi_t \sim \mu(\mathcal{C}_t) \tag{10}$$

where $\mu : \mathbb{R} \to [0, +\infty)$ is a monotonically decreasing function. This function assigns high weights to low cost particles and low weights to high cost particles. Once, the pmf is obtained, one can resort to a standard resampling procedure. Recently, a simplified CRPF was proposed which replaces the standard resampling by a sorting scheme. The justification for this is that since CRPF does not approximate distributions, the selection of promising particles can be achieved with simpler schemes. For example, at time instant t, we simply compute and sort the risks, remove the $M-(M/N)$ worst particles ($N = 2, 3, ...$), and replicate the surviving particles N times. By doing this, we avoid calculating the pmf and drawing samples from the so obtained pmf.

Finally, CRPF also requires generation of new of particles. In CRPF, we propagate particles using an appropriate distribution, usually a Gaussian probability density function (pdf) (other pdfs can also be employed).

In summary, CRPF is implemented as follows. Particles are first initialized and assigned zero costs and then at each time instant t, the following steps are executed:

1. **Particle selection**: Compute the risks according to (9) for $i = 1, 2, \cdots, M$. The particles are resampled according to some pmf, or sorted, in order to keep the promising particles (the ones with low costs). The obtained particle set is
$$\tilde{\chi}_{t-1} = \left\{\tilde{\mathbf{x}}_{t-1}^{(i)}, \tilde{\mathcal{C}}_{t-1}^{(i)}\right\}_{i=1}^M.$$
2. **Particle propagation**: Propagate each particle by
$$\mathbf{x}_t^{(i)} \sim p_t(\mathbf{x}_t|\tilde{\mathbf{x}}_{t-1}^{(i)})$$
where $p_t(\cdot)$ is an appropriate pdf for which $E(\mathbf{x}_t|\tilde{\mathbf{x}}_{t-1}^{(i)}) = f(\tilde{\mathbf{x}}_{t-1}^{(i)})$. Then the corresponding costs are evaluated by
$$\mathcal{C}_t^{(i)} = \lambda\tilde{\mathcal{C}}_{t-1}^{(i)} + \Delta\mathcal{C}_t(\mathbf{x}_t^{(i)}|\mathbf{y}_t).$$
3. **State estimation**: One possible estimate is the mean-square error (MSE) estimate. Compute the pmfs corresponding to the costs by
$$\pi_t^{(i)} \propto \mu(\mathcal{C}_t^{(i)}).$$
With this pmf, we can compute the state estimate as
$$\hat{\mathbf{x}}_t = \sum_{i=1}^M \mathbf{x}_t^{(i)}\pi_t^{(i)}.$$
Other estimation methods that do not require calculation of the pmf can also be used.

4. CRPF COMBINED WITH KALMAN FILTERING

For the system (3)-(5), we propose an algorithm that combines CRPF with Kalman filtering. With the proposed method, we estimate the nonlinear states with CRPF and the conditionally linear ones with Kalman filtering. We only assume that the noise processes have zero-means.

First, let a linear system be given by

$$\begin{aligned}\mathbf{x}_t &= \mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{u}_t \\ \mathbf{y}_t &= \mathbf{B}_t\mathbf{x}_t + \mathbf{v}_t,\end{aligned}$$

where the means of the noises are zero, and the covariance matrices of $\mathbf{u}_t$ and $\mathbf{v}_t$ are $\mathbf{Q}_u$ and $\mathbf{Q}_v$, respectively. Then the standard

Kalman filter updates the state vector and the error covariance matrix sequentially according to

$$\text{State propagation:} \left\{ \begin{array}{l} \hat{\mathbf{x}}_{t|t-1} = \mathbf{A}_t \hat{\mathbf{x}}_{t-1} \\ \mathbf{P}_{t|t-1} = \mathbf{A}_t \mathbf{P}_{t-1} \mathbf{A}_t^\top + \mathbf{Q}_u \end{array} \right. \quad (11)$$

$$\text{Kalman gain: } \mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{B}_t^\top \left(\mathbf{B}_t \mathbf{P}_{t|t-1} \mathbf{B}_t^\top + \mathbf{Q}_v \right)^{-1} \quad (12)$$

$$\text{State update:} \left\{ \begin{array}{l} \hat{\mathbf{x}}_t = \hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t (\mathbf{y}_t - \mathbf{B}_t \hat{\mathbf{x}}_{t|t-1}) \\ \mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{B}_t) \mathbf{P}_{t|t-1}. \end{array} \right. \quad (13)$$

It is clear that the filter requires the knowledge of the noise covariances $\mathbf{Q}_u$ and $\mathbf{Q}_v$. However, a modified version of the Kalman filter can be used where the unknown statistical variables can be estimated simultaneously with the system state and error covariance matrix [8]. We refer to this methodology as to adaptive Kalman filtering (AKF).

An unbiased estimator of $\mathbf{Q}_v$ is obtained by first numerically estimating the covariance of the observation residual,

$$\mathbf{S}_{r_t} = \frac{1}{t-1} \sum_{k=1}^{t} (\mathbf{r}_k - \bar{\mathbf{r}})(\mathbf{r}_k - \bar{\mathbf{r}})^\top, \quad (14)$$

where $\mathbf{r}_t = \mathbf{y}_t - \mathbf{B}_t \hat{\mathbf{x}}_{t|t-1}$, and $\bar{\mathbf{r}} = \frac{1}{t} \sum_{k=1}^{t} \mathbf{r}_k$, and then estimating $\mathbf{Q}_v$ by

$$\hat{\mathbf{Q}}_{v,t} = \mathbf{S}_{r_t} - \frac{1}{t} \sum_{k=1}^{t} \mathbf{B}_k \mathbf{P}_{k|k-1} \mathbf{B}_k^\top. \quad (15)$$

Similarly, the state noise covariance can be estimated by

$$\hat{\mathbf{Q}}_{u,t} = \mathbf{S}_{q_t} - \frac{1}{t} \sum_{k=1}^{t} (\mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^\top - \mathbf{P}_k) \quad (16)$$

$$\mathbf{S}_{q_t} = \frac{1}{t-1} \sum_{k=1}^{t} (\mathbf{q}_k - \bar{\mathbf{q}})(\mathbf{q}_k - \bar{\mathbf{q}})^\top, \quad (17)$$

where $\mathbf{q}_t = \hat{\mathbf{x}}_t - \mathbf{A}_t \hat{\mathbf{x}}_{t-1}$, and $\bar{\mathbf{q}} = \frac{1}{t} \sum_{k=1}^{t} \mathbf{q}_k$.

AKF starts with some initial value of $\mathbf{x}_0$, $\mathbf{P}_0$, $\hat{\mathbf{Q}}_{u,0}$, $\hat{\mathbf{Q}}_{v,0}$, $\mathbf{S}_{r_0}$, $\mathbf{S}'_{r_0}$, $\mathbf{S}_{q_0}$, and $\mathbf{S}'_{q_0}$, updates them recursively, and substitutes them into (11)-(13). The steps of AKF are outlined in Table 1.

Now we return to our system (3)-(5). As already suggested, we apply CRPF for estimating the nonlinear states, and since we do not know the noise covariances, we use AKF for estimating the conditionally linear states. The new algorithm can be implemented as follows:

1. **Particle selection**: Predict the states according to

$$\begin{aligned} \breve{\mathbf{x}}_t^{n,(i)} &= f_t^n(\mathbf{x}_{t-1}^{n,(i)}) + \mathbf{A}_t^n(\mathbf{x}_{t-1}^{n,(i)}) \mathbf{x}_{t-1}^{l,(i)} \\ \breve{\mathbf{x}}_t^{l,(i)} &= f_t^l(\mathbf{x}_{t-1}^{n,(i)}) + \mathbf{A}_t^l(\mathbf{x}_{t-1}^{n,(i)}) \mathbf{x}_{t-1}^{l,(i)}, \end{aligned}$$

and compute the risks of the particles $\breve{\mathbf{x}}_t^{(i)}$ by (9). Sort all the risks in ascending order, and select the first M/N particles (those with lowest risks) and replicate them N times; discard the rest of the particles. Let the new set of particles be denoted by $\tilde{\mathbf{x}}_{t-1}^{(i)}$.

2. **Particle propagation of nonlinear states**: Propagate the nonlinear states by using an appropriate pdf, for example, a Gaussian,

$$\mathbf{x}_t^{n,(i)} \sim p_t(\mathbf{x}_t^n | \tilde{\mathbf{x}}_{t-1}^{(i)}).$$

State propagation: $\hat{\mathbf{x}}_{t\|t-1} = \mathbf{A}_t \hat{\mathbf{x}}_{t-1}$ $\mathbf{P}_{t\|t-1} = \mathbf{A}_t \mathbf{P}_{t-1} \mathbf{A}_t^\top + \hat{\mathbf{Q}}_{u,t-1}$ *Observation noise*: $\mathbf{r}_t = \mathbf{y}_t - \mathbf{B}_t \hat{\mathbf{x}}_{t\|t-1}, \quad \bar{\mathbf{r}}_t = \frac{t-1}{t} \bar{\mathbf{r}}_{t-1} + \frac{1}{t} \mathbf{r}_t$ $\mathbf{S}_{r_t} = \frac{t-1}{t} \mathbf{S}_{r_{t-1}} + \frac{1}{t} (\mathbf{r}_t - \bar{\mathbf{r}}_t)(\mathbf{r}_t - \bar{\mathbf{r}}_t)^\top$ $\mathbf{S}'_{r_t} = \frac{t-1}{t} \mathbf{S}'_{r_{t-1}} + \frac{1}{t} \mathbf{B}_t \mathbf{P}_{t\|t-1} \mathbf{B}_t^\top$ $\hat{\mathbf{Q}}_{v,t} = \mathbf{S}_{r_t} - \mathbf{S}'_{r_t}$ *Kalman gain*: $\mathbf{K}_t = \mathbf{P}_{t\|t-1} \mathbf{B}_t^\top \left(\mathbf{B}_t \mathbf{P}_{t\|t-1} \mathbf{B}_t^\top + \hat{\mathbf{Q}}_{v,t} \right)^{-1}$ *State update*: $\hat{\mathbf{x}}_t = \hat{\mathbf{x}}_{t\|t-1} + \mathbf{K}_t \mathbf{r}_t$ $\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{B}_t) \mathbf{P}_{t\|t-1}$ *State noise*: $\mathbf{q}_t = \hat{\mathbf{x}}_t - \mathbf{A}_t \hat{\mathbf{x}}_{t-1}, \quad \bar{\mathbf{q}}_t = \frac{t-1}{t} \bar{\mathbf{q}}_{t-1} + \frac{1}{t} \mathbf{q}_t$ $\mathbf{S}_{q_t} = \frac{t-1}{t} \mathbf{S}_{q_{t-1}} + \frac{1}{t} (\mathbf{q}_t - \bar{\mathbf{q}}_t)(\mathbf{q}_t - \bar{\mathbf{q}}_t)^\top$ $\mathbf{S}'_{q_t} = \frac{t-1}{t} \mathbf{S}'_{q_{t-1}} + \frac{1}{t} \left(\mathbf{A}_t \mathbf{P}_{t-1} \mathbf{A}_t^\top - \mathbf{P}_t \right)$ $\hat{\mathbf{Q}}_t = \mathbf{S}_{q_t} - \mathbf{S}'_{q_t}$

Table 1. Adaptive Kalman filtering

3. **Estimation of the linear states**: Given the nonlinear state $\mathbf{x}_t^{n,(i)}$, estimate the linear states $\mathbf{x}_t^{l,(i)}$ by AKF (Table 1).

4. **Estimation of the state**: First compute the costs of particles $\mathcal{C}_t^{(i)}$, then obtain the weights of particles by $\pi_t^{(i)} \propto \mu(\mathcal{C}_t^{(i)})$. The MSE estimate is obtained by

$$\hat{\mathbf{x}}_t = \sum_{i=1}^{M} \mathbf{x}_t^{(i)} \pi_t^{(i)}.$$

5. COMPUTER SIMULATIONS

In this section, we present computer simulations that illustrate the validity of our proposed method. We considered a four-dimensional state $\mathbf{x}_t = [x_{1,t}\ x_{2,t}\ x_{3,t}\ x_{4,t}]^\top$, which followed a random walk scheme. Given $x_{4,t}$, the observation vector $\mathbf{y}_t$ was a linear function of the first three components $[x_{1,t}\ x_{2,t}\ x_{3,t}]$, i.e.,

$$\begin{aligned} \mathbf{x}_t &= \mathbf{x}_{t-1} + \mathbf{u}_t \\ \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \\ y_{4,t} \end{bmatrix} &= \begin{bmatrix} x_{1,t} x_{4,t} \\ x_{2,t} x_{4,t} \\ x_{3,t} x_{4,t} \\ x_{4,t}^3 \end{bmatrix} + \mathbf{v}_t, \end{aligned}$$

where $\mathbf{u}_t$ and $\mathbf{v}_t$ were independent Gaussian noise processes. Therefore, the state was considered to contain a nonlinear part, $\mathbf{x}_t^n = [x_{4,t}]$, and a linear part, $\mathbf{x}_t^l = [x_{1,t}\ x_{2,t}\ x_{3,t}]^\top$. In the simulations, we set $T = 200$, $\mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_u)$, $\mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_v)$, with $\mathbf{Q}_u = 4\mathbf{I}$, and $\mathbf{Q}_v = diag\{4, 4, 4, 9\}$.

We applied the standard CRPF and the proposed combination of CRPF and AKF. The incremental cost function was specified by

$$\Delta \mathcal{C}_t(\mathbf{x}_t | \mathbf{y}_t) = \left\| \mathbf{y}_t - h_t(\mathbf{x}_t^n) - \mathbf{B}_t(\mathbf{x}_t^n) \mathbf{x}_t^l \right\|^2$$

and $\lambda = 0$. Once the risks of the particles were computed, they were ranked in ascending order, and one half of the particles with lowest risks were replicated ($N = 2$). The surviving particles were propagated according to

$$\mathbf{x}_t^{(i)} \sim \mathcal{N}\left(\tilde{\mathbf{x}}_{t-1}^{(i)}, \sigma_0^2 \mathbf{I}\right)$$

where σ_0^2 can be estimated as proposed in [1]. Finally, the states were estimated by computing a pmf based on the obtained cost

$$\pi_t^{(i)} \propto \mu(\mathcal{C}_t^{(i)}) = \frac{1}{(\mathcal{C}_t^{(i)} - \min_k(\mathcal{C}_t^{(k)}) + \delta)^{\beta}},$$

where $\delta = 0.1$ and $\beta = 2$.

For comparison purposes, we also implemented the SPF and the MPF as proposed in [6]. Table 2 summarizes the details of the MPF algorithm. All the filters were run with $M = 100$ particles. The performance of the algorithms was measured in terms

Initialization
For $i = 1 \cdots M$
$\mathbf{x}_0^{n,(i)} \sim p_0(\mathbf{x}_0^n)$, set $\{\mathbf{x}_0^{l,(i)}, \mathbf{P}_0^{(i)}\}$
Recursive update
For $t = 1$ to T, for $i = 1 \cdots M$
Nonlinear prediction $\tilde{\mathbf{x}}_t^{(i)} \sim p(\mathbf{x}_t^n | \mathbf{x}_{t-1}^{(i)}, \mathbf{y}_{t-1})$
Linear prediction
$\tilde{\mathbf{x}}_{t|t-1}^{l,(i)} = f_t^l(\mathbf{x}_{t-1}^{n,(i)}) + \mathbf{A}_t^{l,(i)} \mathbf{x}_{t-1}^{l,(i)}$
$\tilde{\mathbf{P}}_{t|t-1}^{(i)} = \mathbf{A}_t^{l,(i)} \mathbf{P}_{t-1}^{(i)} (\mathbf{A}_t^{l,(i)})^\top + \mathbf{Q}_u$
Calculate weight $w_t^{(i)} = p(\mathbf{y}_t | \tilde{\mathbf{x}}_t^{n,(i)}, \tilde{\mathbf{x}}_{t|t-1}^{l,(i)})$
Resample to obtain
$\left\{\mathbf{x}_t^{n,(i)}, \mathbf{x}_{t|t-1}^{l,(i)}, \mathbf{P}_{t|t-1}^{(i)}, w_t^{(i)} = \frac{1}{M}\right\}_{i=1}^{M}$
Linear update by Kalman filter
$\mathbf{K}_t^{(i)} = \mathbf{P}_{t|t-1}^{(i)} (\mathbf{B}_t^{(i)})^\top \left(\mathbf{B}_t^{(i)} \mathbf{P}_{t|t-1}^{(i)} (\mathbf{B}_t^{(i)})^\top + \mathbf{Q}_v\right)^{-1}$
$\mathbf{x}_t^{l,(i)} = \mathbf{x}_{t|t-1}^{l,(i)} + \mathbf{K}_t^{(i)} \left(\mathbf{y}_t - h_t^n(\mathbf{x}_t^{n,(i)}) - \mathbf{B}_t^{(i)} \mathbf{x}_{t|t-1}^{l,(i)}\right)$
$\mathbf{P}_t^{(i)} = \mathbf{P}_{t|t-1}^{(i)} - \mathbf{K}_t^{(i)} \mathbf{B}_t^{(i)} \mathbf{P}_{t|t-1}^{(i)}$
State estimation
$\hat{\mathbf{x}}_t = \sum_{i=1}^{M} \mathbf{x}_t^{(i)} w_t^{(i)}$

Table 2. Marginalized particle filtering algorithm.

of the root mean square (RMS) errors of both the nonlinear and conditionally linear states. The RMS was obtained by averaging over 500 independent simulation trials, i.e.,

$$RMS_t^n = \sqrt{\frac{1}{500} \sum_{k=1}^{500} \|\hat{\mathbf{x}}_{k,t}^n - \mathbf{x}_{k,t}^n\|^2},$$

$$RMS_t^l = \sqrt{\frac{1}{500} \sum_{k=1}^{500} \|\hat{\mathbf{x}}_{k,t}^l - \mathbf{x}_{k,t}^l\|^2},$$

where $\mathbf{x}_{k,t}$ was the true state at time t in the k-th run, and $\hat{\mathbf{x}}_{k,t}$ was the corresponding estimate obtained by the filter.

Figure 1 shows the RMS errors of all the compared methods. It is clear from the plots that the performances of the proposed CRPF (noted as K-CRPF in the figure) and the MPF were much better than the original CRPF and SPF for the conditionally linear state. All the methods, however, performed similarly for the nonlinear state. It is also important to note that the newly proposed CRPF had similar performance as the MPF.

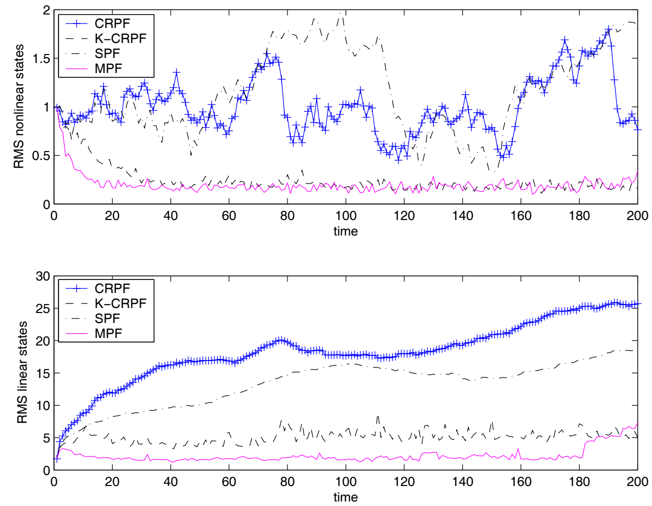

Fig. 1. RMS error comparison

6. CONCLUSIONS

In this paper we present a tracking method for dynamic systems with conditionally linear states that combines cost-reference particle filtering and adaptive Kalman filtering. The new method does not make any assumptions about the noises in the system except that they are zero mean and stationary. The proposed method resembles the Rao-Blackwellized particle filter from standard particle filtering theory. Simulation results show its improved performance over the standard cost-reference particle filter.

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