A GEOMETRIC METHOD FOR BLIND SEPARATION OF DIGITAL SIGNALS WITH FINITE ALPHABETS

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ABSTRACT

We consider the problem of blind separation of discrete sources with finite alphabets. More specifically, multiple-amplitude-shift-keying (M-ASK) alphabet and complex quadrature amplitude modulation (QAM) alphabet are studied. The proposed separation method exploits the geometry of the received data constellation. The method relies on a finite-step non-iterative algorithm, and therefore it is free from any convergence problem. Numerical simulation results are included to illustrate the performance of the proposed algorithm.

Index Terms— Blind separation, MIMO, finite alphabets

1. INTRODUCTION

The problem of blind discrete sources separation is of considerable interest in wireless digital communications and other fields. Past work on this problem include [1–8]. Among them, [1, 4, 5] are iterative methods and suffer from local optima. They, usually, require a good initialization in order to minimize the problem of local minima. In [3], an analytical method based on a generalized eigenvalue decomposition was developed for the constant modulus sources. Some techniques that rely directly on HOS cumulants were introduced in [6–8]. In [6], an adaptive separation algorithm which is free of undesired stationary point for an arbitrary number of users was derived from a constrained multiuser kurtosis optimization criterion. However, the HOS technique often requires a large number of observation samples for the accuracy of the numerical result, and the computational cost is comparably large. Furthermore, the source signal must be non-Gaussian, and their kurtosis must have the same sign [10]. A geometric approach for the blind separation of instantaneous mixtures of digital signals was proposed in [2]. However, this method is specific only to the BPSK signals.

In this paper, we propose a geometric non-iterative method that separates signals with the M-ASK or QAM digital format. We focus on the non-iterative algorithm development for the whitened real case. We compare our proposed method to the hyperplane-based algorithm [4] which has been shown to be a fast algorithm with similar performance as iterative least squares with projection (ILSP) [1]. The kurtosis-based algorithm [6] is also simulated as a comparison. It is shown that our proposed algorithm achieves a lower SER that both [4] and [6].

2. PROBLEM FORMULATION

Suppose we have p narrowband M-ASK or QAM digital signals impinging on an array with q sensors ($q \geq p$). We assume that no intersymbol interference is present. Thus the array output vector is an instantaneous mixture of the p transmitted source signals. The array output vector $\mathbf{x}(n)$ can be written as

$$\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{w}(n) \tag{1}$$

where

$$\mathbf{x}(n) \triangleq [x_1(n)\ x_2(n)\ \cdots\ x_q(n)]^T \tag{2}$$

$$\mathbf{s}(n) \triangleq [s_1(n)\ s_2(n)\ \cdots\ s_p(n)]^T \tag{3}$$

$$\mathbf{w}(n) \triangleq [w_1(n)\ w_2(n)\ \cdots\ w_q(n)]^T \tag{4}$$

where $\mathbf{s}(n)$ is the vector of symbols from the alphabet $\mathcal{S}$ generated by p sources, $\mathbf{w}(n)$ is a vector of q dimensional additive noise, $\mathbf{H}$ is an $q \times p$ unknown instantaneous mixture matrix. To recover all source signals, it is assumed that $\mathbf{H}$ is full column rank. If we concatenate N snapshots of the received data as $\mathbf{X} = [\ \mathbf{x}(1)\quad \mathbf{x}(2)\quad \cdots\quad \mathbf{x}(N)\ ]$, then we have

$$\mathbf{X} = \mathbf{H}\mathbf{S} + \mathbf{W} \tag{5}$$

where $\mathbf{S} \triangleq [\mathbf{s}(1)\ \cdots\ \mathbf{s}(N)]$ and $\mathbf{W} \triangleq [\mathbf{w}(1)\ \cdots\ \mathbf{w}(N)]$. Here we assume that N is large enough to satisfy the so-called "sufficient excitation condition", i.e. every combination vector of length p with elements from M-ASK or QAM alphabet $\mathcal{S}$ appears at least once in $\mathbf{S}$. Our objective is to recover $\mathbf{H}$ and $\mathbf{S}$ up to a permutation matrix and a diagonal matrix from the received data $\mathbf{X}$ only.

3. PROPOSED SOURCE SEPARATION ALGORITHM

3.1. Real Case: M-ASK Alphabets

We first consider the M-ASK signals, i.e.

$$\mathcal{S}_{\mathrm{M-ASK}} = \{\pm 1, \pm 3, \cdots, \pm(M-1)\}$$

Also, we assume that the channel matrix $\mathbf{H}$ and the noise matrix $\mathbf{W}$ are real since the complex equation $\mathbf{X} = \mathbf{HS} + \mathbf{W}$ can be easily converted into the following real equation

$$\begin{bmatrix} \mathbf{X}^R \\ \mathbf{X}^I \end{bmatrix} = \begin{bmatrix} \mathbf{H}^R \\ \mathbf{H}^I \end{bmatrix} \mathbf{S} + \begin{bmatrix} \mathbf{W}^R \\ \mathbf{W}^I \end{bmatrix} \quad (6)$$

where the superscripts $[\cdot]^R$ and $[\cdot]^I$ denote real and imaginary parts of the matrices, respectively. We now enumerate the steps for our source separation algorithm.

3.1.1. Clustering

In the presence of channel noise $\mathbf{w}(n)$, the observed data constellation is a union of clusters centered around the points $\tilde{\mathbf{x}}_i = \mathbf{H}\tilde{\mathbf{s}}_i, i = 1, \ldots, d$, where d is the number of clusters. Without loss of generality, the channel matrix $\mathbf{H}$ is assumed to be full column rank, and hence the number of clusters $d = M^p$. Define $\mathbf{S}_d \triangleq [\tilde{\mathbf{s}}_1 \ \cdots \ \tilde{\mathbf{s}}_d]$ which represents the $p \times d$ matrix containing exactly d distinct column vectors with elements from the M-ASK alphabet. The cluster centers $\tilde{\mathbf{x}}_i$ can be extracted by using the unsupervised clustering algorithms such as the Neural gas algorithm [9] and the smallest distance clustering algorithm [2] (See [9] for a comprehensive treatment of unsupervised clustering methods). If we concatenate the extracted cluster vectors $\tilde{\mathbf{x}}_i$ as $\mathbf{X}_d \triangleq [\tilde{\mathbf{x}}_1 \ \cdots \ \tilde{\mathbf{x}}_d]$, theoretically, the matrix has the form: $\mathbf{X}_d = \mathbf{HS}_d$.

3.1.2. Whitening

We now whiten the data set $\mathbf{X}_d$ by utilizing the following easily verified property: $\mathbf{S}_d\mathbf{S}_d^T = K_r\mathbf{I}$, where $K_r = 2M^{(p-1)}(1^2 + 3^2 + \cdots + (M-1)^2)$. Let $\mathbf{H} = \mathbf{UDV}^T$ be the singular value decomposition (SVD) of $\mathbf{H}$, thus we have

$$\frac{\mathbf{X}_d\mathbf{X}_d^T}{K_r} = \mathbf{HH}^T = \mathbf{UDD}^T\mathbf{U}^T = \bar{\mathbf{U}}\Sigma\bar{\mathbf{U}}^T \quad (7)$$

where $\bar{\mathbf{U}}$ denotes the submatrix of $\mathbf{U}$ from 1^{st} column to p^{th} column, $\Sigma \triangleq \mathrm{diag}(\sigma_1^2, \cdots, \sigma_p^2)$, σ_i denotes i^{th} singular value of $\mathbf{H}$. The whitening matrix is defined as $\mathbf{W} \triangleq \Sigma^{-\frac{1}{2}}\bar{\mathbf{U}}^T$. We can then form the whitened data set as

$$\mathbf{Z}_d = \mathbf{WX}_d = \mathbf{WHS}_d = \mathbf{QS}_d \quad (8)$$

where $\mathbf{Q}$ is a $p \times p$ real unitary channel matrix to be determined.

3.1.3. Geometric Approach for Channel Estimation and Source Recovery

The objective of this step is to estimate $\mathbf{Q}$ from the whitened data set $\mathbf{Z}_d$. Let $\mathrm{dis}(\tilde{\mathbf{z}}_i, \tilde{\mathbf{z}}_j) \triangleq \|\tilde{\mathbf{z}}_i - \tilde{\mathbf{z}}_j\|$ denotes the Euclidean distance in $\mathbb{R}^p$ between two constellation points. It is clear that we have the following

$$\begin{aligned} \mathrm{dis}(\tilde{\mathbf{z}}_i, \tilde{\mathbf{z}}_j) &= \|\tilde{\mathbf{z}}_i - \tilde{\mathbf{z}}_j\| = \|\mathbf{Q}(\tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j)\| = \|\tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j\| \\ &= \mathrm{dis}(\tilde{\mathbf{s}}_i, \tilde{\mathbf{s}}_j) \end{aligned} \quad (9)$$

Notice that for any $i \neq j$, $\mathrm{dis}(\tilde{\mathbf{s}}_i, \tilde{\mathbf{s}}_j)$ is minimized if and only if $\tilde{\mathbf{s}}_i$ and $\tilde{\mathbf{s}}_j$ differ only in one bit by 2, i.e. $\tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j = \pm 2\mathbf{e}_k, k \in \{1, \ldots, p\}$, where $\mathbf{e}_k$ denotes the unit vector with its k^{th} entry equal to one, and its other entries equal to zero. Therefore, for each pair of $\{\tilde{\mathbf{z}}_i, \tilde{\mathbf{z}}_j\}$ that minimizes $\mathrm{dis}(\tilde{\mathbf{z}}_i, \tilde{\mathbf{z}}_j)$, we have

$$\tilde{\mathbf{z}}_i - \tilde{\mathbf{z}}_j = \mathbf{Q}(\tilde{\mathbf{s}}_i - \tilde{\mathbf{s}}_j) = \pm 2\mathbf{Q}\mathbf{e}_k \ \ k \in \{1, \ldots, p\} \quad (10)$$

Thus some column of the unitary matrix $\mathbf{Q}$ can be determined up to a sign. It is clear that for each constellation point $\tilde{\mathbf{z}}_i$, there exist p nearest neighboring vectors $\tilde{\mathbf{z}}_{j_k}, k = 1, \ldots, p$ that allow us to recover all p distinct columns of $\mathbf{Q}$ up to a sign and a permutation of the columns. This implies that the received data constellation geometry is very rich in information pertaining to the channel. In this case, it is desirable to find a way that can extract the channel information from the constellation geometry more accurately at a moderate or low SNR. Notice that we have $\|\tilde{\mathbf{z}}_i\| = \|\tilde{\mathbf{s}}_i\|$ and usually, the constellation points with maximum vector norm contains the highest signal power, and hence achieves the highest SNR. Thus, these points are less likely to be confused. Therefore we can summarize our geometric approach as follows

1. Choose 2^p vectors $\tilde{\mathbf{z}}_{i_k}, k = 1, \ldots, 2^p$ that have maximum vector norm.
2. Choose one vector from $\tilde{\mathbf{z}}_{i_k}, k = 1, \ldots, 2^p$ as a reference vector such that $\|\tilde{\mathbf{z}}_{i_{ref}}\| - \sqrt{p}(M-1)$ is minimal.
3. Choose p nearest neighboring vectors $\tilde{\mathbf{z}}_{nb_k}, k = 1, \ldots, p$ of $\tilde{\mathbf{z}}_{i_{ref}}$ from $\tilde{\mathbf{z}}_{i_k}, k = 1, \ldots, 2^p$ by computing the Euclidean distance between $\tilde{\mathbf{z}}_{i_{ref}}$ and $\tilde{\mathbf{z}}_{i_k}$.
4. The unitary matrix $\mathbf{Q}$ is then estimated as

   $$\mathbf{Q}_e = \begin{bmatrix} \tilde{\mathbf{z}}_{i_{ref}} - \tilde{\mathbf{z}}_{nb_1} & \cdots & \tilde{\mathbf{z}}_{i_{ref}} - \tilde{\mathbf{z}}_{nb_p} \end{bmatrix} \quad (11)$$

   The column normalized $\mathbf{Q}_e$ is an estimate of $\mathbf{Q}$ up to a sign and a permutation of the columns.
5. The input symbols are estimated as $\mathbf{S}_e = \mathbf{Q}_e^{-1}\mathbf{WX}$.

3.2. Extension to The Complex Case: QAM Alphabets

We now discuss the extension of our source separation algorithm to the QAM alphabets, i.e. $\mathcal{S}_{\mathrm{QAM}} = \{\alpha + j\beta : \alpha, \beta \in$

$\mathcal{S}_{\mathrm{M-ASK}}\}$. The complex extension is described as follows. As for the QAM alphabets, we still have $\mathbf{S}_d\mathbf{S}_d^H = K_c\mathbf{I}$, where K_c is a constant implicitly determined by the alphabets and the number of sources p. Therefore the observed data set $\mathbf{X}_d$ can be whitened by following the same way as in the real case. We have $\mathbf{Z}_d = \mathbf{Q}\mathbf{S}_d$, where $\mathbf{Q}$ is a $p \times p$ complex unitary matrix (note that $\mathbf{H}$ is allowed to be a complex matrix). In order to apply the geometric approach presented in previous subsection, we transform the complex equation $\mathbf{Z}_d = \mathbf{Q}\mathbf{S}_d$ into the following real equation

$$\begin{bmatrix} \mathbf{Z}_d^R \\ \mathbf{Z}_d^I \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^R & -\mathbf{Q}^I \\ \mathbf{Q}^I & \mathbf{Q}^R \end{bmatrix} \begin{bmatrix} \mathbf{S}_d^R \\ \mathbf{S}_d^I \end{bmatrix} \triangleq \mathbf{Q}_T \begin{bmatrix} \mathbf{S}_d^R \\ \mathbf{S}_d^I \end{bmatrix} \quad (12)$$

where $\mathbf{Q}_T$ is a $2p \times 2p$ real unitary matrix. Thus we have successfully converted the QAM source separation problem into M-ASK source separation problem and can further estimate $\mathbf{Q}_T$ by using our proposed geometric approach. The construction of $\mathbf{Q}$ from the estimated $\mathbf{Q}_T$ is detailed in [4].

4. DISCUSSION

Our work can be considered as a further development of work [2] where the latter only permits BPSK signals. Both works are clustering-based and operate on a whitened data space. However, in contrast to the work [2] that is essentially an assignment algorithm, our work focuses on extracting the rich channel information hidden in the constellation geometry, as developed in [4]. This fundamental difference accounts for the wider applicability of our geometric approach. It has been shown that, in our work, the channel information is only related to the relative difference between two constellation points while irrespective of the exact positions of the constellation points. In contrast, when assigning the received constellation points to the corresponding source vectors, the constellation points themselves as well as their relationship with other points have to be considered. In other words, we conclude that the information needed for channel estimation is less than that needed for an appropriate assignment algorithm.

We consider the computational complexity of our proposed algorithm. The proposed algorithm involves three steps: clustering, whitening and geometric approach for channel identification and signal recovery. The clustering algorithm adopted in our simulations is the smallest distance clustering algorithm [2], which is self starting, uses each received data vector only once and works very well at moderately high SNR. The complexity for clustering is NC_p, where C_p are the computations required per iteration. Treating addition and multiplication equally, i.e. counting the flops, we find that the computations required per iteration are $\mathcal{O}(qd^2)$. Thus the complexity for clustering is $\mathcal{O}(Nqd^2)$. The complexity of step 2 and step 3 are respectively dominated by the correlation matrix $\mathbf{X}_d\mathbf{X}_d^T$ and signal recovery $\mathbf{S}_e = \mathbf{Q}_e^{-1}\mathbf{W}\mathbf{X}$, which have complexity $\mathcal{O}(q^2 d)$ and $\mathcal{O}(Nqp)$, respectively. Therefore, by combining all these steps, our proposed algorithm has an overall complexity $\mathcal{O}(Nqd^2 + q^2 d + Nqp)$. Since $d = M^p$, the computational complexity is exponential with respect to p. This suggests that our proposed algorithm is practical for separating a small number of discrete sources. In fact, the computational complexity is the most prohibitive issue in almost all geometric methods. On the other hand, the hyperplane-based algorithm does not need to do the clustering. The cost for signal whitening is identical to our proposed algorithms, $\mathcal{O}(q^2 d)$, while the cost for all the iterations is $\mathcal{O}(Iq^2 d)$, where I is the total number of iterations till the global convergence. Hence, the total computational cost of the hyperplane-based algorithm is $\mathcal{O}(q^2 d + Iq^2 d + Nqp)$.

5. SIMULATION RESULTS

We now present simulation results to illustrate the performance of our proposed algorithm. We compare our method to the iterative hyperplane-based algorithm proposed in [4] and the kurtosis-based algorithm proposed in [6]. For the hyperplane-based and kurtosis-based algorithms, the gradient search may converge to local minima. The magnitude of the residual $\frac{1}{Nq}\|\mathbf{X} - \mathbf{H}_e\mathbf{S}_e\|_F^2$ is a good measure to test the converged solution of the gradient search [1]. For the cases where the residual is not reduced to the noise power level, we restart the gradient search until the residual is decreased to the noise power level. In our simulations, we consider $p = 2$ source signals drawn from the 4-ASK alphabet $\{-3, -1, 1, 3\}$ arriving at $q = 2$ sensors. The entries of the tested channel matrices are independently chosen from a white Gaussian process. We Totaly tested 100 independent channels, with 1000 Monte Carlo runs for each channel realization. The symbol error rate shown in the figure is the overall average of them. In each run, we collect $N = 100$ data samples. Fig. 1 shows the symbol error rate (SER) of the respective algorithms as a function of SNR. We can see that our proposed algorithm achieves a slightly lower SER than the hyperplane-based algorithm and the kurtosis-based algorithms, especially at a moderately high SNR. In fact, the performance of our proposed algorithm is closely related to the adopted clustering algorithm. Hence, more accurate clustering techniques, particularly at low SNR, result in better performance of our proposed algorithm.

6. CONCLUSION

It has been shown that the received data constellation geometry contains rich information pertaining to the channel. Based on this observation, we develop a practical non-iterative algorithm for blind separation of digital signals with M-ASK and QAM alphabets. The proposed algorithm compares favorably with the existing hyperplane-based and kurtosis-based algorithms. Since only a small fraction of the constellation geometry is exploited in our geometric approach for channel estimation, it is desirable for us to devise an efficient and more

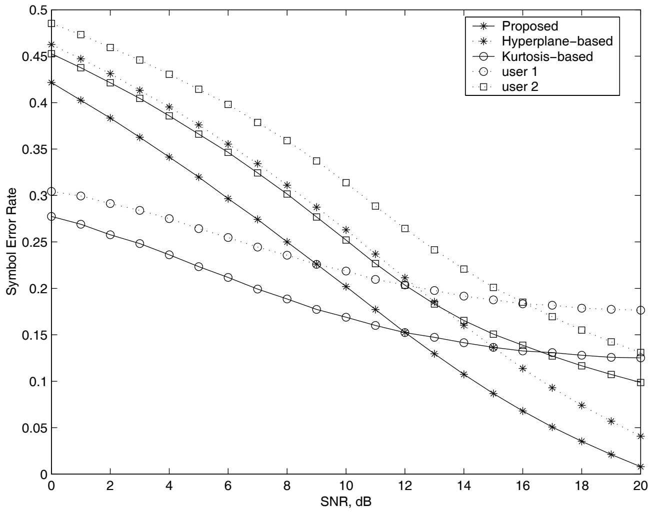

Fig. 1. Symbol Error Rate (SER) Versus SNR

accurate geometric channel estimation approach in the future by utilizing the constellation geometry to a full extent.

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