DIRECTION FINDING OF NONSTATIONARY SIGNALS USING SPATIAL TIME-FREQUENCY DISTRIBUTIONS AND MORPHOLOGICAL IMAGE PROCESSING

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ABSTRACT

We consider the problem of direction finding for nonstationary signals impinging on an array of sensors. Making use of a time-frequency representation of the data, we are able to exploit the non-stationary nature of the source signals. We employ morphological image processing to estimate time-frequency signature segments of each source. Optional overlapping segments are splitted and recombined after direction finding. The proposed method also allows direction finding for the underdetermined case, i.e. when there are more sources than the number of array sensors.

***Index Terms*—** Time-frequency analysis, Direction of arrival estimation, Image processing

1. INTRODUCTION

Nonstationary signals such as frequency modulated signals arise in a number of fields including sonar, radar and telecommunications. These signals are well localizable in the time-frequency (TF) domain. Recently, the application of TF analysis to sensor array processing for non-stationary signals has received significant attention in the literature. The use of spatial time-frequency distribution (STFD) matrices in particular has emerged as a natural means for exploiting both the spatial diversity and TF localization properties of nonstationary sources impinging on a sensor array [1].

From subspace analysis of STFD matrices [2], it was shown that performance improvement with respect to traditional approaches is significant when separately averaging over the TF signatures of each source. Therefore we try to extract segments of TF signatures with image processing techniques, especially morphological ones such as opening, closing or thinning. The extracted IF segments can be effectively used for direction finding, by means of STFD matrix averaging, if they belong to a single source. After direction finding, multiple IF segments of the same source can be recombined according to their spatial signature.

Recently, the application of various image processing techniques has received attention in the TF and STFD literature. Peak detection and component linking has been used for IF estimation. A line detection method, commonly used in road network extraction in SAR images, has been applied to underdetermined blind source separation [3]. Another approach for blind source separation considers a vector classification approach based on the spatial structure of the signal eigenvectors [4]. Such an approach requires the eigen-decomposition of many TF matrices which is computationally demanding.

For direction finding, FM parameter estimation in the generalized Hough space has been used to obtain the source TF signature for the STFD averaging [5] and has shown to have good performance at low SNR. This approach requires the functional form of the IF to be known a priori. If the number of parameters to describe the IF is large, the approach becomes computationally intractable.

We propose to employ morphological image processing to estimate time-frequency signature segments of each source. This approach is more computationally attractive than existing methods and does not require parametric modelling of the signal IF. In Section 2, we outline the signal model used and in Section 3 briefly review the idea behind direction finding using STFDs. Section 4 discusses the proposed direction finding algorithm based on IF extraction with morphological image processing and in Section 5, simulations depicting the performance of the proposed method are included. Section 6 discusses critical issues and summarizes the important conclusions drawn from this work.

2. SIGNAL MODEL

In narrowband array processing, the baseband signal model for n signals, arriving at an m-element array is given by

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \quad t \in \mathbb{R} \tag{1}$$

Here, $\boldsymbol{A}$ is the $m \times d$ mixing matrix, $\boldsymbol{x}(t)$ is the $m \times 1$ sensor array output vector and $\boldsymbol{s}(t)$ is the $1 \times d$ source signal vector. For the problem of direction finding, the mixing matrix takes the form $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), ..., \boldsymbol{a}(\theta_d)]^T$ where $\boldsymbol{a}(\theta_k)$ is the k-th array steering vector and $\boldsymbol{\theta}$ is the parameter of interest, consisting of the source directions of arrival (DOA). The noise vector $\boldsymbol{n}(t)$ is assumed to be stationary, spatially and temporally white, zero-mean, circular, complex Gaussian.

As source signals, we consider constant amplitude FM signals of the form $s_k(t) = A_k \mathrm{e}^{j2\pi\Phi_k(t)}$ for $k = 1, ..., d$ and their corresponding instantaneous frequency (IF) given by $f_k(t) = \mathrm{d}\Phi_k(t)/(2\pi \mathrm{d}t)$ which is to be extracted (in segments) for the STFD averaging. We assume N snapshots of $\boldsymbol{x}(t)$ are available for estimation.

3. STFD MATRICES AND DIRECTION FINDING

We make use of the idea by Amin *et al.* [1] for direction finding based on a spatial TFD matrix, defined in terms of the auto- and cross-TFDs of the sensors as

$$[\boldsymbol{D}_{\boldsymbol{xx}}(t,f)]_{ij} = D_{x_i x_j}(t,f;\varphi) \tag{2}$$

where $D_{x_i x_j}(t,f;\varphi)$ is assumed to be a bilinear TFD of Cohen's class, for which the kernel function is φ. By averaging the STFD matrix over a subset of signal IF signatures, the source direction can be estimated by using subspace decomposition [1].

Our proposed approach performs IF signature estimation by means of morphological image processing. The extracted IF segments are nonoverlapping subsets of single source signals with TF points $(t_n, f_n) \in \mathcal{S}_k$ for segments $k = 1, ..., p$. For each segment, the averaged STFD matrix is computed as

$$\boldsymbol{D}_k = \frac{1}{\#\mathcal{S}_k} \sum_{n \in \mathcal{S}_k} \boldsymbol{D}_{\boldsymbol{xx}}(t_n, f_n) \tag{3}$$

where $\#\mathcal{S}_k$ is the number of TF points in $\mathcal{S}_k$ and a discrete-time TFD is used in the above.

Compared with the sample covariance matrix of the array output, the matrix in (3) provides an effective improvement in SNR by amplification of the source eigenvalues with respect to the noise eigenvalues [2]. The final estimate for $\boldsymbol{\theta}$ is obtained by successively estimating the DOAs of each segment and combining DOAs, which belong to the same source. The DOA estimation procedure is summarized in Table 1.

Details on the IF segment extraction (Step 2) is treated in the following section. Steps 3 to 5 describe the TF MUSIC direction finding. Step 7 requires the choice of a threshold γ for testing if two DOAs belong to the same source. We found an ad-hoc measure, which depends on the number of TF averaging points $\#\mathcal{S}_k$, the signal power at the selected points and the noise power σ^2. The selection of γ based on asymptotic or estimated statistical distributions will be considered in future work.

4. IF SEGMENT EXTRACTION WITH THINNING

Conventional IF extraction is based on the fact that the ridge of a TFD approximately represents the IF of a signal. A possible approach would be peak-finding then component-linking or reassignment, respectively. These methods are generally not well applicable for overlapping IFs.

Assuming that the medial axis of a thresholded, binary TFD image roughly lies on the IF of the signal, the proposed thinning approach could be applied. As we will see later, a thin-line representation of an elongated pattern, such as ridges in a TFD image, allows simple detection of critical features, like end points, junction points or connections between components.

The input of the image processing should be a TFD image as 'clean' as possible. Therefore spatial averaging and optionally TF averaging is used to reduce cross-term effects. After thresholding, opening and closing operations produce smooth objects and diminish noise spots on the TFD image. Opening and closing are both combinations of erosion and dilation, which are basic morphological image processing operations [6].

After the described 'pre-processing' thinning can be applied. For better understanding some properties of the thin-

1. Calculate the spatially averaged TFD as $D_{\boldsymbol{xx}}(t,f) = \mathsf{Tr}\left[\boldsymbol{D}_{\boldsymbol{xx}}(t,f)\right]/m$.
2. Morphological image processing for IF extraction:
   (a) Compute the binary TF image by thresholding $D_{\boldsymbol{xx}}(t,f)$.
   (b) Perform opening and closing operations.
   (c) Obtain the IF skeleton by thinning operation.
   (d) Remove junction points, spurious branches and short segments. Resulting set of IF segments is $\mathcal{S}_k$ for $k = 1, ..., p$.
3. Compute $\boldsymbol{D}_k$ by averaging over TF points of segment $(t_n, f_n) \in \mathcal{S}_k$ as in (3).
4. Obtain an estimate of the noise subspace, $\hat{\boldsymbol{E}}_n$ from an eigen-decomposition of $\boldsymbol{D}_k$.
5. Estimate the DOA, θ_k as a maximum of the MUSIC spectrum
   $$P_{\mathrm{MU}}(\theta) = \|\hat{\boldsymbol{E}}_n^H \boldsymbol{a}(\theta)\|^{-2}.$$
6. Repeat Steps 3 to 5 for each segment $\mathcal{S}_k$.
7. Combine segments $\mathcal{S}_k$ and $\mathcal{S}_l$, if $|\theta_k - \theta_l| < \gamma$.

Table 1. The proposed approach for direction finding with IF segments extraction

ning result, basic notation is necessary. In a digital image, the 8-connected neighborhood $N_8(p)$ of pixel p is defined as the set of neighboring pixels x_i for $i = 1, ..., 8$ as shown below

$$\begin{matrix} x_4 & x_3 & x_2 \\ x_5 & p & x_1 \\ x_6 & x_7 & x_8 \end{matrix}$$

For a binary image, foreground and background pixels take values 1 and 0 respectively. Let us denote $B(p)$ the number of forground pixels in $N_8(p)$ as $B(p) = p + x_1 + ... + x_8$.

Basically, one iteration of thinning removes or retains a foreground pixel p from the boundary of an object, based on $N_8(p)$ by means of the hit-and-miss transform [6]. If applied until stability, the thinning results in a one-pixel thick line which approximates the medial axis. Thinning is connectivity preserving and allows straightforward detection of crossing components and end points according to

$$B(p) \begin{cases} > 3 & \text{crossing} \\ \leq 3 & \text{line point} \\ = 2 & \text{end point} \end{cases}$$

Based on $B(p)$ the IF skeleton is split into nonoverlapping segments. Spur removal and component labeling techniques are used to enhance the image processing result [7].

As a demonstration of the IF extraction method, we consider the following example. Figure 1 (top) shows the spatially averaged pseudo Wigner Ville Distribution (PWVD) of three signals impinging on an array of 8 sensors. The IF segment extraction result is shown in Figure 1 (bottom) with the true signal IFs. For better visibility of the segment-pixels, a small number of snapshots ($N = 128$) was used.

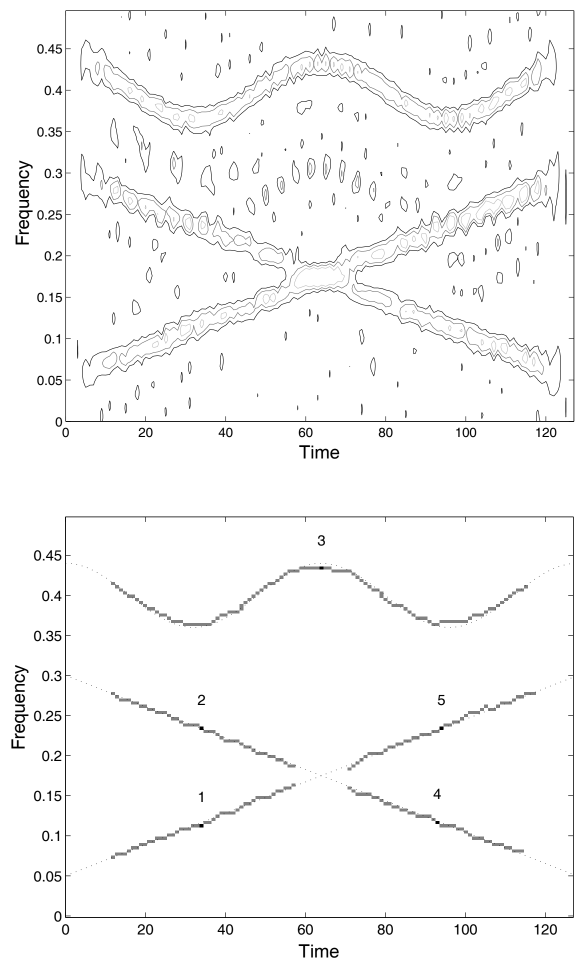

Fig. 1. Sensor-averaged TFD for 3 sources with DOAs $[-10\ 5\ 15]$ degrees at SNR of 0 dB (top). Result of IF segment extraction method (bottom).

5. SIMULATIONS

To evaluate the performance of the proposed approach for DOA estimation using IF segments, Monte Carlo simulations have been conducted. Regarding the image processing parameters, the threshold to obtain the binary image has been set to the 10% of the maximum value of the TFD, the structuring element for the opening and closing operations has been diamond shape with radius of 2 pixels. After the removal of junction points in the thinned image, 4 (6) spurious pixels and segments smaller than 8 (12) pixels have been removed for the first (and second) example.

In the first simulation example, 3 sources as in Figure 1 with DOAs $[-10\ 5\ 15]$ degrees are impugning on a uniform linear array with 8 sensors. All sources have the same power w.r.t. the noise and $N = 128$ observations have been used. For the spatially averaged auto TFD (Step 1, Table 1) we consider the modified B-distribution [8] with window length 21. For the TF averaging (Step 3, Table 1) a PWVD with window length 25 is chosen. Figure 2 shows the Comparison of the RMSE between TF-MUSIC with known IF and estimated IF segments, and covariance-based MUSIC versus SNR.

The second, underdetermined simulation example considers 3 non-overlapping FM signals (linear, quadratic and sinusoidal) with DOAs $[-20\ 5\ 25]$ degrees, impugning on 2 sensors. The number of snapshots is constant at $N = 256$. For the spatially averaged auto TFD for the image processing, we considered the modified B-distribution kernel function and a rectangular lag domain window of length 41. For the TF averaging a pseudo Wigner-Ville with a window length of 49 has been used. The comparison of the RMSE between TF-MUSIC with known IF and estimated IF segments for 1 source is shown in Figure 3.

6. DISCUSSION AND CONCLUSION

When carrying out the simulation, an estimation bias for higher SNR has occurred when averaging over individual TF signatures. This is due to the small residual contribution of other

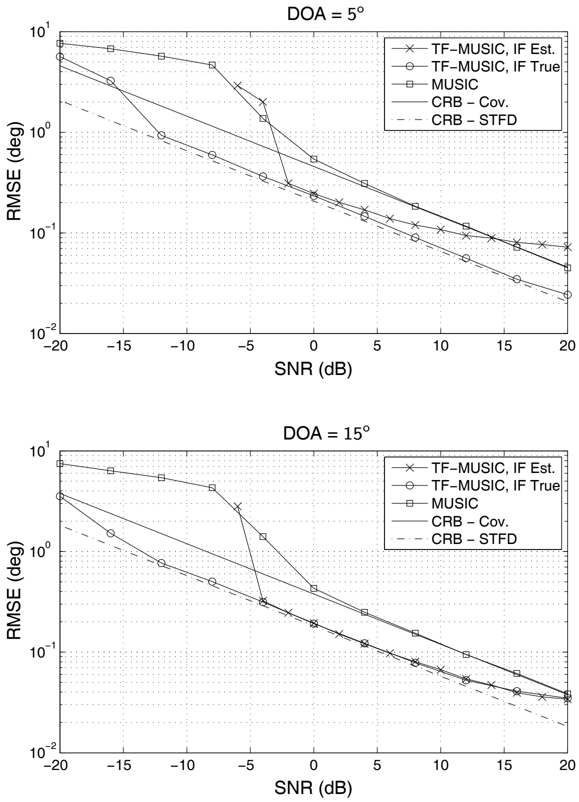

Fig. 2. Comparison of RMSE vs. SNR of TF-MUSIC (IF known and estimated) and MUSIC.

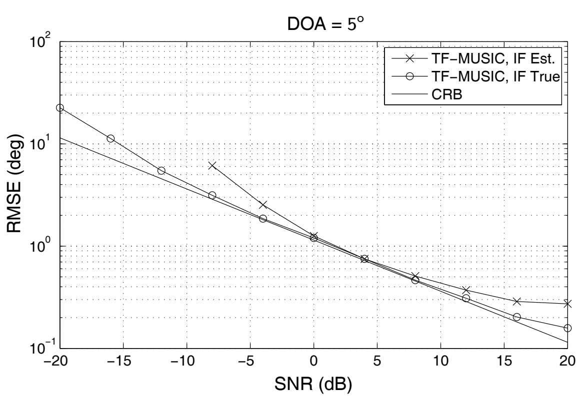

Fig. 3. Comparison of RMSE vs. SNR of TF-MUSIC for IF known and IF estimated.

sources present, i.e. when performing estimation with MUSIC, one assumes the dimension of the signal subspace is 1, although there is actually a very small contribution from other sources which results in slight mis-modelling. The bias is significant w.r.t. variance at higher SNR for closely separated sources, either spatially or in the TF plane. Therefore the direction finding of crossing components may be done jointly to minimize the mis-modelling.

The proposed method is more computationally attractive than [4, 3] while following similar strategies. This is because eigen-decomposition is done only once for each segment, as opposed to once for each peak in the TF plane. Compared to the Hough method of [5] it is not as good at low SNR, where image processing and DOA grouping of segments fails. However, the proposed approach is nonparametric and can handle unknown IF models. It is therefore more computationally attractive than the Hough transform, when higher order models are required, or different types of FM are present simultaneously.

In conclusion, the proposed approach for DOA estimation with IF segments performs better than MUSIC at moderate SNR. It additionally estimates the number of sources, which is the number of remaining DOA estimates after the recombination of segments. Furthermore the challenge of DOA estimation for the underdetermined case can be tackled.

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