PROFILE LIKELIHOOD ESTIMATOR FOR PASSIVE SCAN-BASED EMITTER LOCALIZATION

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ABSTRACT

This paper is concerned with the geolocation of a scanning emitter from time of intercept measurements of the rotating emitter beam. The problem of estimating the emitter location is cast into a profile likelihood estimation framework by treating the unknown scan rate of the emitter as a nuisance parameter. A grid search technique is developed for initializing iterative profile likelihood estimation algorithms. The grid spacing is determined from an estimate of the Lipschitz constant of the profile likelihood cost function. Maxima of directional derivatives of the cost function are fitted to a Weibull distribution to estimate the Lipschitz constant. The performance of the profile likelihood estimator is illustrated with simulation examples.

Index Terms— Passive localization, profile likelihood estimation, grid search methods, extreme value theory.

1. INTRODUCTION

Passive emitter localization is an important research problem with many civilian and military applications such as mobile user localization in wireless mobile communication systems, and target location and tracking in electronic warfare systems. Several techniques have been developed for passive localization of an emitter, each with certain advantages and disadvantages. This makes it very difficult to identify a single best localization approach for all applications. Consequently the localization research has mainly focused on the development of high-performance techniques tailored for particular applications. Most passive localization techniques utilize angle of arrival (bearings), time of arrival, time difference of arrival (TDOA), Doppler shift and received signal energy measurements. Hybrid approaches employing a combination of these are also available.

Passive TDOA localization techniques are well suited for localization of uncooperative emitters. However the main limitation of TDOA techniques is the requirement of high-precision time of arrival measurements coupled with highly synchronized clocks at multiple receivers. Furthermore the receivers are required to detect the same signal from significantly different angles, often along sidelobes of the signal. The scan-based localization technique, which is the passive localization technique considered in this paper, does away with these limitations and is particularly effective for mechanically scanning radars, whose azimuth beamwidth is narrow [1].

This paper presents a profile likelihood estimator for scan-based emitter localization for unknown scan rates. The proposed method is well suited for geolocation of radars performing circular or sector scans [2]. The unknown emitter scan rate is treated as a nuisance parameter and is eliminated by the profile likelihood method. Grid search is used for initializing iterative estimators where the grid spacing is determined by estimating the Lipschitz constant of the profile likelihood cost function.

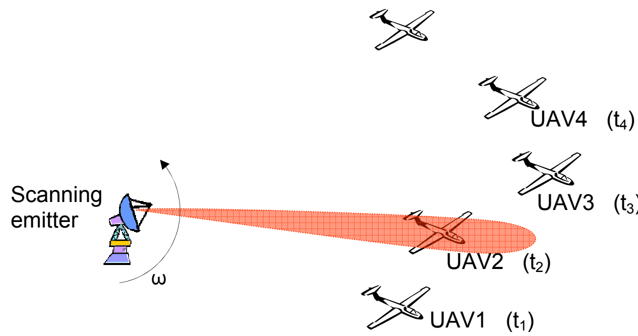

Fig. 1. Scan-based geolocation where the t_k correspond to time instants the emitter beam is intercepted by UAV receivers.

2. PROBLEM DEFINITION AND ASSUMPTIONS

Fig. 1 shows an illustration of a typical operational scenario where the scan-based localization technique is used to locate a stationary emitter scanning its main antenna beam at a constant scan rate ω. The main beam sweeps across a number of RF receivers on-board UAVs equipped with GPS receivers.

The RF receivers sense the incoming emitter signals (modulated by the antenna beam pattern) and pass the measured data to a processing unit, which estimates the scan intercept times at the peaks of the rotating beam. The instantaneous UAV positions associated with the beam peaks are also recorded.

The main underlying idea of the scan-based localization technique is to deduce the emitter location by exploiting the constraint imposed on the emitter location by the uniform rotation of the antenna beam. This paper extends the scan-based localization method developed in [1] to estimate the emitter location from scan intercept time measurements at recorded receiver positions, dispensing with the requirement of prior knowledge of the scan rate. This is particularly useful in scenarios where the emitter is only performing a sector scan [2], which complicates the problem of scan rate estimation.

We assume that the scan intercept time measurements t_k recorded at receiver positions $\boldsymbol{p}_k$ are subject to additive white Gaussian noise with zero mean and variance σ_t^2. At least $N = 4$ receivers are necessary to estimate the unknown parameters. We assume that the receivers and the emitter do not form a circle as this prevents unique estimation of the emitter location.

3. PROFILE LIKELIHOOD ESTIMATOR

The information about the emitter location $\boldsymbol{p} = [p_x, p_y]^T$ (here ^{T} denotes transpose) and scan rate is carried by intercept time differences $t_{1i} = t_i - t_1$. The likelihood function of $\boldsymbol{\tau} = [t_{12}, t_{13}, , \cdots, t_{1N}]^T$ is given by its conditional joint probability density function:

$$f(\boldsymbol{\tau}|\boldsymbol{p}, \omega) = \frac{1}{(2\pi)^{(N-1)/2}|\boldsymbol{\Sigma}|^{1/2}} \times \exp\left\{-\frac{1}{2}(\boldsymbol{\tau} - \boldsymbol{t}(\boldsymbol{p}, \omega))^T \boldsymbol{\Sigma}^{-1}(\boldsymbol{\tau} - \boldsymbol{t}(\boldsymbol{p}, \omega))\right\} \quad (1)$$

where $\boldsymbol{t}(\boldsymbol{p}, \omega)$ is the mean vector of $\boldsymbol{\tau}$

$$\boldsymbol{t}(\boldsymbol{p}, \omega) = \frac{1}{\omega}\boldsymbol{v}(\boldsymbol{p}), \quad \boldsymbol{v}(\boldsymbol{p}) = \begin{bmatrix} \cos^{-1} \frac{(\boldsymbol{p}_1-\boldsymbol{p})^T(\boldsymbol{p}_2-\boldsymbol{p})}{\|\boldsymbol{p}_1-\boldsymbol{p}\|\|\boldsymbol{p}_2-\boldsymbol{p}\|} \\ \vdots \\ \cos^{-1} \frac{(\boldsymbol{p}_1-\boldsymbol{p})^T(\boldsymbol{p}_N-\boldsymbol{p})}{\|\boldsymbol{p}_1-\boldsymbol{p}\|\|\boldsymbol{p}_N-\boldsymbol{p}\|} \end{bmatrix} \quad (2)$$

and $\boldsymbol{\Sigma}$ is the $(N - 1) \times (N - 1)$ covariance matrix of $\boldsymbol{\tau}$

$$\boldsymbol{\Sigma} = E\{\boldsymbol{n}^T\boldsymbol{n}\} = \sigma_t^2\boldsymbol{Q}. \quad (3)$$

Here $\boldsymbol{n} = [n_{12}, n_{13}, \ldots, n_{1N}]^T$ is the intercept time difference noise vector in $\boldsymbol{\tau} = \boldsymbol{t}(\boldsymbol{p}, \omega) + \boldsymbol{n}$, and

$$\boldsymbol{Q} = \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 2 \end{bmatrix}.$$

Maximizing the log-likelihood function over $[\boldsymbol{p}, \omega]$ gives

$$[\hat{\boldsymbol{p}}_{\mathrm{ML}}, \hat{\omega}_{\mathrm{ML}}] = \arg\min_{\boldsymbol{p},\, \omega} J_{\mathrm{ML}}(\boldsymbol{p}, \omega) \quad (4)$$

where $\hat{\boldsymbol{p}}_{\mathrm{ML}}$ and $\hat{\omega}_{\mathrm{ML}}$ are the ML estimates of the emitter location and the scan rate, respectively, and $J_{\mathrm{ML}}(\boldsymbol{p}, \omega)$ is the ML cost function:

$$J_{\mathrm{ML}}(\boldsymbol{p}, \omega) = \boldsymbol{e}^T(\boldsymbol{p}, \omega)\boldsymbol{\Sigma}^{-1}\boldsymbol{e}(\boldsymbol{p}, \omega), \quad \boldsymbol{e}(\boldsymbol{p}, \omega) = \boldsymbol{\tau} - \boldsymbol{t}(\boldsymbol{p}, \omega). \quad (5)$$

The ML estimation problem does not have a closed-form solution and requires numerical search techniques such as the Gauss-Newton (GN) algorithm. Due to the nonlinear nature of the ML cost function the GN algorithm can get stuck in local minima or become unstable if it is not initialized appropriately. Grid search, which is a non-iterative search technique over $[\boldsymbol{p}, \omega]$ in 3D space, can be prohibitively expensive.

It is desirable to reduce the dimension of the search space by eliminating nuisance parameters. In scan-based geolocation the scan rate ω can be considered a nuisance parameter. To eliminate ω we employ the profile likelihood method. The profile likelihood estimate of the emitter location is given by

$$\hat{\boldsymbol{p}} = \arg\min_{\boldsymbol{p}} \hat{J}(\boldsymbol{p}), \quad \hat{J}(\boldsymbol{p}) = \min_{\omega} J_{\mathrm{ML}}(\boldsymbol{p}, \omega). \quad (6)$$

To minimize $J_{\mathrm{ML}}(\boldsymbol{p}, \omega)$ over ω, consider

$$\frac{\partial J_{\mathrm{ML}}(\boldsymbol{p}, \omega)}{\partial \omega} = \frac{2}{\sigma_t^2\omega^2}\boldsymbol{v}^T(\boldsymbol{p})\boldsymbol{Q}^{-1}\left(\boldsymbol{\tau} - \frac{1}{\omega}\boldsymbol{v}(\boldsymbol{p})\right)\Bigg|_{\omega=\hat{\omega}} = 0 \quad (7)$$

which yields

$$\hat{\omega} = \frac{\boldsymbol{v}^T(\boldsymbol{p})\boldsymbol{Q}^{-1}\boldsymbol{v}(\boldsymbol{p})}{\boldsymbol{v}^T(\boldsymbol{p})\boldsymbol{Q}^{-1}\boldsymbol{\tau}}. \quad (8)$$

Substituting $\hat{\omega}$ into $J_{\mathrm{ML}}(\boldsymbol{p}, \omega)$ gives

$$\hat{J}(\boldsymbol{p}) = \left(\boldsymbol{\tau} - \frac{1}{\hat{\omega}}\boldsymbol{v}(\boldsymbol{p})\right)^T \boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\tau} - \frac{1}{\hat{\omega}}\boldsymbol{v}(\boldsymbol{p})\right) \quad (9a)$$

$$= \boldsymbol{\tau}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\tau} - \frac{(\boldsymbol{v}^T(\boldsymbol{p})\boldsymbol{\Sigma}^{-1}\boldsymbol{\tau})^2}{\boldsymbol{v}^T(\boldsymbol{p})\boldsymbol{\Sigma}^{-1}\boldsymbol{v}(\boldsymbol{p})}. \quad (9b)$$

The inverse of the covariance matrix can be written as

$$\boldsymbol{\Sigma}^{-1} = \frac{1}{\sigma_t^2}\left(\boldsymbol{I} - \frac{\boldsymbol{1}}{N}\right) \quad (10)$$

where $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{1}$ is the matrix of ones. After substituting (10) into (9b) and carrying out some vector/matrix algebra, we finally obtain

$$\hat{J}(\boldsymbol{p}) = \frac{1}{\sigma_t^2}\left(\|\boldsymbol{\tau}\|^2 - \frac{\tau_s^2}{N} - \frac{\left(\boldsymbol{\tau}^T\boldsymbol{v}(\boldsymbol{p}) - \tau_s v_s(\boldsymbol{p})/N\right)^2}{\|\boldsymbol{v}(\boldsymbol{p})\|^2 - v_s^2(\boldsymbol{p})/N}\right) \quad (11)$$

where τ_s and v_s are the sums of the elements of $\boldsymbol{\tau}$ and $\boldsymbol{v}(\boldsymbol{p})$, respectively.

Noting that the minimization of the profile likelihood cost function is not affected by σ_t^2 and the first two terms within the brackets, the cost function $\hat{J}(\boldsymbol{p})$ can be simplified to

$$\hat{J}_S(\boldsymbol{p}) = -\frac{\left(\boldsymbol{\tau}^T \boldsymbol{v}(\boldsymbol{p}) - \tau_s v_s(\boldsymbol{p})/N\right)^2}{\|\boldsymbol{v}(\boldsymbol{p})\|^2 - v_s^2(\boldsymbol{p})/N} \tag{12}$$

which has a complexity of $\mathcal{O}(N)$ compared with $\mathcal{O}(N^2)$ for $\hat{J}(\boldsymbol{p})$ in (9b). This complexity reduction is especially important for grid search.

The profile likelihood estimator does not lead to a closed-form solution either. Thus a numerical search technique such as GN, the Nelder-Mead simplex method [3] or grid search is required. The nonconvex topology of the profile likelihood cost function (i.e., the existence of local minima) necessitates the availability of good initial guesses for iterative techniques such as the GN algorithm. Grid search can be employed to generate an initial guess for iterative search algorithms.

4. GRID SEARCH FOR PROFILE LIKELIHOOD ESTIMATOR

Supposing that the search region for the emitter is known *a priori*, the grid search for the profile likelihood estimator involves evaluation of $\hat{J}_S(\boldsymbol{p})$ over discrete points on a 2D grid and then selection of the grid point with the minimum cost function value as the emitter location estimate. While grid search avoids convergence difficulties associated with iterative techniques such as the GN algorithm, it can suffer from high complexity.

In order to ensure that the global minimum is not missed, the grid points must be separated by $\epsilon/M\sqrt{2}$ [4] where ϵ is the required accuracy for determining the global minimum and M is the Lipschitz constant of $\hat{J}_S(\boldsymbol{p})$ satisfying

$$|\hat{J}_S(\boldsymbol{s}_1) - \hat{J}_S(\boldsymbol{s}_2)| \leq M\|\boldsymbol{s}_1 - \boldsymbol{s}_2\| \tag{13}$$

for all $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ within the search region.

The Lipschitz constant of $\hat{J}_S(\boldsymbol{p})$ needs to be estimated to determine the spacing of the grid points. An estimate of the Lipschitz constant M can be obtained by fitting largest directional derivatives of $\hat{J}_S(\boldsymbol{p})$ to a reverse Weibull distribution [5]. The location parameter of the reverse Weibull distribution then gives an estimate of M. Under certain mild conditions, the maxima of directional derivatives were shown to converge to the Type III extreme value distribution (reverse Weibull) in [5].

The Lipschitz constant estimation procedure is summarized below:

1. Sample D randomly distributed points $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_D$ over the region of interest;
2. Evaluate directional slopes at sampled points:

$$M_i = \left\| \frac{\partial \hat{J}_S(\boldsymbol{s}_i)}{\partial \boldsymbol{p}} \right\|, \quad i = 1, \ldots, D.$$

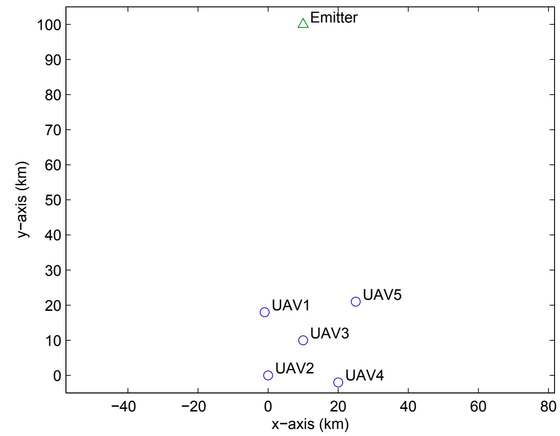

Fig. 2. Simulated localization scenario.

3. Find the maximum of $\{M_1, \ldots, M_D\}$:

$$m = \max_i M_i.$$

4. Repeat steps 1–3 L times to produce $m_1, \ldots, m_L$;
5. Fit a three-parameter reverse Weibull distribution to m_1, $\ldots, m_L$;
6. The location parameter estimate of the reverse Weibull distribution gives an estimate of the Lipschitz constant.

We use a moment estimation method for determining the location parameter of the reverse Weibull distribution [6].

5. SIMULATION EXAMPLES

The simulated geolocation scenario is depicted in Fig. 2. Five UAVs at $\boldsymbol{p}_1 = [-1, 18]^T$, $\boldsymbol{p}_2 = [0, 0]^T$, $\boldsymbol{p}_3 = [10, 10]^T$, $\boldsymbol{p}_4 = [20, -2]^T$, and $\boldsymbol{p}_5 = [25, 21]^T$ attempt to locate a scanning emitter at $\boldsymbol{p} = [10, 100]^T$ by utilizing scan intercept time measurements. The scan rate of the emitter, which is unknown by the UAVs, is $\omega = \pi/2$ rad/s.

The first simulation studies the local minima of the profile likelihood cost function $\hat{J}_S(\boldsymbol{p})$. The scan time intercept measurement noise standard deviation is set to $\sigma_t = 10^{-3}$ s. Several locations were used as an initial guess for the profile likelihood estimator when implemented as a Nelder-Mead simplex search algorithm. For a single realization of the scan time intercept measurements the following minima on the cost function surface were discovered:

$$\hat{\boldsymbol{p}}_1 = [12.9349, 17.7397]^T \quad \hat{J}_S(\hat{\boldsymbol{p}}_1) = -0.0271$$
$$\hat{\boldsymbol{p}}_2 = [44.1425, -254.2713]^T \quad \hat{J}_S(\hat{\boldsymbol{p}}_2) = -0.0288$$
$$\hat{\boldsymbol{p}}_3 = [10.5358, 107.6101]^T \quad \hat{J}_S(\hat{\boldsymbol{p}}_3) = -0.0289$$

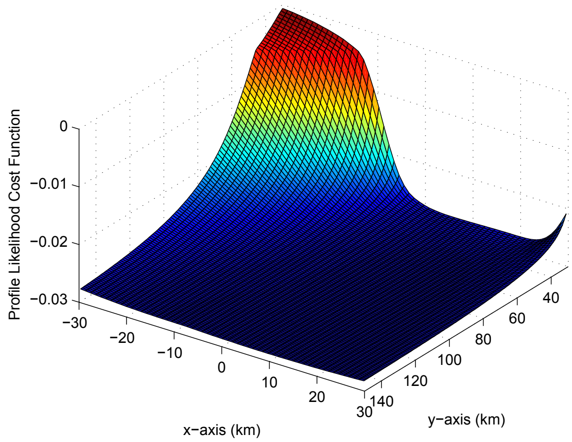

Fig. 3. Cost function grid.

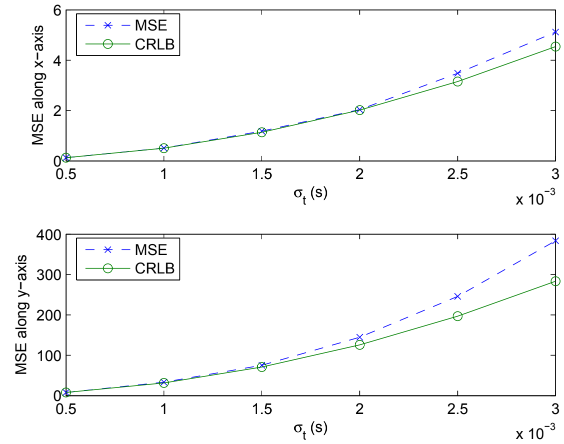

Fig. 4. MSE performance.

Out of these $\hat{\boldsymbol{p}}_3$ is the global minimum corresponding to the emitter location and the other two are local minima. The existence of undesirable local minima increases the significance of good initial guess selection.

We use the grid search algorithm proposed in Section 4 to obtain an initial guess for the Nelder-Mead profile likelihood estimator. The grid search region (region of interest) was chosen to be $[-30, 30] \times [30, 150]$. For one realization of the scan time intercept measurements with $\sigma_t = 10^{-3}$ s, the Lipschitz constant estimate obtained from Weibull fit using $D = 20$ and $L = 20$ was $\hat{M} = 0.0066$, which for $\epsilon = 10^{-2}$ gives a grid spacing of $\epsilon / M\sqrt{2} = 1.0635$. The resulting grid search estimate was $\hat{\boldsymbol{p}}_G = [10.4136, 107.6366]^T$ compared with the profile likelihood estimate $\hat{\boldsymbol{p}} = [10.5358, 107.6101]^T$. Initializing the profile likelihood estimator to the grid search estimate would require only a few iterations of the search algorithm to find the final estimate $\hat{\boldsymbol{p}}$. Fig. 3 shows a plot of $\hat{J}_S(\boldsymbol{p})$ over the grid points obtained from the Lipschitz constant estimate.

To assess the performance of the profile likelihood estimator we conducted Monte Carlo simulations. Fig. 4 shows the MSE and CRLB curves for 5000 simulation runs. The profile likelihood estimator was implemented as a Nelder-Mead simplex search algorithm and initialized to the grid search estimate. The profile likelihood estimator is seen to perform well for $\sigma_t < 2$ ms. As the noise becomes larger the MSE performance begins to deviate from the CRLB.

6. CONCLUSION

A scan based profile likelihood estimator was proposed for passive emitter localization. The developed estimator does not require explicit estimation of the scan rate of the emitter antenna. Being a nonlinear least-squares estimator, the profile likelihood estimator does not have a closed-form solution, thus requiring the use of an iterative numerical search algorithm. The existence of local minima in the profile likelihood cost function necessitates a good initial guess. Since no closed-form solutions are available even for suboptimal estimation, a grid search technique was employed to initialize iterative search algorithms. The grid spacing was determined from an estimate of the Lipschitz constant of the cost function using maxima of directional slopes fitted to the reverse Weibull distribution. The performance of the developed algorithm was demonstrated and compared with the CRLB for the joint emitter and scan rate estimation problem.

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