UNDERDETERMINED SOURCE SEPARATION FOR NON-STATIONARY SIGNAL

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ABSTRACT

This paper proposes a new blind source separation (BSS) technique for underdetermined MIMO system using space-time-frequency distributions. The sparseness in time-frequency domain is exploited for underdetermined BSS. The paper also proposed a new detection technique for auto sources time-frequency points, which is incorporated into the BSS technique. The proposed BSS technique applicable to the situations where sources time-frequency signatures overlap at certain points in time-frequency plane and even in situations where there is source with multicomponent signal. In these situations, where time-frequency planes cannot be partitioned (masked) into groups of fewer or equal number sources than antennas, the existing overdetermined time-frequency BSS techniques could not be applied. Simulation results are included to show its effectiveness.

***Index Terms*—** Time-frequency distributions, blind sources separation, MIMO systems, underdetermined system

1. INTRODUCTION

Blind source separation has been used in many applications, for example in processing signal received by ill-calibrated antenna array and separating different audio sounds in speech processing. BSS for non-stationary signals were introduced in [1, 2]. They are based on time-frequency (TF) method. However, they are only applicable to determined or overdetermined system, where the original sources are unmixed by multiplying inverse or pseudoinverse of the mixing matrix to the received signal vector. BSS of underdetermined system is a challenging problem even after blind identification of the wide mixing matrix, because separating sources by the inversion of the mixing matrix is impossible. Hence, obtaining the unmixed source signals would require additional assumptions and steps. In this paper, considering only non-stationary signals, the sparseness of the signal in time-frequency (TF) domain could be exploited for underdetermined BSS. The methods in [1, 2] could be extended for the underdetermined system under condition that the signals' signatures in the TF plane could be masked or partitioned into groups, so that each group contains fewer or equal number of signals than antenna sensors. Following masking or partitioning, the BSS techniques [1, 2] are then applied to each of the partitioned group. In addition, there are some extra algorithmic steps that are essential to mitigate the cross-terms (CTs) between groups. In this paper we proposed a method for source separation of underdetermined system with possibility that signal signatures in TF plane is non-disjoint, in which the above mentioned extended BSS technique is inapplicable.

Authors in [3] proposed to mitigate BSS problem with disjoint signal signatures in TF plane. The algorithm uses clustering algorithm and exploits the spatial time-frequency distribution (STFD) [2] structure at single auto points (SAPs), i.e. the location in TF plane where individual source exists alone. Also, the same algorithm was applied to signals with few overlappings in their TF signatures, and it performs well except at the multiple auto points (MAP), i.e. the TF location where time-frequency distributions (TFDs) of two or more sources intersect [3]. In [4], the authors proposed a new subspace-based algorithm to perform separation on both SAPs and MAPs, assuming at MAPs less number sources that overlaps than number of sensors. However application of this subspace-based algorithm to each points of SAPs and MAPs all together could be expensive.

In this paper, we proposed a separation technique that relies on pseudoinverse of the virtual array structure [5] of the vectorized STFD matrices of SAPs. It assumed that the mixing matrix is obtained through other means, such as [8]. In this paper we also extend the method in [4] to be applied to cross points (CPs). The CPs are the locations of CTs. In addition to that, we also proposed a new method for selecting mixtures of CPs and MAPs. With the mixture of MAPs and CPs selected, STFD matrices at these TF points are processed similar to [4], but at the lower computational cost due to less points processed, because only MAPs and CPs, not SAPs, are processed. Due to lower computation cost and extensibility of subspace method, we have the luxury to use Wigner-Ville (WV)-based STFD. WV-based STFD has many unsuppressed CTs, which are advantageous for source synthesis of multicomponent signal from any single source, such as in audio sources that contain harmonic.

2. SIGNAL MODEL

Assume instantaneous mixing matrix $\mathbf{A} \triangleq [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_K]$ with M sensors and K narrow band signals impinging on them. Since we are dealing with underdetermined system, threfore $K > M$. It is also assumed any M columns of $\mathbf{A}$ are linearly independent. The received signal is modeled as

$$\mathbf{x}(t) \triangleq \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t), \tag{1}$$

where $\mathbf{w}(t)$ is the $M \times 1$ vector of additive noise. The vector $\mathbf{s}(t) \triangleq [s_1(t), \ldots, s_K(t)]^T$ is source signal vector of size $K \times 1$ and each of $s_i(t)$ is a non-stationary source signal. Without loss of generality, the first row of $\mathbf{A}$ is assumed real-valued and each column of $\mathbf{A}$ is normalized. This is to provide uniqueness in estimating mixing matrix. Before looking into sources assumptions, we first define spatial time-frequency distribution of the received signals, $\mathbf{x}(t)$, as follows:

$$\mathbf{D}_{\mathbf{xx}}(t,f) \triangleq \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \phi(m,l)\mathbf{x}(t+m+l)\mathbf{x}^H(t+m-l)e^{-j4\pi fl} \tag{2}$$

where $\phi(m, l)$ is the TFD time-lag kernel which is applied to all received sensors equally and $(\cdot)^H$ denotes Hermitian transpose. There

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are various TFD time-lag kernels to be chosen from, depending on how the cross terms to be suppressed. Assuming no noise for now, which is also a practical assumption because noise power will spread out evenly on the TF plane, the received signal's STFD is related to the source signal's STFD, $\mathbf{D_{ss}}(t,f)$, in the following way,

$$\mathbf{D_{xx}}(t,f) = \mathbf{A}\mathbf{D_{ss}}(t,f)\mathbf{A}^H \tag{3}$$

Basically, elements of the STFDs, e.g. $[\mathbf{D_{ss}}(t,f)]_{i,j} \triangleq D_{s_is_j}(t,f) = \sum_l \sum_m \phi(m,l) s_i(t+m+l) s_j^*(t+m-l) e^{-j4\pi fl}$, is an auto-TFD (if $i=j$) or cross-TFD (if $i \neq j$).

Definition 1 *The two sources, $s_i(t)$ and $s_j(t)$, are disjoint if and only if $\Omega_i \cap \Omega_j = \varnothing$, where Ω_k is the TF support*[1] *of the source k's TFD. Conversely, the two sources are called non-disjoint.*

Definition 2 *Suppose that two sources, $s_i(t)$ and $s_j(t)$, are non-disjoint, then $(t,f) \in \{\Omega_i \cap \Omega_i\}$ is called MAP. The $(t,f) \in \{(\Omega_i \cup \Omega_j) - (\Omega_i \cap \Omega_i)\}$ is called SAP (regardless sources are disjoint or not).*

Source signals in this paper are allowed to be disjoint or non-disjoint. It is assumed that SAPs of each source exists, which is the requirement for estimation of $\mathbf{A}$. We further assumed that at most $M-1$ sources intersect at any MAPs, i.e. $\Omega_{i_1} \cap \Omega_{i_2} \cap \ldots \cap \Omega_{i_M} = \varnothing$ for any sets of M sources. This assumption is essential for estimating TFDs of sources at MAPs. Before we proceed to the next section, we define the following,

Definition 3 *The (t,f) point such that $D_{s_is_j}(t,f) \neq 0$, for $i \neq j$, and is not a MAP, is defined as CP. The $D_{s_is_j}(t,f)$ both evaluated at CP and at MAP is called CT.*

It is important to note that, CP and CT in definition above are due to two sources. In TF literatures, CT also arises due to multicomponent signal within one source, however, it will appear at SAPs. From now on, unless it is specifically mentioned, CP and CT refer to the definition above and CT due to multicomponent signal is processed the same as processing STFD at SAPs.

3. SELECTION OF TIME-FREQUENCY POINTS

3.1. Properties of STFDs at SAP, MAP and CP

Firstly, we will look at the property of STFDs at SAPs. It has been studied in [6] that $\mathbf{D_{ss}}(t,f)$ will have a diagonal structure only at SAPs. In fact, only at the i-th diagonal element where $D_{s_is_i}(t,f)$ is non-zero, the rest of the entries are zero. Hence, Eqn. (3) becomes $\mathbf{D_{xx}}(t,f) = \mathbf{A}\,diag\{[0\ldots,0,D_{s_is_i}(t,f),0,\ldots 0]\}\mathbf{A}^H = \mathbf{a}_i\mathbf{a}_i^H D_{s_is_i}(t,f)$, which is rank one and semi-positive definite due to positiveness of $D_{s_is_i}(t,f)$. Note that CT due to multicomponent signal will have this property as well. Secondly, we will look at the property of $\mathbf{D_{ss}}(t,f)$ at MAPs. In general $\mathbf{D_{ss}}(t,f)$ at MAPs are not diagonal because the CTs at MAPs are non-zeros. The $rank\{\mathbf{D_{ss}}(t,f)\} = rank\{\mathbf{D_{xx}}(t,f)\} = k$ if it is MAP of k sources, i.e. $(t,f) \in \{\Omega_{i_1} \cap \Omega_{i_2} \cap \ldots \Omega_{i_k}\}$. In addition to that STFDs at MAPs are in general Hermitian symmetric indefinite. Lastly, we study the property of $\mathbf{D_{ss}}(t,f)$ at CPs. It will have off-diagonal entries only, because only CTs are non-zeros. The $rank\{\mathbf{D_{ss}}(t,f)\} = rank\{\mathbf{D_{xx}}(t,f)\} = k$ if there are CTs of k sources. Note that, regardless of the kernel, CTs near and at MAPs are difficult to be suppressed without suppressing the signal's TFD itself. We will see how these CPs would not affect the performance of the source separation in the section 4. This gives the flexibility to use the original WV distribution without any suppression of CTs.

[1] Assume source $s_i(t)$ has TFD, $D_{s_is_i}(t,f)$, then its TF support is Ω_i, if and only if $\forall(t,f) \in \Omega_i$, $D_{s_is_i}(t,f) \neq 0$.

3.2. TF points for blind identification

Although, this paper does not propose new identification method, we discuss briefly the method of selecting TF points for blind identification algorithms, such as [8]. The objective is to have sufficient TF points such that their STFDs posses diagonal form. This implies STFDs at SAPs posses this property. Since, only sufficient and small numbers of SAPs are needed for blind identification, one could use detection scheme that has low error probability in detection of SAPs, such as [6]. This also lowers the computation cost of blind identification due to less STFDs being processed.

4. UNDERDETERMINED SOURCE SEPARATION

4.1. Overview

The objective of source separation is to estimate individual source signals (in time domain). However, if one has the source's TFD, one could invert it uniquely, up to a complex constant, to yield source signal in time domain [7]. Thus, estimation of sources' TFDs from STFDs is the main issue in this paper. Initially, TF points that consist noise only need to be ignored and their TFDs to be zeroed out. This is called *noise-thresholding*. Following that TF points that left could be either SAPs or MAPs or CPs. Here, we proposed method to treat STFDs at SAPs to obtain individual source's TFDs at SAPs. Apparently, the separation method also prompted a new method to separate SAPs from MAPs and CPs. Now, only STFDs at MAPs and CPs are untreated yet. Subspace method, which are originally meant for TFD separation at MAPs and SAPs only [4], are analyzed at CPs and then its property are exploited to process STFDs at mixture of MAPs and CPs that remained from previous step. Finally, one could form individual TFDs at SAPs, MAPs and CPs (the other TF points are zeros after *noise-thresholding*) and inverting them to obtain estimated sources' signal in time domain.

4.2. Proposed simultaneous TFDs separation at SAPs

Preceding any processing, the *noise-thresholding* is performed and could be done by selecting TF points that satisfy the following,

$$trace\{\mathbf{D_{xx}}(t,f)\} \geq \epsilon_1 \underset{(t,f)}{mean}\{trace\{\mathbf{D_{xx}}(t,f)\}\} \tag{4}$$

where value of ϵ_1 typically is 1 (see [6]). Following the *noise-thresholding*, blind identification of $\mathbf{A}$ is performed as discussed in subsection 3.2. Thereafter, the proposed source separation algorithm is performed on SAPs, in which method of selecting SAPs and mixture of MAPs and CPs will be shown in next subsection. The algorithm exploits the diagonal structure at SAPs, and hence, by vectorizing Eqn. (3), it would gives

$$\mathbf{y}(t,f) \triangleq vec\{\mathbf{D_{xx}}(t,f)\} = (\mathbf{A}^* \odot \mathbf{A})\mathbf{z}(t,f) \tag{5}$$

where $\mathbf{z}(t,f) \triangleq diag\{\mathbf{D_{ss}}(t,f)\}$ is the vector that contain diagonal entries of sources STFD, and $\odot$ is the Khatri-Rao product (see [10]). Note that the size of the *virtual array*, $\mathbf{A}^* \odot \mathbf{A}$, is $M^2 \times K$. Even when $M < K$, the condition $M^2 > K$ is easily achievable to form full rank *virtual array* matrix [5], and hence solving for $\mathbf{z}(t,f)$ in Eqn (5) is the full-rank (overdetermined) least square problem now. For example, with only three sensors, it is possible to do separation of TFDs up to eight sources at SAPs. Mathematically, the estimate of separated TFDs at SAPs is just

$$\hat{\mathbf{z}}(t,f) = (\hat{\mathbf{A}}^* \odot \hat{\mathbf{A}})^\dagger \mathbf{y}(t,f) \tag{6}$$

where $(\hat{\mathbf{A}}^*\odot\hat{\mathbf{A}})^\dagger$ is pseudoinverse of the *virtual array* in this full-rank least square case, i.e. $(\hat{\mathbf{A}}^* \odot \hat{\mathbf{A}})^\dagger = [(\hat{\mathbf{A}}^* \odot \hat{\mathbf{A}})^H(\hat{\mathbf{A}}^* \odot \hat{\mathbf{A}})]^{-1}(\hat{\mathbf{A}}^* \odot \hat{\mathbf{A}})^H$. Note also, one could stack $\mathbf{y}(t, f)$ from different SAPs column-wise into matrix $\mathbf{Y}$, and hence batch processes it to obtain the stacked $\hat{\mathbf{z}}(t, f)$ from different SAPs, $\hat{\mathbf{Z}}$, just by one matrix multiplication, i.e. $\hat{\mathbf{Z}} = (\hat{\mathbf{A}}^* \odot \hat{\mathbf{A}})^\dagger\mathbf{Y}$.

4.3. Proposed detection of SAPs and mixture of MAPs and CPs

Now, suppose that pseudoinverse of the *virtual array* is applied to the vectorized STFD matrix at MAPs or CPs as in Eqn. (6), then it will lead to the following equation

$$\hat{\mathbf{z}}(t, f) = (\hat{\mathbf{A}}^* \odot \hat{\mathbf{A}})^\dagger(\tilde{\mathbf{A}}^* \otimes \tilde{\mathbf{A}})\mathbf{v}(t, f) \tag{7}$$

where $\otimes$ is the Kronecker product and without lost of generality, we have assumed the MAPs or CPs are the points overlap of the first K' sources, and hence $\tilde{\mathbf{A}} \triangleq [\mathbf{a}_1, \ldots, \mathbf{a}_{K'}]$ and the vectorized non-diagonal K'-sources' STFD matrix at MAPs or CPs is $\mathbf{v}(t, f) \triangleq vec\{\tilde{\mathbf{D}}_{\mathbf{ss}}(t, f)\}$. The Kronecker product arises due to non-diagonal structure of the sources' STFD at MAPs and CPs. Assuming perfect estimation of $\mathbf{A}$, some of the columns of *virtual array* $\mathbf{A}^* \odot \mathbf{A} = [\mathbf{a}_1^* \otimes \mathbf{a}_1, \mathbf{a}_2^* \otimes \mathbf{a}_2, \ldots, \mathbf{a}_K^* \otimes \mathbf{a}_K]$ are contained in $\mathbf{A}^* \otimes \mathbf{A} = [\mathbf{a}_1^* \otimes \mathbf{a}_1, \mathbf{a}_1^* \otimes \mathbf{a}_2, \ldots, \mathbf{a}_1^* \otimes \mathbf{a}_{K'}, \ldots, \mathbf{a}_{K'}^* \otimes \mathbf{a}_{K'}]$. This leads to

$$(\mathbf{A}^* \odot \mathbf{A})^\dagger(\mathbf{A}^* \otimes \mathbf{A}) = [\mathbf{e}_1, \star, \ldots, \star, \mathbf{e}_2, \star, \ldots, \star, \mathbf{e}_{K'}] \tag{8}$$

where $\star$'s are arbitrary column vectors and $\mathbf{e}_k = [0, \ldots, 0, 1, 0, \ldots 0]^T$ is vector with all elements are zeros except at the k-th row. At every k^2-th column of the result of post-multiplication with pseudo-inverse above in Eqn. (8) gives $\mathbf{e}_k$. Substituting Eqn. (8) into Eqn. (7) gives

$$\hat{\mathbf{z}}(t, f) = \begin{bmatrix} D_{s_1s_1}(t, f) + \text{cross terms} \\ D_{s_2s_2}(t, f) + \text{cross terms} \\ \vdots \\ D_{s_{K'}s_{K'}}(t, f) + \text{cross terms} \end{bmatrix} \tag{9}$$

at MAPs and similarly for CPs except that $D_{s_ks_k}(t, f) = 0$ for all k. This prompts a new way of selecting MAPs and CPs out from the mixture of MAPs, CPs and SAPs. Keep (t, f) as SAPs and $\max_i\{\hat{z}_i(t, f)\}$ as $\hat{D}_{s_{i_{max}}s_{i_{max}}}(t,f)$ if,

$$\frac{\max_i\{\hat{z}_i(t, f)\}}{\sum_i |\hat{z}_i(t, f)|} \geq 1 - \epsilon_2, \tag{10}$$

otherwise keep (t, f) as mixture of MAPs and CPs. Here, $\hat{z}_i(t, f)$ is the i-th element of $\hat{\mathbf{z}}_i(t, f)$ and i_{max} is the index that maximizes numerator of the Eqn (10). The ϵ_2 is chosen to be small value less than 1, typically can be chosen $0.1 \sim 0.5$. We will see in the next subsection the reason that this value are not that critical.

4.4. Subspace method and its properties at MAPs and CPs

Originally, subspace method is intended for SAPs and MAPs not CPs as in [4]. However, here we will show that it is applicable at CPs as well. This means suppression of CTs are not that crucial. Another advantage when dealing with a source with multicomponent signal is in estimating its original TFD that contains CTs. Hence suppressing CTs in this case is disadvantageous. This also means choice of ϵ_2 are not that crucial as well since all SAPs, MAPs and CPs could be processed by subspace method. But the subspace algorithm processes STFDs at each TF points and hence computationally expensive. Thus, processing more SAPs by the proposed method in subsection 4.2 could reduce the computational load due to its batch processing nature in Eqn. (6).

Now, we will observe the property of subspace method at MPs and CPs. It is assumed that the number of sources signal involved in the CT at CPs, K', are less than $M-1$, which is the same assumption for MAPs taken previously. Thus, at any MPs or CPs, we perform the eigenvalue decomposition to obtain the subspace of $\tilde{\mathbf{A}}$,

$$\mathbf{D}_{\mathbf{xx}}(t, f) = \tilde{\mathbf{A}}\tilde{\mathbf{D}}_{\mathbf{ss}}(t, f)\tilde{\mathbf{A}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H \tag{11}$$

where $\mathbf{U}$ corresponds to the K' largest magnitude of the eigenvalues. Magnitude is of eigenvalues is used due to the Hermitian symmetric indefiniteness of MPs and CPs. Next, $\tilde{\mathbf{A}}$ could be identified by

$$\tilde{\mathbf{A}} = \min_{\{i_1, \ldots, i_{K'}\}} \|(\mathbf{I} - \mathbf{U}\mathbf{U}^H)\mathbf{a}_i\| \tag{12}$$

which, basically finding a set of K $\mathbf{a}_i$'s, which is obtained from $\hat{\mathbf{A}}$, such that their orthogonal projection to subspace of $\tilde{\mathbf{A}}$ are minimized. Following that, the TFDs at the MAPs or CPs could be extracted from the diagonal elements of the following,

$$\tilde{\mathbf{D}}_{\mathbf{ss}}(t, f) = \tilde{\mathbf{A}}^\dagger\mathbf{D}_{\mathbf{xx}}(t, f)(\tilde{\mathbf{A}}^\dagger)^H \tag{13}$$

If it is STFD of CPs, then the diagonal entries will be small near to zero. This is the property at allow the subspace method to be applied to STFDs at CPs. Note that CT due to multicomponent signal will not be zero because it has the spatial structure of STFDs at SAPs as mentioned in previous section.

4.5. Synthesis of sources

Finally the source separated TFDs could be formed as follows,

$$\hat{D}_{s_rs_r}(t, f) = \begin{cases} \hat{z}_r(t, f) & \text{at SAPs by (6)} \\ \tilde{D}_{s_{i_r}s_{i_r}}(t, f) & \text{at MAPs/CPs by (13)} \\ 0 & \text{elsewhere} \end{cases} \tag{14}$$

where the all the STFDs used in the algorithm is chosen to be Wigner-Ville-based (WV) or Modified WV-based (MWV) [9], which is needed in order to perform the inversion. Finally, sources signals could be synthesized from the separated TFDs, by inverting the WVD as follows, $s_i(t) = \frac{1}{s_i^*(0)}\int_{-\infty}^{\infty} D_{s_is_i}(\frac{t}{2}, f)e^{j2\pi ft}df$ where its discrete time implementation could be found in [7]. It is also noteworthy to use WVD rather than MWVD in the case that sources multicomponent signal, because MWVD suppress the CTs while WVD does not. The algorithm is summarized in the following box

Table 1. Summary of the new STFD-based underdetermined BSS

Given sensors output $\mathbf{x}(n)$
1. Compute WV-based or MWV-based STFD in Eqn. (2)
2. Noise thresholding using Eqn. (4) to obtain signals' TF points
3. Select STFDs at SAPs by [6] and estimate $\mathbf{A}$ by [8]
4. Separate MAPs/CPs from SAPs for BSS by applying Eqn. (6) and (10) to the STFDs at signals' TF points obtained from step 2
6. Obtain source separated TFDs at SAPs using $\max_i \hat{z}_i(t, f)$
7. Obtain TFDs at MAPs/CPs using Eqn. (11), (12) and (13)
8. Form source separated TFDs as in Eqn. (14)
9. Synthesize the source separated signals by inverting TFDs [7]

5. SIMULATION RESULTS

In this section, the simulations are performed to show effectiveness of the proposed algorithm and it is compared to algorithm similar to that in [4]. In the proposed algorithm WV-based STFD is used

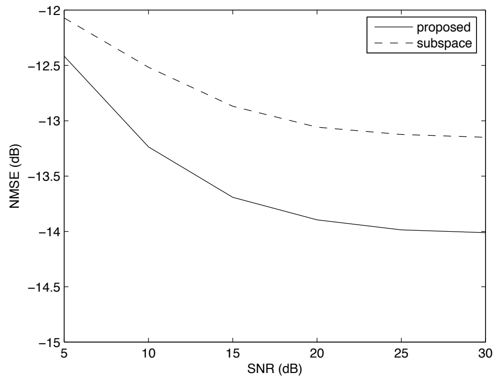

Fig. 1. NMSE of all linear FM sources

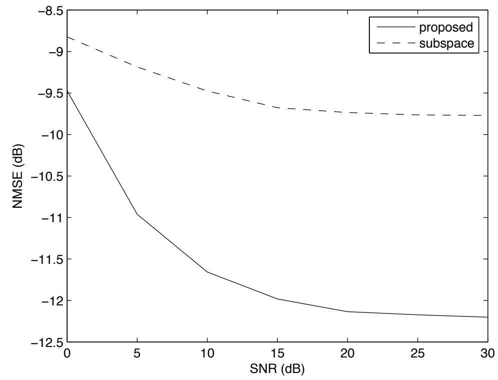

Fig. 2. NMSE of 3 linear FM sources and one multicomponent signal source

while in [4] MWV-based STFD is used. In order to make a fair comparison, both algorithms are assumed to have perfect estimation of $\mathbf{A}$, which is randomly generated. Furthermore, values of ϵ_1 and ϵ_2 are tuned such that the computational speed of both algorithms are the same (using MATLAB profiling function). There are four sources and three sensors. Minimal of three sensors are needed for both algorithms to work because there are two sources involved at MPs/CPs. The additive noise is assumed to be zero-mean Gaussian and the SNR is varied up to $30dB$. There are 256 number of snapshot collected each second. In the first example, the sources are one single tone with frequency of $0.2\frac{rad}{s}$ and three linear FM signals with instantaneous frequency of $0.0074\frac{rad}{s}$, $0.008t + 2.2\frac{rad}{s}$ and $-0.008t + 2.7\frac{rad}{s}$. In the second example, the sources are two single tone with frequency of $0.2\frac{rad}{s}$ and $2.2\frac{rad}{s}$, one linear FM signals with instantaneous frequency of $-0.008t + 2.7\frac{rad}{s}$ and one multicomponent source that consist of two linear FM signal criss-crossing each other with instantaneous frequencies of $0.009t\frac{rad}{s}$ and $-0.009t + 1.148\frac{rad}{s}$. With these kind of sources, it is impossible to mask or to partition the TF plane into TF planes that contain sources less than four sources without partitioning any source's TF signature. Thus, one could not use the method in [2] onto each partition.

The Figure 1 and 2, shows the normalized mean square error (NMSE) performance over $N_{mc} = 100$ Monte Carlo runs. The NMSE is defined as

$$NMSE = \frac{1}{N_{mc}} \sum_{r=1}^{N_{mc}} \frac{\|\hat{\mathbf{s}}_r - \mathbf{s}\|^2}{\|\mathbf{s}\|^2}$$

In the Figure 1, result of the first example shows the proposed method is better than existing subspace method [4] by about 1dB at SNR 30dB. The sources in this case are non-disjoint however each of them are still linear FM. The result of the proposed method is even more dramatic in the second example as it is shown in Figure 2. The performance gain at SNR 20dB to 30dB are almost 3dB. This is because of the proposed algorithm uses WV-based STFD. This is possible because, proposed algorithm speed could be maintained at the same speed as existing subspace algorithm by controlling ϵ_1 and ϵ_2 while allowing more CPs into the subspace processing in subsection 4.4.

6. DISCUSSION AND CONCLUSION

This paper has demonstrated a better underdetermined source separation with time-frequency technique. The performance gain without expense of the computational speed is mainly due two combined factors. Firstly, due to the use of WV-based STFD which do not suppress CTs due to different sources or multicomponent signal. The spatial structure of STFD could reveal between these two types of CT by the exploitation of subspace method at CPs, which was not exploited in [4]. By now the computational speed is severely reduced if one would use subspace algorithm alone because many CPs are included for processing. Finally, with the batch processing of (6), the computational burden of subspace method could be off-loaded especially for SAPs.

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