DESIGN OF OVERSAMPLED FILTER BANK BASED WIDELY LINEAR OQPSK EQUALIZER

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ABSTRACT

In this paper, the design and analysis of multirate filter bank (FB) based OQPSK widely linear (WL) oversampled equalizers for frequency selectively channels (FSCs) are studied. We focus on OQPSK signal transmissions in FSCs with various amount of fractionally spaced equalization using WL processing under the framework of FB. We explicitly represent a OQPSK communications system as a FB which allows the study of conditions for perfect reconstruction using a finite length polynomial equalizer. Furthermore, WL FB based MMSE oversampled equalizer designs operating at various sampling rates are presented. Computer simulations are simulated to study the performance of various types of equalizers in FSCs.

Index Terms— Filter Bank, OQPSK, Equalizer

1. INTRODUCTION

In this paper, the design and analysis of multirate filter bank (FB) [1] based OQPSK [2] widely linear (WL) [3, 4] oversampled equalizers for frequency selective channels (FSCs) are studied. In a OQPSK communications system, the I-channel and the Q-channel are offset by a half symbol period. The offset induces improper [5] property in the transmitted signal which can be exploited by receivers (Rxs) through widely linear processing (WLP). Many other signals are also improper such as BPSK, OQAM, etc. The works in [6, 3, 5] provide good backgrounds and applications of WLP. There are some existing works [3, 7] of improper signal equalization in FSCs. The authors in [7] focused on using improper signal property and non-redundant precoder with WLP to equalize OQPSK signal in a FSC. The authors in [3] focused on improper signal transmission in a FSC. They proposed a WL-DFE structure which provides substantial gains compared with the corresponding linear processing part. These equalizers are WL as they linearly combine the received signal and its conjugate[5].

We focus on a OQPSK communications system with oversampling at the Rx[1] with WLP under the framework of FB [1, 8, 9]. Since oversampling provides better multipaths resolution in a FSC, this motivates us to represent the overall communications system with an oversampled equalizer as a FB. When a OQPSK system is employed in a FSC, channel equalization is required. Direct equalization at symbol level does not exploit the structure of the transmitted OQPSK symbols. To illustrate this point, we use a FB to illustrate this offset, such that the overall transmitter (Tx) and the communications channel can be considered as an analysis polyphase matrix [1]. At the Rx, the sampled received sequence and its conjugated version are simultaneously processed by the synthesis polyphase matrix. It can be shown that, due to the OQPSK signal property, the conjugate operation does not change the transmitted sequence, but only the transfer function is conjugated. Hence, the Rx experiences 2 different channels from the Tx to the Rx. This contributes the resultant communications system as a non-maximally decimated FB [1]. To demonstrate this redundancy, the resultant block channel matrix is decomposed by Smith Normal Form Decomposition [1] to analyze when the channel can have a FIR inverse.

We further explore oversampling at the equalizer. Through oversampling, there are more virtual channels between the Tx and the Rx. The number of channels is equal to the number of rows of an analysis polyphase matrix. Various FB based oversampled WL MMSE equalizers are simulated in different FSC scenarios. Our simulations show that under certain channel conditions, higher oversampling MMSE OQPSK equalizers can recover the original transmitted signal better than those equalizers operating at a lower OSR.

2. PROBLEM FORMULATION

Fig. 1 shows a OQPSK communication model in a FSC.

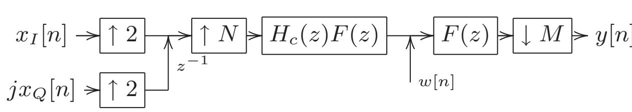

Fig. 1. An oversampled OQPSK communication model.

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[1] In this work, we assume the symbol period is T. The equalizer is operated at rate $\frac{2N}{MT}$, with $2N$ is divisible by M. The oversampling ratio (OSR) in this case is $\frac{2N}{M}$.

In this figure, $x_I[n]$ and $x_Q[n]$ together form the QPSK signal constellation with a period T. These two sequences are upsampled by 2. The Q-channel output is multiplied by j and delayed by $T/2$ period. The outputs of these two sequences are then combined to give $S(z) = X_I(z^2) + X_Q(z^2)z^{-1}j$. $S(z)$ is upsampled by N and filtered by the root raise cosine (RRC) filter $F(z)$ before transmission. The transmitted sequence is distorted by the channel $H_c(z)$ and corrupted by the additive noise $w[n]$. At the Rx, the received signal is sampled at a rate of $2N/T$ and then filtered by the RRC filter $F(z)$ to obtain $r_F[n]$. $r_F[n]$ is downsampled by a factor of M (assuming $2N$ is divisible by M) to obtain $y[n]$. Our goal is to design the equalizer which processes $y[n]$ to estimate the transmitted symbols $x_I[n]$ and $x_Q[n]$. Noticed that through controlling the downsampling rate M, one can operate the equalizer at various oversampling ratios (versus the symbol rate). Also through controlling the upsampling rate N, one can simulate multipath position at a higher resolution.

To derive the underlying FB structure for this OQPSK system, we rewrite overall input-output transfer function as $H(z) = F(z)H_c(z)F(z)$. If $H(z)$ is moved through the upsampler, then it can be represented as a pseudo circulant matrix (PCM) [10] $\mathbf{H}(z)$ of dimension $2N$ by 2. The resultant structure is shown in Fig. 2.

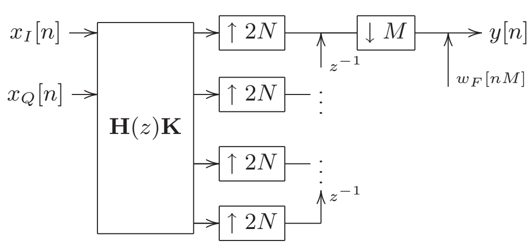

Fig. 2. The PCM form for the overall channel matrix.

From this figure, notice that the complex multiplication of j on Q-channel is absorbed into the matrix $\mathbf{K} = diag(1, j)$. Moreover, the filtered noise sequence $w_F[n]$ is downsampled to $w_F[nM]$.

The next step is to simplify the upsampling and downsampling operations in Fig. 2. There are $2N$ outgoing branches sum together. We assume $2N$ is divisible by M. For each of these $2N$ branches, the downsampling and upsampling operators can be combined if the number of delay in between is a multiple of $2N/M$, otherwise that branch is open. This effect can be modeled by the matrix $\mathbf{D}_M$: The product $\mathbf{D}_M\mathbf{H}(z)\mathbf{K}$ only keeps every Mth row of the matrix $\mathbf{H}(z)\mathbf{K}$. At the Rx, $y[n]$ is blocked into the vector $\mathbf{y}[n] = [y[n]\; y[n-1] \;\cdots y[n-2N/M+1]]^T$ and its z-transform is $\mathbf{Y}(z)$. The overall input-output equation is:

$$\mathbf{Y}(z) = \mathbf{D}_M\mathbf{H}(z)\mathbf{K}\mathbf{X}(z) + \mathbf{B}(z), \tag{1}$$

where $\mathbf{B}(z)$ is the z-transform of the vector $\mathbf{b}[n] = [w_F[nM]\; w_F[(n-1)M] \;\cdots\; w_F[(n-2N/M-1)M]]$. Since $\mathbf{X}(z) = \mathbf{X}^*(z^*)$ and $\mathbf{D}_M = \mathbf{D}_M^*$, the input-output equation for $\mathbf{y}^*[n]$ in z-transform is:

$$\mathbf{Y}^*(z^*) = \mathbf{D}_M\mathbf{H}^*(z^*)\mathbf{K}^*\mathbf{X}(z) + \mathbf{B}^*(z^*). \tag{2}$$

Stacking (1) and (2) yield

$$\tilde{\mathbf{Y}}(z) = \tilde{\mathbf{H}}(z)\mathbf{X}(z) + \tilde{\mathbf{B}}(z), \tag{3}$$

where $\tilde{\mathbf{Y}}(z) = \begin{bmatrix} \mathbf{Y}(z) \\ \mathbf{Y}(z^*)^* \end{bmatrix}$, $\tilde{\mathbf{H}}(z) = \begin{bmatrix} \mathbf{H}(z)\mathbf{K} \\ \mathbf{H}(z^*)^*\mathbf{K}^* \end{bmatrix}$, $\tilde{\mathbf{B}}(z) = \begin{bmatrix} \mathbf{B}(z) \\ \mathbf{B}(z^*)^* \end{bmatrix}$, $\tilde{\mathbf{y}}[n] = \begin{bmatrix} \mathbf{y}[n] \\ \mathbf{y}^*[n] \end{bmatrix}$ and $\tilde{\mathbf{b}}[n] = \begin{bmatrix} \mathbf{b}[n] \\ \mathbf{b}^*[n] \end{bmatrix}$. Let the equalizer length be L_G, then the time domain equation of (3) is:

$$\tilde{\mathcal{Y}}[n] = \tilde{\mathcal{H}}\mathcal{X}[n] + \tilde{\mathcal{B}}[n], \tag{4}$$

where $\tilde{\mathcal{Y}}[n]^H = [\tilde{\mathbf{y}}[n]^H \;\; \tilde{\mathbf{y}}[n-1]^H \;\; \cdots \;\; \tilde{\mathbf{y}}[n-L_G-1]^H]$, $\mathcal{X}[n]^H = [\mathbf{x}[n]^H \;\; \mathbf{x}[n-1]^H \;\; \cdots \;\; \mathbf{x}[n-L_G-L_H]^H]$, $\tilde{\mathcal{B}}[n]^H = [\tilde{\mathbf{b}}[n]^H \;\; \tilde{\mathbf{b}}[n-1]^H \;\; \cdots \;\; \tilde{\mathbf{b}}[n-L_G-1]^H]$ and $\tilde{\mathcal{H}} = \begin{bmatrix} \tilde{\mathbf{H}}[0] & \cdots & \tilde{\mathbf{H}}[L_H-1] & \mathbf{0} & \cdots \\ \mathbf{0} & \tilde{\mathbf{H}}[0] & \cdots & \tilde{\mathbf{H}}[L_H-1] & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{H}}[0] & \cdots & \tilde{\mathbf{H}}[L_H-1] \end{bmatrix}$. At the Rx, the decision variable $\hat{\mathbf{x}}[n] = \mathcal{G}^H \cdot \mathcal{Y}[n]$ is constructed to estimate $x_I[n]$ and $x_Q[n]$ where $\mathcal{G}^H = [\mathbf{G}[0]^H \;\; \mathbf{G}[1]^H \;\cdots\; \mathbf{G}[L_G-1]^H]$. The overall OQPSK equivalent FB structure is shown in Fig. 5.

3. FILTER BANK BASED EQUALIZERS

From (3), ignoring the noise for the moment, the matrix $\tilde{\mathbf{H}}(z)$ can be viewed as an analysis matrix [1]. If there exists a matrix $\mathbf{G}^H(z^*)$ such that $\mathbf{G}^H(z^*) \cdot \tilde{\mathbf{H}}(z) = z^{-N_d} \cdot \mathbf{I}_2$, then this system achieves perfect reconstruction (PR) [1]. It is noted that PR constraint is the same as ZF criteria. Performance-wise, performance of ZF equalizers may suffer due to noise enhancement [2]. Therefore it is more favorable to use MMSE type equalizers.

3.1. MMSE FB Equalizers

Using the proposed FB structure as the equalizer, the estimation error is given by $\mathbf{e}[n] = \mathcal{G}^H\mathcal{Y}[n] - \begin{bmatrix} x_I[n-N_d] \\ x_Q[n-N_d] \end{bmatrix} = \begin{bmatrix} \hat{x}_I[n] - x_I[n-N_d] \\ \hat{x}_Q[n] - x_Q[n-N_d] \end{bmatrix}$, where N_d is some positive integer to account for the casuality of the system. Assuming $\sigma_x^2 = 1$, the MMSE solution which minimizes trace($\mathbb{E}[\mathbf{e}[n]\mathbf{e}[n]^H]$) is

$$\mathcal{G}_{MMSE} = (\tilde{\mathcal{H}}\tilde{\mathcal{H}}^H + \mathcal{R}_{\tilde{\mathcal{B}},\tilde{\mathcal{B}}})^{-1}\tilde{\mathcal{H}}\mathbf{\Delta}_{N_d}, \tag{5}$$

where $\mathcal{R}_{\tilde{\mathcal{Y}},\tilde{\mathcal{Y}}} = \tilde{\mathcal{H}}\tilde{\mathcal{H}}^H + \mathcal{R}_{\tilde{\mathcal{B}},\tilde{\mathcal{B}}}$, $\mathcal{R}_{\tilde{\mathcal{Y}},\tilde{\mathcal{X}}} = \tilde{\mathcal{H}}\mathbf{\Delta}_{N_d}{}^2$, and $\mathcal{R}_{\tilde{\mathcal{B}},\tilde{\mathcal{B}}} = \mathbb{E}[\tilde{\mathcal{B}}[n]\tilde{\mathcal{B}}[n]^H]$. Alternatively, we can rewrite (5) using matrix inversion lemma [11] to obtain:

$$\mathcal{G}_{MMSE} = \mathcal{R}^{-1}_{\tilde{\mathcal{B}},\tilde{\mathcal{B}}}\mathcal{H}(\tilde{\mathcal{H}}^H\mathcal{R}^{-1}_{\tilde{\mathcal{B}},\tilde{\mathcal{B}}}\tilde{\mathcal{H}} + \mathbf{I})^{-1}\mathbf{\Delta}_{N_d}. \tag{6}$$

It is noted that $\mathcal{R}_{\tilde{\mathcal{B}},\tilde{\mathcal{B}}}$ is assumed to be non-singular. In practice, this may be singular, so a small diagonal term is added to regularize the matrix. Depending on $\mathcal{H}$, (4) maybe over or underdetermined (depending on the OSR). If $\mathcal{H}$ is a tall matrix, (5) can be used, otherwise, (6) is used. Using the $\mathcal{G}_{MMSE}$, the resultant error magnitude is: $\sigma^2_{MMSE} = \text{trace}(\mathbf{I}_2 - \mathcal{G}^H_{MMSE}\mathcal{R}^{-1}_{\tilde{\mathcal{Y}},\tilde{\mathcal{Y}}}\mathcal{G}_{MMSE})$. The SNR at the MMSE equalizer output is $\text{SNR}_{MMSE} = (\text{trace}(\mathbf{I}_2 - \mathcal{G}^H_{MMSE}\mathcal{R}^{-1}_{\tilde{\mathcal{Y}},\tilde{\mathcal{Y}}}\,\mathcal{G}_{MMSE}))^{-1}$.

4. CONDITIONS OF EQUALIZATIONS

From Fig. 5, all the transfer functions of the system is described by $\tilde{\mathbf{H}}(z)$. This matrix is a $\frac{2N}{M}$ by 2 polynomial matrix. Notice that if a finite length polynomial matrix inverse of $\tilde{\mathbf{H}}(z)$ exists, that may lead to good equalizer performance. For the case of oversampling by 2, (i.e. $N = 1$), $\tilde{\mathbf{H}}(z)$ has the form: $\tilde{\mathbf{H}}(z) = \begin{bmatrix} H_0(z) & jH_1(z)z^{-1} \\ H_1(z) & jH_0(z) \\ H_0^*(z^*) & -jH_1^*(z^*)z^{-1} \\ H_1^*(z^*) & -jH_0^*(z^*) \end{bmatrix}$. It can be shown that[3] the above system $\tilde{\mathbf{H}}(z)$ achieves PR with a FIR polynomial matrix if and only if $H_0(z)$, $H_1(z)$, $H_0^*(z^*)$, $H_1^*(z^*)$ are all relatively prime. If $gcd(H_0(z), H_1(z), H_0^*(z^*), H_1^*(z^*)) = \alpha(z)$, one can use Smith Form Decomposition to decompose $\tilde{\mathbf{H}}(z)$ into $\alpha(z)\mathbf{U}(z)\begin{bmatrix}\mathbf{I}_2 \\ \mathbf{0}_2\end{bmatrix}\mathbf{V}(z)$ for some unimodular matrices $\mathbf{U}(z)$ and $\mathbf{V}(z)$. When $\alpha(z)$ is a nontrivial FIR filter, it is not possible to have a finite length FIR polynomial matrix equalizer. To resolve this problem, the equalizer may need to operate at a higher sampling rate. This may resolve the multipaths at a higher accuracy and potentially lead to a FIR polynomial matrix inverse.

5. SIMULATIONS AND DISCUSSION

In this section, various OQPSK equalizers are simulated. The channel between the Tx and Rx is simulated at 8 times the symbol rate. The RRC filter at the Rx is operated at 8 times the symbol rate followed by downsampling to appropriate rate at the equalizer. In all simulations, the equalizer length L_G and the delay N_d is set to 64 and 32 respectively. Rxs are assumed to have ideal channel estimates. The RRC filter has a roll off of 0.22 and length of 17. 500 channel realizations are simulated for each SNR in order to estimate the average BER performance of various equalizers.

[2] $\mathbf{\Delta}_{N_d} = [\delta_{N_d}\ \delta_{N_d+1}]$ and δ_{N_d} is a column vector in the form $[\delta_{N_d}]_{k,1} = \delta[k - N_d]$.

[3] Proof is omitted due to page limitation.

Ex 1. In this example, the OQPSK signal is transmitted through a relative mild multipath channel. The channel delay profile is $0, 1.25T, 2.35T$. The power of these multipaths are $0, -3, -6$ (dB) which their total power is normalized to unity. The BER performance is shown in Fig. 3.

Ex 2. In this example, the OQPSK signal is transmitted through a relative poor multipath channel. The channel delay profile is $0, 2.35T, 3.45T$. The power of these multipaths are $0, 0, 0$ (dB) which their total power is normalized to unity. Its performance is shown in Fig. 4.

In both examples, ZF equalizers are simulated as a reference.

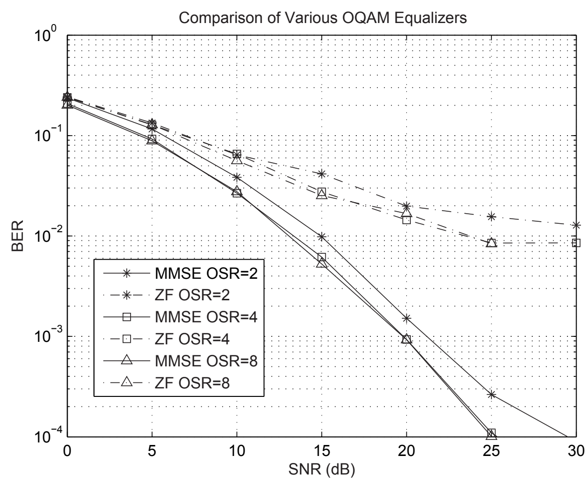

Fig. 3. BER curves for various OQPSK equalizers in Ex. 1

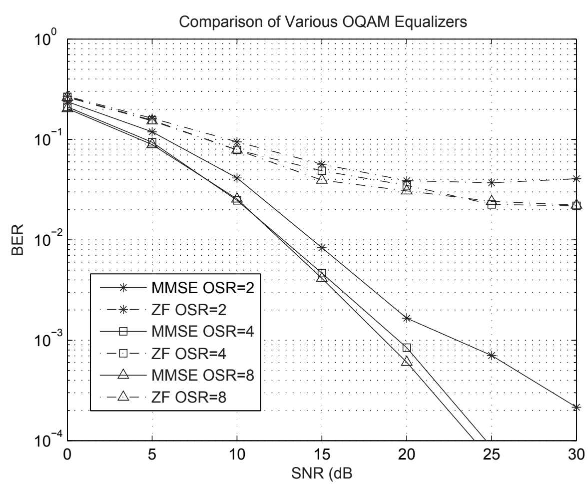

Fig. 4. BER curves for various OQPSK equalizers in Ex. 2

From Fig. 3 and Fig. 4, we observe that the performance of MMSE equalizers is much better than that of ZF equalizers. The BER curves of ZF equalizers reach error floors much earlier than those of the MMSE equalizers. This performance limitation is due to noise enhancement of ZF equalizer. The noise enhancement problem aggravates when the communications channel is ill conditioned. In oversampling case, we notice that about 1dB gain when the MMSE equalizer is operated at OSR=4 versus when it is operated at OSR=2 for medium range of SNR. At a higher SNR, the SNR gain widens further. This may due to the equalizer can better resolve the multipaths at a higher OSR. Also, it is more likely to have a FIR type equalizer to equalize the channel perfectly under noiseless condition. At a higher OSR, no noticeable gain can be obtained as most of the gain is already captured by the equalizers.

6. CONCLUSIONS

In this paper, we presented the design and analysis of multirate FB based OQPSK equalizers for a FSC. Various sampling rate of the OQPSK FB WL MMSE equalizers can be studied under the framework of FB. We explicitly construct the OQPSK communications system as a FB structure. This allows us to study the conditions of when FIR equalizers exist under no noise scenarios. Through oversampling, more virtual channels between Tx and Rx can be obtained thereby increasing the likelihood of constructing a PR system using a finite length equalizer. Various types of OQPSK equalizers are simulated and their performance are compared. In our simulated scenarios, we observe that higher sampling rate WL MMSE equalizers are effective for OQPSK communications systems employed in FSCs.

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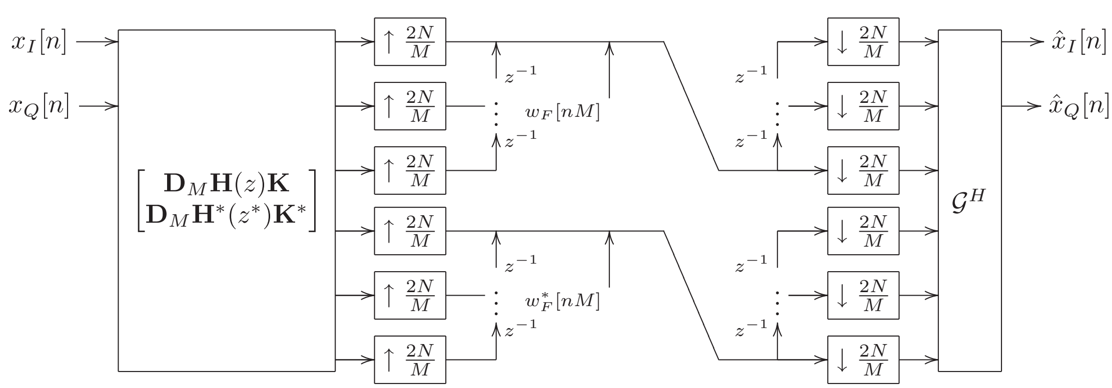

Fig. 5. A general oversampled WL OQPSK communication model.