ESTIMATION OF NEAR-FIELD PARAMETERS USING SPATIAL TIME-FREQUENCY DISTRIBUTIONS

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ABSTRACT

This work deals with the estimation of near-field parameters using passive sensor arrays. A transformation of the array data is proposed which allows the extraction of near-field time-frequency signatures from data containing a mixture of far- and near-field sources. Spatial time-frequency distribution matrices are then used as a means for solving the nearfield parameter estimation problem. The estimation accuracy of the proposed approach is compared to existing methods via simulation analysis. An experimental validation of theoretical ideas is also presented.

Index Terms— Array signal processing, time-frequency analysis, direction of arrival estimation, radar measurements.

1. INTRODUCTION

As discussed in [1], both near-field (NF) and far-field (FF) scatterers may be present in certain applications such as sur-face-wave radar. In such cases, estimation methods which assume only a given class (FF or NF) of scatterers are present mis-model the data and subsequently result in sub-optimal or erroneous estimation of the parameters of interest.

Characterization of NF scatterers in the presence of FF sources was considered in [1, 2], using a quadratic sensorangle distribution (SAD). The work in [1] assumes that a NF scatterer is illuminated by a cooperative FF source of known DOA. Characterization of the NF scatterer requires one to first suppress the FF source using subspace projection techniques. Parameter estimation using the SAD, and the issue of aliasing, are discussed in [2]. It is noted that the SAD was formulated based on the assumption of a uniform linear array (ULA) and a large number of sensors.

In this paper, a means for distinguishing NF and FF sources is proposed, based on time-frequency analysis. The proposed approach is not restricted to a ULA geometry or large number or sensors and the DOA of FF sources need not be known *a priori*. Estimation of NF parameters is proposed

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based on spatial time-frequency distribution (STFD) matrices, which have previously been applied to both blind source separation [3] and direction-of-arrival (DOA) estimation [4] problems.

2. SIGNAL MODEL

We consider an array of M sensors observing narrowband signals, of which K are in the NF and L in the FF of the array. The baseband signal model is given by

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{b}(r_k, \phi_k) s_k(t) + \sum_{l=1}^{L} \boldsymbol{a}(\theta_l) z_l(t) + \boldsymbol{v}(t), \quad (1)$$

where $\boldsymbol{x}(t)$ is the vector of sensor outputs at time t, $\{s_k(t)\}$ are the NF source signals, $\{z_l(t)\}$ are the FF source signals and $\boldsymbol{v}(t)$ is an additive noise process. The vector $\boldsymbol{b}(r, \phi)$ denotes the array response to a NF source at range r and angle ϕ with respect to the reference sensor. The vector $\boldsymbol{a}(\theta)$ denotes the array response to a FF source at angle θ . For convenience, we denote the array response to the kth NF source as \boldsymbol{b}_k and to the lth FF source as \boldsymbol{a}_l .

It is assumed that the sources $\{s_k(t)\}$ and $\{z_l(t)\}$ are FM signals and that the noise is a complex white process of variance σ_v^2 . N observations, $\{\boldsymbol{x}[n]\}_{n=0}^{N-1}$, are collected for estimation.

3. NEAR-FIELD PARAMETER ESTIMATION

In this Section, we investigate the use of STFD matrices, for the purpose of NF parameter estimation. The STFD matrix of the sensor data is defined as:

$$\boldsymbol{D}_{\boldsymbol{x}\boldsymbol{x}}[n,\omega) = \sum_{l=0}^{N-1} \{\varphi[n,l] *_n [\boldsymbol{x}[n+l]\boldsymbol{x}^{\mathsf{H}}[n-l]] \} \mathrm{e}^{-\mathrm{j}2\omega l},$$
(2)

whose elements are the auto- and cross-TFDs¹ of the sensor data.

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 $^{{}^{1}\}varphi$ is a kernel function defining the distribution [5] and $*_{n}$ denotes convolution w.r.t. n.

Let us assume the sources of interest have a well-defined TF structure, such as constant amplitude FM signals, whose TF signatures are distinct from one another. By averaging Equation (2) across the TF signature of a given source, one is able to isolate the signal energy from that component alone, and perform parameter estimation without the influence of the other signals. This property has been investigated for FF direction-finding in [4].

Consider that the kth NF source has an instantaneous frequency (IF), denoted $\omega_{i,k}(t)$, for $k = 1, \ldots, K$. Calculating the averaged STFD for source k yields:

$$D_{k} = \frac{1}{N} \sum_{n=0}^{N-1} D_{xx}[n, \omega_{i,k}[n])$$
(3)
$$\approx \boldsymbol{b}(r_{k}, \phi_{k}) \left[\frac{1}{N} \sum_{n=0}^{N-1} D_{s_{k}s_{k}}[n, \omega_{i,k}[n]) \right] \boldsymbol{b}^{\mathrm{H}}(r_{k}, \phi_{k})$$
+Noise term. (4)

Estimation of the NF parameters for source k may be achieved by applying traditional covariance based estimation methods to D_k , assuming only one source is present. In the following, we shall make use of both the MUSIC estimator and the beamforming approach.

The NF-MUSIC estimator is obtained from a subspace decomposition of the data covariance matrix in [6]. In this work, we apply the same approach to the averaged STFD matrix. An estimate of the range and bearing of the kth NF source is obtained according to:

$$(\hat{r}_k, \hat{\phi}_k) = \arg\max_{(r,\phi)} \frac{1}{\boldsymbol{b}^{\mathrm{H}}(r,\phi)\boldsymbol{U}_k\boldsymbol{U}_k^{\mathrm{H}}\boldsymbol{b}(r,\phi)}, \qquad (5)$$

for k = 1, ..., K. In Equation (5), U_k denotes the noise subspace which is estimated from by taking the M - 1 eigenvectors of D_k corresponding to the M - 1 smallest eigenvalues.

In many applications, the sources are not well modelled by discrete points in space, which means that use of Equation (5) is not appropriate. Instead, the beamformer spectrum may be calculated over a range of parameter values to form an "image" of the NF characteristics of the scatterers (this shall be demonstrated in the following section using experimental data). The beamformer spectrum for the averaged STFD of source k is defined as:

$$P_k(r,\phi) \triangleq \frac{\boldsymbol{b}^{\mathrm{H}}(r,\phi)\boldsymbol{D}_k\boldsymbol{b}(r,\phi)}{\boldsymbol{b}^{\mathrm{H}}(r,\phi)\boldsymbol{b}(r,\phi)}; \quad k = 1,\dots,K.$$
(6)

4. NEAR-FIELD SOURCE DISCRIMINATION

In order to apply Equation (3) for estimation, the IF of the NF sources must be known or estimated. In this section, we outline a means of discriminating the time-frequency signatures

of NF sources from a mixture of NF and FF sources. The proposed approach is based on the TFDs of the sensor data.

We propose that by subtracting the average of sensor TFDs from the TFD computed at each sensor, one may remove the TF contributions of all FF sources. This is motivated by the fact the FF sources are received with the same power at each sensor, while the NF sources are received with varying powers due to the sphericity of the wavefront. Consider the following quantity:

$$\Delta_m[n,\omega) \triangleq D_{x_m x_m}[n,\omega) - \frac{1}{M} \sum_{l=1}^M D_{x_l x_l}[n,\omega), \quad (7)$$

for m = 1, ..., M. Substitution of the model from Equation (1) into (7) yields:

$$\begin{split} \Delta_{m}[n,\omega) &= \sum_{u=1}^{K} \sum_{u=1}^{K} \left[b_{mu} b_{mv}^{*} - \frac{1}{M} \sum_{l=1}^{M} b_{lu} b_{lv}^{*} \right] D_{s_{u}s_{v}}[n,\omega) \\ &+ \sum_{u=1}^{L} \sum_{u=1}^{L} \left[a_{mu} a_{mv}^{*} - \frac{1}{M} \sum_{l=1}^{M} a_{lu} a_{lv}^{*} \right] D_{z_{u}z_{v}}[n,\omega) \\ &+ \sum_{u=1}^{K} \sum_{u=1}^{L} \left[b_{mu} a_{mv}^{*} - \frac{1}{M} \sum_{l=1}^{M} b_{lu} a_{lv}^{*} \right] D_{s_{u}z_{v}}[n,\omega) \\ &+ \sum_{u=1}^{L} \sum_{u=1}^{K} \left[a_{mu} b_{mv}^{*} - \frac{1}{M} \sum_{l=1}^{M} a_{lu} b_{lv}^{*} \right] D_{z_{u}s_{v}}[n,\omega) \\ &+ \text{Noise term}, \end{split}$$

for m = 1, ..., M, where b_{mv} and a_{mu} denote the *m*th element of b_v and a_u respectively.

Through use of a cross-term free TFD such as the spectrogram, one may consider only the auto-terms of Equation (8):

$$\Delta_{m}[n,\omega) = \sum_{u=1}^{K} \left[|b_{mu}|^{2} - \frac{1}{M} \sum_{l=1}^{M} |b_{lu}|^{2} \right] D_{s_{u}s_{u}}[n,\omega) + \sum_{u=1}^{L} \left[|a_{mu}|^{2} - \frac{1}{M} \sum_{l=1}^{M} |a_{lu}|^{2} \right] D_{z_{u}z_{u}}[n,\omega) + \text{Noise term.}$$
(9)

Under the mild assumption that each sensor of the array has the same gain, the magnitude of each element of the FF response vector is equal. The second term in Equation (9) therefore evaluates to zero and the quantity $\Delta_k[n, \omega)$ contains only the TFDs of the near-field sources.

As the weighting of NF source TFDs in Equation (9) may be negative, we propose the following "NF-TFD" for estimation of NF TF signatures:

$$B[n,\omega) \triangleq \left[\frac{1}{M} \sum_{m=1}^{M} \left(\Delta_m[n,\omega)\right)^2\right]^{\frac{1}{2}}.$$
 (10)

The TF signature of the NF sources may be estimated, e.g. by applying peak-finding or multi-component IF estimation techniques to $B[n, \omega)$.

5. SIMULATION RESULTS

The estimation accuracy of the proposed approach was evaluated via Monte Carlo simulations. In the experiment, two NF and two FF chirp signals were present. The NF sources had locations of $(r_1, \phi_1) = (0.5\lambda, 70^\circ)$ and $(r_2, \phi_2) = (5\lambda, 69^\circ)$ respectively, where λ denotes the carrier wavelength. The FF sources had directions $\theta_1 = 75^\circ$ and $\theta_2 = 110^\circ$. A uniform linear array structure of 6 elements with spacing $\lambda/2$ was used. All angles specified herein are w.r.t. the array endfire. All sources had the same power w.r.t. the noise and N = 256 observations were used. Calculation of the NF-TFD was based on a spectrogram with window length 51 samples.

The NF parameters were estimated using the NF-MUSIC algorithm. The RMSE of the estimates of range and direction of the first NF source are shown in Figure 1. For comparison, the estimation was performed using the STFD matrix when the TF signature of the source is known and when it was estimated using Equation (10). The signature was estimated by taking the location of the largest peak of $B[n, \omega)$ at each time-slice. In Figure 1, the accuracy of the MUSIC algorithm based on the covariance matrix and the CRB for estimation of a single NF source are also shown.

The results in Figure 1 indicated that STFD-based estimation is superior to covariance-based estimation at very low SNR. Unfortunately, the estimation of the NF TF signature is not successful at very low SNR where the best performance gain of the STFD-based estimation is achieved. We note, however, that in the range from 2 to 15 dB SNR the method which estimates the TF signature has higher accuracy than the covariance based method. It is expected that better techniques for estimating the TF signature from $B[n, \omega)$, such as the Hough transform [7], will allow one to apply this approach at lower SNR.

6. EXPERIMENTAL RESULTS

The proposed approaches for estimation of NF TF signatures and NF parameters have been applied to experimental data for validation of the theoretical ideas. The experimental system consists of a linear array of 16 antennas, located on the coast close to the land-sea boundary. Three targets are present in the experiment; two ships in the far-field of the array and one target in the near-field. All targets are transmitting linear FM signals of bandwidth 20 KHz and waveform repetition frequency 50 Hz, transmitted at a carrier frequency of 6.41 MHz.

The TF distribution of the signal received at the first sensor of the array and the NF-TFD of the sensor data are shown in Figures 2 and 3 respectively, which have been computed using the spectrogram with a rectangular window of length 61 samples. The NF-TFD clearly shows the TF signature of the NF source, while the TF signatures of the FF sources have been suppressed by over 15 dB. Based on the NF-TFD shown in Figure 3, the IF of the NF source is estimated as the location of the largest peak for each time-slice. Using the estimated IF, the averaged STFD for the NF source is computed according to Equation (3), using the pseudo Wigner-Ville distribution (PWVD) with rectangular window of length 61. The NF beamformer spectrum is obtained according to Equation (6) and plotted in Figure 4. For comparison, the standard beamformer, obtained by substituting the matrix D_k with the sample covariance matrix in Equation (6), is plotted in Figure 5. We note that the use of TF processing to isolate the NF source allows one to more clearly observe the NF characteristics, without the influence of the FF sources.

7. CONCLUSIONS

The use of STFD matrices for NF parameter estimation has been investigated, and shown to provide improved accuracy with respect to covariance-based estimation. It was shown that by selectively averaging the TF signature of only NF sources, one may ignore other FF sources present in the data. A means of discriminating between the NF and FF TF signatures was also proposed and successfully applied to both simulated and experimental data.

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Fig. 1. The RMSE of NF parameter estimation using the MU-SIC algorithm, for estimation of the range (top) and direction (bottom) of the reference source.



Fig. 2. The TFD of the signal received at the first sensor of the experimental system.



Fig. 3. The NF-TFD of the experimental data, showing clearly the TF signature of the NF source.



Fig. 4. The NF beamformer obtained using the averaged STFD.



Fig. 5. The NF beamformer obtained using the sample co-variance matrix.