AN EFFICIENT SDP BASED DESIGN FOR PROTOTYPE FILTERS OF M-CHANNEL COSINE-MODULATED FILTER BANKS

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ABSTRACT

The paper presents an efficient semidefinite programming (SDP) based design for prototype filters of cosine-modulated filter banks (CMFBs). We consider a class of near-perfect reconstruction CMFBs with the linear phase prototype filter, which structurally eliminates the amplitude overall distortion. The prototype filter design problem is then formulated into a convex semi-infinite programming problem. Furthermore, to handle the semi-infinite constraints, we use the linear matrix inequality (LMI) characterization of positive trigonometric polynomials to cast the semi-infinite programming problem into SDP one. Finally, convex duality is applied to transform the SDP into another SDP with the minimal number of additional variables, which is efficiently solved. An additional advantage of the proposed method is that we can precisely control the filter specifications.

Index Terms— cosine-modulated filter bank, semidefinite programming, linear matrix inequality.

1. INTRODUCTION

Multirate filter banks are used in a variety of applications from data compression (speech, audio, image, and video) to communications (multicarrier modulation), and feature detection [1], [2]. Fig.1 illustrates a typical M-channel maximally decimated filter bank, where $H_k(z)$ and $F_k(z)$, $0 \leq k \leq M-1$ are analysis and synthesis filters, respectively. The design of general M-channel filter banks is very complicated. The reason is that a large number of parameters and constraints are required to be handled. However, modulated filter banks have been very attractive in practical applications due to their ease of design and implementation. In CMFBs, all the analysis and synthesis filters can be obtained by modulating the coefficient values of one prototype filter, and hence, the design of the filter bank reduces to that of the prototype filter. Moreover, there are efficient structures with fast transform for modulation implementation, so the cost of the analysis filter bank is the cost of one filter plus modulation overhead [1], [2].

The problem of designing the optimal prototype filter of CMFBs has been extensively studied in [1], [2], [3]. A method that structurally guarantees the perfect reconstruction property of the filter bank is lattice factorization [2]. This method requires a good initial point to optimize angles in a cascade of lattices, and requires solutions of highly nonlinear equations of the relations between the angles and the filter coefficients. Some other design approaches of the prototype are based on (local) nonlinear optimization [3], [4]. These methods can achieve filter banks with high stopband attenuation, low aliasing and amplitude distortion. However, a good initial filter is also required, and of course, the globally optimal solution cannot guaranteed. There are some approaches that avoid nonlinear

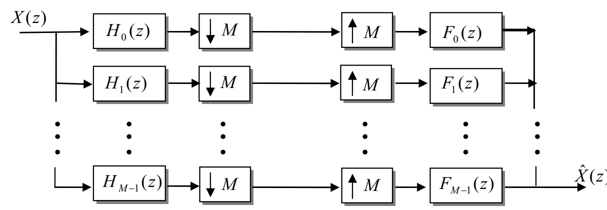

Fig. 1. M-channel maximally decimated filter bank.

optimization. For instance, the one presented in [5] is based on designing the product filter and then spectral factorization to generate the prototype filter. However, the resulting filter banks are not flat at $\omega = 0$ and $\omega = \pi$. Moreover, this method cannot be applied to design linear phase prototype filters. Another method is to limit the search of the prototype filters to the class of filters obtained using Kaiser window [6], which obtained a high stopband attenuation filter. However, the major disadvantage is the lack of control of the edge frequencies and passband ripple. Like other classical filter designs, the resulting filter usually has wide transition bandwidth.

In this paper, we show a class of cosine-modulated QMF banks with a prototype filter which structurally cancels the overall amplitude distortion. Then, the filter bank design problem boils down to designing the prototype filter which has a frequency response satisfying given specifications. By applying LMI characterization of positive trigonometric polynomials, the design problem is recast as a convex SDP problem. Different from the conventional (local) nonlinear optimization based CMFB design, the SDP problem can be efficiently solved, and more importantly, the globally optimal solution can be obtained. In addition, our method has an additional advantage that the tradeoff parameters are flexibly chosen to control the passband ripple and stopband attenuation, and hence, the stopband attenuation of the analysis and synthesis filters, and reconstruction error. The edge frequencies can also be controlled to obtain the filter with narrow transition bandwidth.

Notations : Boldfaced characters denote matrices and column vectors, with upper case used for the former and lower case for the latter. The notation $\mathbf{X} \geq 0$ denotes a (symmetric) positive semi-definite matrix. The inner product $\langle \mathbf{X}, \mathbf{Y} \rangle$ between the matrices $\mathbf{X}$ and $\mathbf{Y}$ is defined as Trace($\mathbf{XY}$), i.e. $\langle \mathbf{X}, \mathbf{Y} \rangle$= Trace($\mathbf{XY}$). For a given set $\mathbf{C} \subset \mathbb{R}^N$ its convex hull (conic hull), denoted by conv($\mathbf{C}$) (cone($\mathbf{C}$)), is the smallest convex set (cone) in $\mathbb{R}^N$ that contains $\mathbf{C}$.

2. PROBLEM FORMULATION

In the M-channel maximally decimated filter bank shown in Fig. 1, the reconstructed signal $\hat{X}(z)$ is given by

$$\hat{X}(z) = X(z)T_0(z) + \sum_{l=1}^{M-1} X(ze^{-j2\pi l/M})T_l(z) \tag{1}$$

where

$$T_l(z) = \frac{1}{M}\sum_{k=0}^{M-1} H_k(ze^{-j2\pi l/M})F_k(z).$$

Here, $T_0(z)$ is the overall distortion transfer function and $T_l(z)$, $l \neq 0$ is the aliasing transfer function. To cancel aliasing and achieve perfect reconstruction, it is required that

$$\begin{array}{ll} T_l(z) = 0 & \text{for } l = 1, 2, ..., M-1 \\ T_0(z) = cz^{-n_d}, & c \neq 0,\ n_d \text{ is positive integer.} \end{array} \tag{2}$$

In cosine-modulated QMF banks, all the analysis and synthesis filters can be generated by just modulating a linear phase lowpass prototype filter $H(z)$ with cutoff frequency $\frac{\pi}{2M}$ as follows:

$$h_k(n) = 2h(n)\cos\left(\frac{\pi}{M}(k+0.5)(n-\frac{N}{2}) + \theta_k\right)$$

$$f_k(n) = 2h(n)\cos\left(\frac{\pi}{M}(k+0.5)(n-\frac{N}{2}) - \theta_k\right)$$

for $0 \leq n \leq N,\ 0 \leq k \leq M-1$.

As the analysis and synthesis filters have narrow transition bands and high stopband attenuation, the overlap between nonadjacent filters is negligible. Moreover, it was shown in [3] that significant aliasing terms from the overlap of the adjacent filters are cancelled by choosing $\theta_k = (-1)^k\pi/4$. Under these circumstances, the overall distortion function is given by

$$T_0(e^{j\omega}) = \frac{e^{-j\omega N}}{M}\sum_{k=0}^{2M-1} |H(e^{j(\omega - k\pi/M - \pi/2M)})|^2. \tag{3}$$

It can be verified that $T_0(e^{j\omega})$ is periodic with period π/M, and so is approximated by

$$\begin{aligned} T_0(e^{j\omega}) \approx &\frac{e^{-j\omega N}}{M}\big[|H(e^{j(\omega - k\pi/M - \pi/2M)})|^2 \\ &+ |H(e^{j(\omega-(k+1)\pi/M-\pi/2M)})|^2\big], \end{aligned} \tag{4}$$

for $\omega \in [(k+\frac{1}{2})\frac{\pi}{M},\ (k+\frac{3}{2})\frac{\pi}{M}]$.

To eliminate amplitude distortion, $|T_0(e^{j\omega})|$ must be constant for all frequencies, i.e.,

$$|H(e^{j(\omega - k\pi/M - \pi/2M)})|^2 + |H(e^{j(\omega-(k+1)\pi/M-\pi/2M)})|^2 = 1,$$

for $\omega \in [(k+\frac{1}{2})\frac{\pi}{M},\ (k+\frac{3}{2})\frac{\pi}{M}]$, or, equivalently,

$$|H(e^{j\omega})|^2 + |H(e^{j(\omega-\pi/M)})|^2 = 1, \tag{5}$$

for $\omega \in [0,\ \pi/M]$.

Now, let $H(e^{j\omega})$ be a lowpass filter with the passband and stopband edges

$$\omega_p = (\frac{\pi}{2M} - \varepsilon), \qquad \omega_s = (\frac{\pi}{2M} + \varepsilon) \tag{6}$$

where $0 < \epsilon < \pi/2M$ decides the transition bandwidth. Assuming that the filter has small ripples in the passband and high attenuation in the stopband, we have

$$|H(e^{j\omega})|^2 + |H(e^{j(\omega-\pi/M)})|^2 \approx 1, \tag{7}$$

for $\omega \in [0,\ \omega_p] \cup [\omega_s,\ \pi/M]$.

In the transition band, if

$$|H(e^{j\omega})| = \cos\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right), \quad \omega \in [\omega_p,\ \omega_s], \tag{8}$$

then

$$\begin{aligned} &|H(e^{j\omega})|^2 + |H(e^{j(\omega-\pi/M)})|^2 \\ &= \cos^2\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right) + \sin^2\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right) = 1 \end{aligned} \tag{9}$$

In summary, the overall amplitude distortion will be completely canceled if the prototype filter has the following magnitude response

$$|H(e^{j\omega})| = \begin{cases} 1 & \omega \in [0,\ \omega_p], \\ \cos\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right) & \omega \in [\omega_p,\ \omega_s], \\ 0 & \omega \in [\omega_s,\ \pi]. \end{cases} \tag{10}$$

For simplicity, we assume that the linear phase prototype filter have even order, i.e., $N = 2L$. Then

$$H(e^{j\omega}) = e^{-j\omega N/2}H_R(\omega) \tag{11}$$

and the amplitude response is

$$H_R(\omega) = \sum_{k=0}^{L} g_k\cos(k\omega) = \mathbf{g}^T\boldsymbol{\varphi}_L(\omega) \tag{12}$$

with $\boldsymbol{\varphi}_L(\omega) = [1,\ \cos\omega,\ \cos 2\omega, ...,\ \cos L\omega]^T$ and

$$\mathbf{g} = [g_0,\ g_1,\ g_2,\ ...,\ g_L]^T = [h_L,\ 2h_{L-1},\ 2h_{L-1}, ...,\ 2h_0]^T.$$

Define the mean square error

$$\begin{aligned} \Phi(\mathbf{g}) =& W_p\int_0^{\omega_p} |H_R(\omega)-1|^2\frac{d\omega}{\pi} \\ &+ W_t\int_{\omega_p}^{\omega_s} |H_R(\omega) - \cos\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right)|^2\frac{d\omega}{\pi} \\ &+ W_s\int_{\omega_s}^{\pi} |H_R(\omega)|^2\frac{d\omega}{\pi} \end{aligned} \tag{13}$$

where $W_p + W_t + W_s = 1$. Here, W_p, W_t, W_s are tradeoff parameters among passband, transition band, and stopband performances. It can be shown that $\Phi(\mathbf{g})$ is a convex quadratic function in $\mathbf{g}$

$$\Phi(\mathbf{g}) = \mathbf{g}^T\mathbf{Q}\mathbf{g} + \mathbf{g}^T\mathbf{q} + r, \tag{14}$$

with a symmetric positive definite matrix $\mathbf{Q}$.

It is well known that the minimization of the mean square error may not always lead the small peak errors. Therefore, the constraints for the peak errors should be imposed.

$$\begin{array}{ll} |H_R(\omega)-1| \leq \delta_p, & \omega \in [0,\ \omega_p], \\ |H_R(\omega) - \cos\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right)| \leq \delta_t, & \omega \in [\omega_p,\ \omega_s], \\ |H_R(\omega)| \leq \delta_s, & \omega \in [\omega_s, \pi]. \end{array} \tag{15}$$

The filter bank design is now formulated as an optimization problem of a convex objective subject to semi-infinite constraints

$$\begin{array}{ll} \min\limits_{\mathbf{g}} & \mathbf{g}^T\mathbf{Q}\mathbf{g}+\mathbf{g}^T\mathbf{q}+r \\ \text{subject to} & -\delta_p \le \mathbf{g}^T\boldsymbol{\varphi}_L(\omega)-1\le\delta_p,\ \omega\in[0,\ \omega_p] \\ & -\delta_t \le \mathbf{g}^T\boldsymbol{\varphi}_L(\omega)-\cos\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right)\le\delta_t, \omega\in[\omega_p,\ \omega_s] \\ & -\delta_s\le \mathbf{g}^T\boldsymbol{\varphi}_L(\omega)\le\delta_s,\quad \omega\in[\omega_s,\ \pi]. \end{array} \tag{16}$$

The next section will show how to efficiently handle these semi-infinite constraints.

3. CONVERSION TO SDP

To express the above semi-infinite constraints by LMIs, we first express the term $\cos\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right)$ by the Kth-order trigonometric polynomial via interpolation:

$$\cos\left(\frac{\pi}{4\varepsilon}(\omega-\omega_p)\right)=\sum_{i=0}^{K}c_i\cos(i\omega) \tag{17}$$

where

$$\begin{bmatrix} 1 & \cos\omega_0 \ldots & \cos K\omega_0 \\ 1 & \cos\omega_1 \ldots & \cos K\omega_1 \\ \vdots & \vdots & \vdots \\ 1 & \cos\omega_K \ldots & \cos K\omega_K \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_K\end{bmatrix}=\begin{bmatrix}\cos\left(\frac{\pi}{4\varepsilon}(\omega_0-\omega_p)\right)\\ \cos\left(\frac{\pi}{4\varepsilon}(\omega_1-\omega_p)\right)\\ \vdots \\ \cos\left(\frac{\pi}{4\varepsilon}(\omega_K-\omega_p)\right)\end{bmatrix}$$

$$\omega_i=\frac{\omega_s-\omega_p}{K}i+\omega_p,\qquad i=0,1,...,K.$$

Define a trigonometric curve

$$\mathbf{C}_{a,b}=\{\boldsymbol{\varphi}_L(\omega):\cos\omega\in[\cos a,\ \cos b]\}\subset\mathbb{R}^{L+1} \tag{18}$$

and its polar $\mathbf{C}^*_{a,b}$ is given by

$$\mathbf{C}^*_{a,b}=\left\{\mathbf{u}\in\mathbb{R}^{L+1}:\langle\mathbf{u},\mathbf{v}\rangle\ge 0\quad\forall\mathbf{v}\in\mathbf{C}_{a,b}\right\}. \tag{19}$$

Then, the semi-infinite linear constraints in (16) are rewritten as

$$\beta_i\mathbf{g}+\mathbf{d}_i\in\mathbf{C}^*_{a_i,b_i}, i=1,2,...,6. \tag{20}$$

where

$$\begin{aligned} [\beta_1,\ \beta_2,\ \beta_3,\ \beta_4,\ \beta_5,\ \beta_6]&=[1,-1,\ 1,-1,\ 1,-1],\\ \mathbf{d}_1&=[\delta_p-1,0,0,...,0]^T,\\ \mathbf{d}_2&=[\delta_p+1,0,0,...,0]^T,\\ \mathbf{d}_3&=[\delta_t-c_0,-c_1,...,-c_K,0,...,0]^T,\\ \mathbf{d}_4&=[\delta_t+c_0,c_1,...,c_K,0,...,0]^T,\\ \mathbf{d}_5&=[\delta_s,0,0,...,0]^T,\\ \mathbf{d}_6&=[\delta_s,0,0,...,0]^T,\\ [a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6]&=[\omega_p,\ \omega_p,\ \omega_s,\ \omega_s,\ \pi,\ \pi],\\ [b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6]&=[0,\ 0,\ \omega_p,\ \omega_p,\ \omega_s,\ \omega_s]. \end{aligned}$$

Then, the optimization problem (16) can be expressed as

$$\begin{array}{ll}\min\limits_{\mathbf{g}} & \mathbf{g}^T\mathbf{Q}\mathbf{g}+\mathbf{g}^T\mathbf{q}+r\\ \text{subject to} & \beta_i\mathbf{g}+\mathbf{d}_i\in\mathbf{C}^*_{a_i,b_i},\ i=1,2,...,6.\end{array} \tag{21}$$

It was shown in [7], [8] that each of constraints $\beta_i\mathbf{g}+\mathbf{d}_i\in\mathbf{C}^*_{a_i,b_i}$ can be expressed as a set of linear equations and two LMI constraints of two symmetric positive semidefinite matrix variables. Therefore, in addition to vector variable $\mathbf{g}\in\mathbb{R}^{L+1}$, the constraints in (21) introduce 12 symmetric matrix variables of dimension roughly $[L/2]\times[L/2]$.

In order to reduce a number of additional variables, we employ convex duality. We first definite Lagrangian function associated with the problem (21):

$$L(\mathbf{g},\mathbf{y}_i)=\mathbf{g}^T\mathbf{Q}\mathbf{g}+\mathbf{q}^T\mathbf{g}+r-\sum_{i=1}^{6}(\beta_i\mathbf{g}+\mathbf{d_i})^T\mathbf{y}_i \tag{22}$$

where vectors $\mathbf{y}_i\in\mathbb{R}^{L+1}$ are dual variables. It means that $\mathbf{y}_i\in(\mathbf{C}^*_{a_i,b_i})^*$. We also note that $(\mathbf{C}^*_{a_i,b_i})^*=((\text{cone}(\mathbf{C}_{a_i,b_i}))^*)^*=\text{cone}(\mathbf{C}_{a_i,b_i})$. The minimum of the Lagrangian over $\mathbf{g}$ is given by

$$\begin{aligned} D(\mathbf{y}_i)&=\min_{\mathbf{g}}L(\mathbf{g},\mathbf{y}_i) \qquad (23)\\ &=-\frac{1}{4}\left(\mathbf{q}-\sum_{i=1}^{6}\beta_i\mathbf{y}_i\right)^T\mathbf{Q}^{-1}\left(\mathbf{q}-\sum_{i=1}^{6}\beta_i\mathbf{y}_i\right)\\ &\quad-\sum_{i=1}^{6}\mathbf{d_i}^T\mathbf{y}_i+r.\end{aligned}$$

Then the dual problem is given

$$\begin{array}{ll}\max\limits_{\mathbf{y}_i} & D(\mathbf{y}_i)\\ \text{subject to} & \mathbf{y}_i\in\text{cone}(\mathbf{C}_{a_i,b_i}), i=1,2,...,6.\end{array} \tag{24}$$

Using Schur's complement [9], the optimization problem above can be rewritten as

$$\begin{array}{ll}\max\limits_{\mathbf{y}_i} & -\eta-\sum_{i=1}^{6}\mathbf{d_i}^T\mathbf{y}_i+r\\ \text{subject to} & \begin{bmatrix}\eta & (\mathbf{q}-\sum_{i=1}^{6}\beta_i\mathbf{y}_i)^T\\ \mathbf{q}-\sum_{i=1}^{6}\beta_i\mathbf{y}_i & 4\mathbf{Q}\end{bmatrix}\ge 0\\ & \mathbf{y}_i\in\text{cone}(\mathbf{C}_{a_i,b_i}), i=1,2,...,6.\end{array} \tag{25}$$

Define the $k-th$ moment trigonometric $\boldsymbol{\mathcal{T}}_k(\omega)$:

$$\boldsymbol{\mathcal{T}}_k(\omega)=\boldsymbol{\varphi}_k(\omega)\boldsymbol{\varphi}_k^T(\omega)\text{ and }\boldsymbol{\mathcal{T}}_{1k}(\omega)=\cos\omega\boldsymbol{\mathcal{T}}_k(\omega) \tag{26}$$

and given a vector variable $\mathbf{y}=[y_0,y_1,...,y_{2k+1}]^T$, a matrix function $\mathbf{T}_{1k}(\mathbf{y})$ is created from $\boldsymbol{\mathcal{T}}_{1k}(\omega)$ by the change of variables

$$\cos\ell\omega\leftarrow y_\ell,\quad \ell=0,1,...,2k+1. \tag{27}$$

The following theorem will show that $\mathbf{y}_i\in\text{cone}(\mathbf{C}_{a_i,b_i})$ are described by LMIs.

Theorem 1 *[7], [8] The conic hull cone$(\mathbf{C}_{a,b})$ of the trigonometric curve $\mathbf{C}_{a,b}$ is fully characterized by LMIs: $\mathbf{y}\in$ cone$(\mathbf{C}_{a,b})$ if and only if it satisfies the LMIs*

$$\cos b\mathbf{T}_{[L/2]}(\mathbf{y})\ge\mathbf{T}_{1[L/2]}(\mathbf{y})\ge\cos a\mathbf{T}_{[L/2]}(\mathbf{y}). \tag{28}$$

The convex hull conv$(\mathbf{C}_{a,b})$ of $\mathbf{C}_{a,b}$ is also fully characterized by LMIs: $\mathbf{y}\in$ conv$(\mathbf{C}_{a,b})$ if and only if it satisfies the LMIs (28) with $y_0=1$.

Note that for L even, by the definition, $\mathbf{T}_{1[L/2]}$ is a matrix function of $[y_0,\ y_1,\ ...,\ y_{L+1}]^T$ and accordingly LMIs (28) are understood for some y_{L+1}.

From the theorem 1, it is clear that each of constraints $\mathbf{y}_i \in \text{cone}(\mathbf{C}_{a_i,b_i})$ can be expressed by two LMI constraints with additional variables of dimension $L+1$. Therefore, the total number of scalar variables now is down to $6(L+1)+1$. The optimization problem (25) is thus in a ready SDP form and its optimal solution can be efficiently computed by available SDP software packages. Then, the globally optimal solution of the primal problem (21) is derived from the solution of the dual one by

$$\mathbf{g}_{opt} = -\frac{1}{2}\mathbf{Q}^{-1}\left(\mathbf{q} - \sum_{i=1}^{6}\beta_i \mathbf{y}_{iopt}\right). \tag{29}$$

4. DESIGN EXAMPLE

In this section, an example is presented to illustrate the effectiveness of the proposed method. The result is compared to that of another method which is based on nonlinear optimization. A 17-channel cosine modulated pseudo-QMF bank with the prototype filter order $N = 102$ is designed. The other specifications are $\omega_p = \frac{3.1176}{10000}\pi$, $\omega_s = 0.0585\pi$, and peak passband ripple $\delta_p = 10^{-3}$, transition band peak error $\delta_t = 10^{-3}$, peak stopband ripple $\delta_s = 10^{-2}$. The magnitude responses of the optimized prototype filter $H(e^{j\omega})$, the corresponding analysis filters $H_k(e^{j\omega})$, the overall distortion function $M|T_0(e^{j\omega})|$ and the aliasing error function $E_a(e^{j\omega}) = \sqrt{\sum_{l=1}^{M-1}|T_l(e^{j\omega})|^2}$ are plotted in Fig.2, respectively. Note that the stopband attenuation of $H(e^{j\omega})$ and $H_k(e^{j\omega})$ is about $-45dB$. The maximum peak to peak ripple of $M|T_0(e^{j\omega})|$, denoted E_{pp}, is $8.574e-03$, and the maximum of the aliasing error, $E_a = 2.856e-04$. The comparison of our result with that of nonlinear based method in [2] is shown in Table 1. As expected, the transition width of the designed filter by [2] is much wider than that settled by our method.

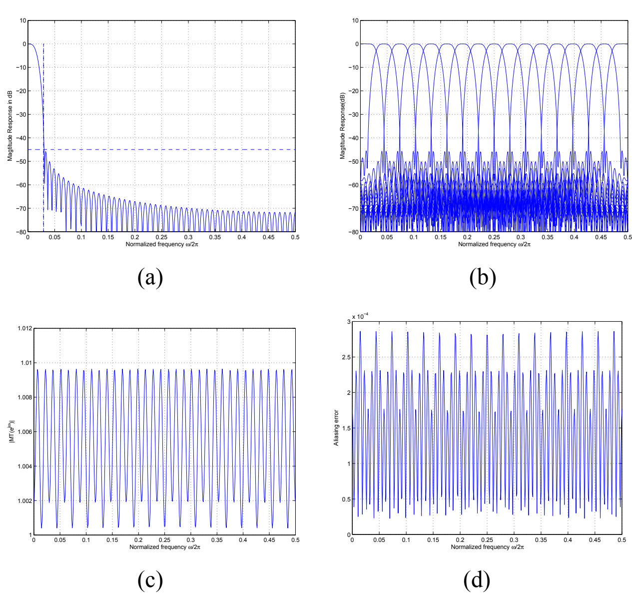

Fig. 2. (a) Magnitude response of the optimized prototype $H(e^{j\omega})$ with N=102; (b) magnitude response plots for the analysis filters $H_k(e^{j\omega})$; (c) magnitude response plot for the overall distortion $M|T(e^{j\omega})|$; (d) magnitude response plot of aliasing error $E_a(e^{j\omega})$.

Table 1. Comparison between our method and the method in [2]

	Method in [2]	Our method
Filter order	101	102
Stopband attenuation (dB)	40.65	45
Stopband edge ω_s	0.0590π	0.0585π
Reconstruction error E_{pp}	$6.790e-03$	$8.574e-03$
Aliasing error E_a	$3.794e-04$	$2.856e-04$

5. CONCLUDING REMARKS

A new design approach for optimizing the prototype filter of CMFBs was presented. The prototype filter design is formulated as a convex SDP problem. Compared to the nonconvex nonlinear optimization based designs in [2], which the resulting prototype filter is sensitive to the initial filter, our method can be obtained the globally optimal prototype filter. Compared to the method presented in [6], our method has an additional advantage that it can totally control the filter specifications such as edge frequencies and peak ripples. In addition, the weights in our objective function can be used to control the tradeoff between the passband ripple, and stopband attenuation, and consequently, reconstruction error. Much more simulation is under way to confirm the theoretical advantage of our method over other existing ones.

6. REFERENCES

[1] G. Strang and T. Q. Nguyen, *Wavelets and Filter banks*, Wellesley, MA: Wellesley-Cambridge, 1997.

[2] P.P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs, NJ: Prentice-Hall, 1993.

[3] T. Q. Nguyen, "Near-perfect-reconstruction pseudo-QMF banks," *IEEE Trans. Signal Processing*, vol. 42, no. 1, pp. 65–76, Jan. 1994.

[4] T. Q. Nguyen, "Digital filter bank design quadratic-constrained formulation," *IEEE Trans. Signal Processing*, vol. 43, no. 9, pp. 2103–2108, Sep. 1995.

[5] R. D. Koilpillai and P. P. Vaidyanathan, "A spectral factorization approach to pseudo-QMF design," *IEEE Trans. Signal Processing*, vol. 41, no. 10, pp. 82–92, Jan. 1993.

[6] Y. P. Lin and P. P. Vaidyanathan, "A Kaiser window approach for the desgin of prototype filters of cosine modulated filterbanks," *IEEE Signal Processing Letter*, vol. 5, no. 6, pp. 132–134, Jun. 1998.

[7] H. H. Kha, H. D. Tuan, B. Vo, and T. Q. Nguyen, "Symmetric orthogonal complex-valued filter bank design by semidefinite programming," *Proc. of 2006 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Toulouse, France*, vol. 3, pp. 221–224, May 2006.

[8] H. D. Tuan, T. T. Son, B. Vo, and T. Q. Nguyen, "LMI characterization for the convex hull of trigonometric curves and applications," *Proc. of 2005 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Philadelphia*, May 2005.

[9] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2003.