OPTIMAL ROBUST BEAMFORMING FOR INTERFERENCE AND MULTIPATH MITIGATION IN GNSS ARRAYS

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ABSTRACT

In this paper, we introduce an optimal robust beamformer for detecting a desired signal in presence of noise, strong interferers of unknown directions-of-arrival (DOA), and multipath. The proposed approach achieves the highest possible signal-to-interference plus noise ratio (SINR) by optimally estimating the interference DOA, followed by a triply constrained robust Capon beamformer. More specifically, we maximize the SINR subject to nulling strong interferers and offer robustness against multipath and steering vector uncertainty. Unlike existing techniques, we examine explicitly the use of robust beamforming for multipath mitigation to enhance acquisition and tracking performance of GPS receivers.

***Index Terms*—** Beamforming, interference, multipath, robustness, GPS.

1. INTRODUCTION

Global Navigation Satellite Systems (GNSS) enables the calculation of the user position, velocity and timing information using the time-of-arrival of signals transmitted by a constellation of satellites, such as the US GPS system, the Russian GLONASS and the European GALILEO [2]. They have been widely used in civil and military applications such as navigation, land surveying and mapping, and synchronization for sensor networks [2]. However, the main challenges are the vulnerability of the GNSS receivers to strong interference and the multipath effects [2]. Conventional GPS interference suppression methods including time domain, frequency domain, and time-frequency domain apply advanced time-varying filtering and signal processing techniques to discriminate against the interference [2, 6]. These methods have the advantages of simple implementations and low cost, but the drawback is that they cannot mitigate multiple narrowband, wideband interferers, or short-delay multipath [2, 6]. Adaptive antenna arrays have been very effective in combating both interference and multipath, owing to their abilities to differentiate between desired signals and interferers by exploiting the direction-of-arrival (DOA) information [1, 5, 10]. More importantly, robust beamformers (RB) have been proposed to avoid performance degradation when some of the underlying look-direction assumptions are violated [3]. However, most of these approaches deal only with steering vector (SV) mismatch and have not considered the coherence effect of multipath.

Unlike the standard adaptive array processing, the potential benefits of robust beamforming in navigation systems have not been investigated thoroughly. Only recently in [4], RB for interference mitigation in GPS receivers was successfully applied. In this paper, we discuss the shortcomings of existing robust beamformers in achieving the highest SINR in the case of unknown interference DOAs, and present a new robust beamformer that incorporates multipath effects using appropriate constraints.

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2. STANDARD ROBUST CAPON BEAMFORMING

2.1. Problem formulation

We consider a GPS receiver equipped with an M-element array. The received signal vector $\mathbf{x}(t_n)$ can be modeled as

$$\mathbf{x}(t_n) = \sum_{k=0}^{K} s_k(t_n - \tau_k)\mathbf{a}_k + \sum_{i=1}^{I} \mathbf{i}_i(t_n)\mathbf{v}(\theta_i) + \mathbf{n}(t_n), \quad (1)$$

where K is the number of multipath components, $s_k(.)$ is the kth path signal including the C/A code, τ_k is the time-delay of the kth component, $\mathbf{a}_k$ is the spatial signature of the kth multipath, I is the number of interferers, $\mathbf{i}_i(.)$ is the waveform of the ith interference, $\mathbf{v}(\theta_i)$ is the spatial signature of the ith interference, and $\mathbf{n}(.)$ denotes the white Gaussian noise of power σ^2. Let $\mathbf{s}_d(t_n) \stackrel{\text{def}}{=} s_0(t_n - \tau_0)\mathbf{a}_0 \stackrel{\text{def}}{=} s_0\mathbf{a}_0$ denote the desired GPS signal. Then, Eq. (1) can be expressed as

$$\mathbf{x}(t_n) = \mathbf{s}_d(t_n) + \mathbf{s}_r(t_n) + \mathbf{V}(\theta)\mathbf{i}(t_n) + \mathbf{n}(t_n), \quad (2)$$

where $\mathbf{s}_r(.)$ denotes the contribution of the K multipath reflections, $\mathbf{V}(\theta) = [\mathbf{v}(\theta_1), \cdots, \mathbf{v}(\theta_I)]$ is a $M \times I$ matrix formed by the SVs from different interference directions $\theta = [\theta_1, \cdots, \theta_I]^T$, $\mathbf{i}(t_n) = [\mathbf{i}_1(t_n), \cdots, \mathbf{i}_I(t_n)]$ is the vector of complex amplitudes of interference. In this paper, $E\{.\}$ denotes the expectation operator, $(.)^T$ the transpose, $(.)^H$ the conjugate transpose, and $\Re(.)$ the real part of a complex.

Under the assumption that GPS signals, interference, and noise are uncorrelated, the covariance matrix of the received data vector has the form $\mathbf{R} \stackrel{\text{def}}{=} \mathbf{R}_s + \mathbf{R}_u$, where $\mathbf{R}_s$ is the covariance matrix of all the GPS signals, i.e., $\mathbf{s}(t_n) \stackrel{\text{def}}{=} \mathbf{s}_d(t_n) + \mathbf{s}_r(t_n)$, and $\mathbf{R}_u$ is the sum of the interference and noise covariance matrices. The robust beamforming approaches extend the Capon beamformer so as to improve array output SINR even when only an imprecise knowledge of steering vector $\mathbf{a}_0$ is available [3]. It minimizes the output power of beamformer subject to an uncertainty constraint on the array SV. Interestingly, most of these robust methods are equivalent and belong to the extended class of diagonal loading approaches of form [3] : $\mathbf{w}_r = \kappa(\hat{\mathbf{R}}_u + \delta\mathbf{I})^{-1}\mathbf{a}_0$, where δ denotes the diagonal level, and κ is a scaling factor which is immaterial for SINR [3]. Their differences lie in the distinct forms of κ and in the choices of δ. For the considered recent class of RB [3], the parameters κ and δ are directly linked to the SV uncertainty set which allows the determination of diagonal loading level value. Here, we assume that the only knowledge we have on $\mathbf{a}_0$ is that it belongs to the following uncertainty ellipsoid $\mathbf{a}_0 \in \{\mathbf{a} \mid (\mathbf{a}_0 - \bar{\mathbf{a}})\mathbf{C}^{-1}(\mathbf{a}_0 - \bar{\mathbf{a}}) \leq 1\}$, where $\mathbf{C}$ is a given positive definite matrix and $\bar{\mathbf{a}}$ is the assumed SV of the desired GPS signal. The doubly constrained robust Capon beamformer (DCRCB) [3] was recommended for applications requiring high SINR, and as such, is adopted below through the constrained optimization formulation

$$\min_{\mathbf{a}} \mathbf{a}^H \mathbf{R}_u^{-1} \mathbf{a} \quad s.t. \quad \begin{cases} (\mathbf{a} - \bar{\mathbf{a}})\mathbf{C}^{-1}(\mathbf{a} - \bar{\mathbf{a}}) \leq 1, \\ \|\mathbf{a}\|^2 = M. \end{cases} \tag{3}$$

In practice, the interference-plus-noise covariance matrix $\mathbf{R}_u$ is estimated by $\hat{\mathbf{R}} = (1/L)\sum_{n=1}^{L} \mathbf{x}(t_n)\mathbf{x}(t_n)^H$, where all received signals have zero means and the L samples are independent. For GPS applications, $\hat{\mathbf{R}}$ is an efficient maximum-likelihood estimator (MLE) of $\mathbf{R}_u$ because the GPS signal is negligible in the received data.

2.2. Highest SINR robust beamformer

In this section, we consider the multipath-free case, i.e., $K = 1$ in Eq. (1). In the case of strong interference with unknown angles-of-arrivals θ, we establish the following results.

Proposition 1 *In the presence of a desired signal in strong interference with unknown angles-of-arrivals θ, robust beamformers achieve the maximum SINR if and only if the angles estimator $\hat{\theta}$ is asymptotically efficient.*

Proof : Let $\hat{\theta}$ be the available interference's DOA estimate vector which characterizes the interference subspace. Similar to the case of conventional optimal weight [9], we can write the robust weight vector in the form $\mathbf{w}_r^H = \frac{\tilde{\kappa}}{\sigma^2}\mathbf{a}_0^H \mathbf{P}_I^{\perp}(\hat{\theta})$, where $\tilde{\kappa}$ is a related scale factor depending on κ and δ, and $\mathbf{P}_I^{\perp}(\hat{\theta})$ is the projection matrix onto a subspace orthogonal to the interference subspace. Accordingly, the output of the RB is expressed as follows

$$\begin{aligned} \mathbf{y}(t_n) &= \mathbf{w}_r^H \mathbf{x}(t_n) = \mathbf{w}_r^H \left[s_0\mathbf{a}_0 + \mathbf{V}(\theta_0)\mathbf{i}(t_n) + \mathbf{n}(t_n)\right] \\ &= \frac{\tilde{\kappa}}{\sigma^2}\mathbf{a}_0^H \mathbf{P}_I^{\perp}(\hat{\theta})\left[s_0\mathbf{a}_0 + \mathbf{V}(\theta_0)\mathbf{i}(t_n) + \mathbf{n}(t_n)\right] \\ &\stackrel{\text{def}}{=} \mathbf{y}_s(t_n) + \mathbf{y_i}(t_n) + \mathbf{y_n}(t_n), \end{aligned} \tag{4}$$

where θ_0 is the vector of true values of interference DOA. The parameter $\tilde{\kappa}/\sigma^2$ can be omitted, since it is insignificant for SINR computations. We replace $\mathbf{P}_I^{\perp}(\hat{\theta})$ by $\mathbf{P}_I^{\perp}(\theta_0)$, as $\hat{\theta} \rightarrow \theta_0$ for high INR.

- The signal output $\mathbf{y}_s(n)$ is given by

$$\mathbf{y}_s(n) = s_0\mathbf{a}_0^H \mathbf{P}_I^{\perp}(\theta_0)\mathbf{a}_0 = s_0\|\mathbf{P}_I^{\perp}(\theta_\mathbf{0})\mathbf{a}_0\|^2. \tag{5}$$

- The noise output is $\mathbf{y_n}(t_n) = \mathbf{a}_0^H \mathbf{P}_I^{\perp}(\theta_0)\mathbf{n}(t_n)$, and the noise power is expressed as

$$E\{|\mathbf{y_n}(t_n)|^2\} = \sigma^2\mathbf{a}_0^H \mathbf{P}_I^{\perp}(\theta_0)\mathbf{a}_0 = \sigma^2\|\mathbf{P}_I^{\perp}(\theta_0)\mathbf{a}_0\|^2. \tag{6}$$

- The interference output $\mathbf{y_i}(t_n) = \mathbf{a}_0^H \mathbf{P}_I^{\perp}(\hat{\theta})\mathbf{V}(\theta_0)\mathbf{i}(t_n)$, is equal to zero if $\hat{\theta}$ is the actual interference direction vector θ_0. However, in a practical situation, these angles are not perfectly known. The estimation mismatch causes residual interference $\mathbf{y_i}(t_n)$ at the beamformer output. We apply Taylor-expansion to $\mathbf{V}(\theta)$ around the actual DOAs, θ_0,

$$\mathbf{V}(\hat{\theta})\mathbf{i}(t_n) = \mathbf{V}(\theta_0)\mathbf{i}(t_n) + \mathbf{V}'(\theta_0)\mathrm{diag}[\mathbf{i}(t_n)](\hat{\theta} - \theta_0),$$

where $\mathbf{V}'(\theta_0) \stackrel{\text{def}}{=} [\mathbf{v}'(\theta_1), \cdots, \mathbf{v}'(\theta_1)]$ with $\mathbf{v}'(\theta_i)$ being the derivative of $\mathbf{v}(\theta_i)$. Then,

$$\mathbf{V}(\theta_0)\mathbf{i}(t_n) = \mathbf{V}(\hat{\theta})\mathbf{i}(t_n) - \mathbf{V}'(\theta_0)\mathrm{diag}[\mathbf{i}(t_n)](\hat{\theta} - \theta_0).$$

By substituting the latter into $\mathbf{y_i}(t_n)$ we get $\mathbf{y_i}(t_n) = \mathbf{a}_0^H \mathbf{P}_I^{\perp}(\hat{\theta})\{\mathbf{V}(\hat{\theta})\mathbf{i}(t_n) - \mathbf{V}'(\theta_0)\mathrm{diag}[\mathbf{i}(t_n)](\hat{\theta} - \theta_0)\}$. Because of $\mathbf{P}_I^{\perp}(\hat{\theta})\mathbf{V}(\hat{\theta}) = 0$, we obtain

$$\mathbf{y_i}(t_n) = -\mathbf{a}_0^H \mathbf{P}_I^{\perp}(\hat{\theta})\mathbf{V}'(\theta_0)\mathrm{diag}[\mathbf{i}(t_n)](\hat{\theta} - \theta_0).$$

By ignoring high order terms when replacing $\hat{\theta}$ by θ_0, we get

$$\begin{aligned} \mathbf{y_i}(t_n) &\cong -\mathbf{a}_0^H \mathbf{P}_I^{\perp}(\theta_0)\mathbf{V}'(\theta_0)\mathrm{diag}[\mathbf{i}(t_n)](\hat{\theta} - \theta_0) \\ &\cong -\mathbf{b}^H(\hat{\theta} - \theta_0), \end{aligned}$$

where $\mathbf{b}^H = \mathbf{a}_0^H \mathbf{P}_I^{\perp}(\theta_0)\mathbf{V}'(\theta_0)\mathrm{diag}[\mathbf{i}(t_n)]$. Accordingly, the interference output power is given by

$$E\{|\mathbf{y_i}(t_n)|^2\} = \mathbf{b}^H E\{(\hat{\theta} - \theta_0)(\hat{\theta} - \theta_0)^T\}\mathbf{b}. \tag{7}$$

When $\hat{\theta}$ is an efficient estimator, $E\{(\hat{\theta}-\theta_0)(\hat{\theta}-\theta_0)^T\}$ achieves asymptotically the Cramer-Rao lower bound $CRB(\theta)$. Hence, the lowest power of the residual interference is

$$E\{|\mathbf{y_i}(t_n)|^2\} = \mathbf{b}^H CRB(\theta)\mathbf{b}. \tag{8}$$

Finally, by combining (5), (6) and (8), the maximum achieved SINR by optimally estimating the interference angles is

$$SINR_{\max} = \frac{E\{|\mathbf{y}_s(n)|^2\}}{E\{|\mathbf{y_i}(t_n)|^2\} + E\{|\mathbf{y_n}(t_n)|^2\}}. \tag{9}$$

Proposition 2 *RBs are asymptotically equivalent to nullsteering for interference cancellation in which the DOA estimates are provided by the MUSIC algorithm. Therefore, they fail to achieve the maximum SINR.*

Proof : Using similar analysis, Taylor series expansion of $\mathbf{V}(\theta)$ in the MUSIC estimator θ_m allows us to show that in existing RBs, i.e., without accurate DOAs estimates, the RB interference output power is given by

$$E\{|\tilde{\mathbf{y}}_{\mathbf{i}}(t_n)|^2\} = \mathbf{b}^H E\{(\hat{\theta}_m - \theta_0)(\hat{\theta}_m - \theta_0)^T\}\mathbf{b}. \quad (10)$$

Eq. (10) prove that RBs are equivalent to the beamformer in which nulls are placed at the MUSIC DOA estimator. Since $E\{(\hat{\theta}_m - \theta_0)(\hat{\theta}_m - \theta_0)^T\} > E\{(\hat{\theta} - \theta_0)(\hat{\theta} - \theta_0)^T\}$ is a known result for the MLE $\hat{\theta}$ [9], then $E\{|\mathbf{y}_{\mathbf{i}}(t_n)|^2\} < E\{|\tilde{\mathbf{y}}_{\mathbf{i}}(t_n)|^2\}$. Therefore, the highest possible $SINR_r$ for a robust beamformer satisfy $SINR_r < SINR_{max}$.

3. OPTIMAL ROBUST CAPON BEAMFORMING

3.1. Interference mitigation

As the GPS signals are typically 20 dB bellow the noise floor, the received signal is dominated by the interference component. In this case, the actual SV of the desired GPS signal lies in the subspace orthogonal to the interference subspace. This suggests a third additional constraint on the cost function by minimizing the projection of the signal steering vector onto the interference subspace. Therefore, we introduce a new subspace based robust beamformer as a solution of the following constrained optimization problem

$$\min_{\mathbf{a}} \mathbf{a}^H \mathbf{P}_I \mathbf{a} \quad s.t. \quad \begin{cases} (\mathbf{a} - \bar{\mathbf{a}})\mathbf{C}^{-1}(\mathbf{a} - \bar{\mathbf{a}}) \leq 1, \\ \|\mathbf{a}\|^2 = M \end{cases} \quad (11)$$

The key idea is to search the minimizer in a much smaller feasible set to estimate the actual SV of the desired signal, which could improve the SINR. The optimization problem in (11) can be solved by the Lagrange multiplier method. Without loss of generality, we consider $\mathbf{C} = \varepsilon \mathbf{I}$, where ε is the uncertainty level and $\mathbf{I}$ is the identity matrix. Define a function

$$\begin{aligned} f(\mathbf{a}, \lambda, \mu) \quad = \quad & \mathbf{a}^H \mathbf{P}_I \mathbf{a} + \mu(2M - \varepsilon - \bar{\mathbf{a}}^H \mathbf{a} - \mathbf{a}^H \bar{\mathbf{a}}) \\ & + \lambda(\mathbf{a}^H \mathbf{a} - M), \end{aligned} \quad (12)$$

where $\mu \geq 0$ and $\lambda \geq 0$ are the Lagrange multipliers. Hence, the unconstrained minimization of (12) w.r.t. $\mathbf{a}$, for fixed μ and λ, is given by setting the following gradient to zero

$$\frac{\partial f(\mathbf{a}, \mu, \lambda)}{\partial \mathbf{a}} = 2\mathbf{P}_I \mathbf{a} - 2\mu \bar{\mathbf{a}} + 2\lambda \mathbf{a} = 0. \quad (13)$$

Clearly, the optimal solution is $\hat{\mathbf{a}} = \mu(\mathbf{P}_I + \lambda \mathbf{I})^{-1}\bar{\mathbf{a}}$. Now, we minimize the cost function f w.r.t. μ, we obtain

$$\hat{\mu} = \frac{2M - \varepsilon}{2\bar{\mathbf{a}}^H (\mathbf{P}_I + \lambda \mathbf{I})^{-1}\bar{\mathbf{a}}}. \quad (14)$$

Inserting $\hat{\mathbf{a}}$ and $\hat{\mu}$ and minimizing f w.r.t. λ, we derive the following equation

$$\frac{\bar{\mathbf{a}}^H (\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-2}\bar{\mathbf{a}}}{\left(\bar{\mathbf{a}}^H (\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-1}\bar{\mathbf{a}}\right)^2} = \frac{M}{(M - \frac{\varepsilon}{2})^2}. \quad (15)$$

The solution of $\hat{\lambda}$ can be obtained by solving the above equation using Newton's method. Substituting $\hat{\mu}$ and $\hat{\lambda}$ into $\hat{\mathbf{a}}$ gives

$$\hat{\mathbf{a}} = (M - \frac{\varepsilon}{2}) \frac{(\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-1}\bar{\mathbf{a}}}{\bar{\mathbf{a}}^H (\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-1}\bar{\mathbf{a}}}. \quad (16)$$

To improve the performance according to proposition 1, we exploit the optimally estimated interference DOA information for interference subspace projection matrix estimation as

$$\hat{\mathbf{P}}_I = \mathbf{V}(\hat{\theta})[\mathbf{V}(\hat{\theta})^H \mathbf{V}(\hat{\theta})]^{-1} \mathbf{V}(\hat{\theta})^H. \quad (17)$$

Another advantage of the use of DOA information is the reduction of the computational burdens in estimating $\mathbf{P}_I$ according to Eq. (17) because we avoid the eigendecomposition operation for interference subspace estimation. To summarize, we have the following essential result.

Proposition 3 *The proposed optimal robust weight is computed by Capon beamforming method (ORCB), that is,*

$$\mathbf{w}_{ORCB} = \frac{\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}}{\hat{\mathbf{a}}^H \hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}}, \quad (18)$$

where $\hat{\mathbf{a}}$ *is computed by substituting (17) into (16), and* $\hat{\theta}$ *is a MLE of interference direction of arrivals.*

3.2. Multipath mitigation

Consider one multipath $s_1(t_n)\mathbf{a}_1$ which can be expressed as $s_1(t_n)\mathbf{a}_1 = \alpha_1 s_0(t_n)\mathbf{a}_1$, where α_1 is the relative coefficient consisting of the product of the multipath power and $e^{j2\pi\Delta\tau_1}$, where $\Delta\tau_1$ is the relative delay to the direct path. Hence, the GPS signal is given by $s_0(t_n)\mathbf{a}_0 + s_1(t_n)\mathbf{a}_1 = s_0(t_n)(\mathbf{a}_0 + \alpha_1 \mathbf{a}_1)$ with $\mathbf{a}_1$ is approximately known relative to the direct-path GPS signal SV [4]. Then, $s_1(t_n)$ and $s_0(t_n)$ can be viewed as the desired signal $s_0(t_n)$ with another spatial signature $\mathbf{a}_0 + \alpha_1 \mathbf{a}_1$. Using a Capon beamformer, the signal is therefore interpreted as an interference, and as such, is attenuated. Thus, we observe that multipath reflection causes a similar problem as the SV errors. To subtract the contribution of the multipath spatial signature offset, we replace $\mathbf{a}$ by $(\mathbf{a} + \alpha_1 \mathbf{a}_1)$ in the objective function in Eq. (11). We solve the resultant optimization problem using Lagrange multipliers method in the same way as in the above subsection for the free-multipath case. Thus, we obtain

$$\hat{\mathbf{a}}_m = (M - \frac{\varepsilon}{2}) \frac{(\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-1}\bar{\mathbf{a}}}{c} - \alpha_1 (\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-1}\mathbf{P}_I \mathbf{a}_1, \quad (19)$$

where $c = \bar{\mathbf{a}}^H (\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-1}\bar{\mathbf{a}} + \alpha_1 \Re\left\{\bar{\mathbf{a}}^H (\mathbf{P}_I + \hat{\lambda}\mathbf{I})^{-1}\mathbf{P}_I \mathbf{a}_1\right\}$. Note that Eq. (19) is equivalent to estimating $\mathbf{a}$ using the ORCB method while subtracting the contribution of the reflected path. Finally, we compute the multipath optimal robust Capon beamformer (MORCB) by substituting (19) into (18), as

$$\mathbf{w}_{MORCB} = \frac{\hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}_m}{\hat{\mathbf{a}}_m^H \hat{\mathbf{R}}^{-1}\hat{\mathbf{a}}_m}. \quad (20)$$

4. SIMULATION RESULTS

A linear uniform array of $M = 7$ sensors with half wavelength spacing is used in the simulation. All results are achieved via 200 Monte Carlo trials with comparison to the Capon beamformer, DCRCB [3], as well as the optimal theoretical limit. The theoretical limit is the highest SINR in Eq. (9) where no SV mismatch is present and using the CRB given by [9]

$$CRB(\theta) = \frac{\sigma^2}{2L}\left[\Re\{[\mathbf{V}'(\theta_0)^H(\mathbf{I}-\mathbf{P_I})\mathbf{V}'(\theta_0)]\otimes\hat{\Gamma}\}\right]^{-1}$$

where $\hat{\Gamma} = (1/L)\sum_{n=1}^{L}[\mathbf{i}^*(t_n) \;\; \mathbf{i}^T(t_n)]$, with $(.)^*$ denoting the conjugate operator, and $\otimes$ the Hadamard product.

• First experiment. We first examine the performance of the proposed algorithm ORCB in the case of a strong jammer and SV error. The satellite signal is located at 30° with SNR = -20 dB, and the non-coherent interference is located at 20° with SIR = -50 dB. The exact direction of arrival of the desired GPS signal is ϕ_0. The assumed value is $\phi_0 \pm \Delta$, i.e., $\bar{\mathbf{a}}(\phi_0) = \mathbf{a}(\phi_0 \pm \Delta)$. The uncertainty parameter was set $\varepsilon = 5.5$ and $\Delta = 3^\circ$. Figure 1 shows the array output SINRs versus the SNR of the GPS signal when the number of snapshots is set to be $L = 300$. It is clear that the performance of the proposed ORCB significantly outperforms both the DCRCB and Capon beamformer. It is also evident that the output SINR of the proposed ORCB is close to the optimal limit value.

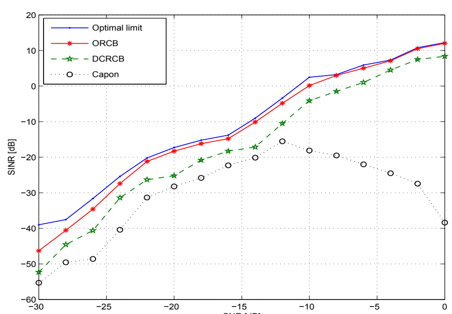

Fig. 1. SINR beamformers output versus SNR of the desired GPS signal.

• Second experiment. We consider the same scenario as experiment 1 with an additional reflected path from the direction $-\phi_0$, a typical situation in vertical GPS arrays [4]. The multipath power is one half of the direct-path signal power. To calculate the SINR, we use the following appropriate formula

$$SINR = s_0^2|\mathbf{w}_{MORCB}^H\hat{\mathbf{a}}_m|^2/\mathbf{w}_{MORCB}^H\hat{\mathbf{R}}\mathbf{w}_{MORCB}$$

Figure (2) plots the SINRs output of the Capon beamformer, DCRCB and the proposed MORCB. We observe that the best performance is achieved by the proposed MORCB algorithm in all input SNRs. Also, since Capon beamformer doesn't take into account SV errors, whereas DCRCB is not designed for multipath situations, their respective performances degrade severely.

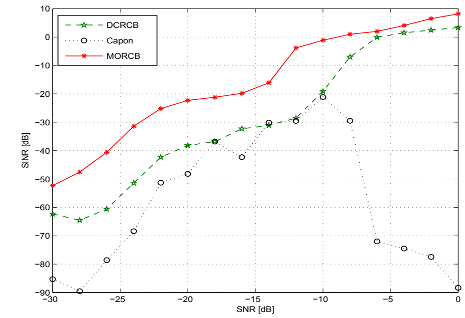

Fig. 2. SINR beamformers output versus SNR of the desired GPS signal.

5. CONCLUSION

We have shown that without interference DOA information, RBs are only suboptimal, i.e., fail to achieve the highest SINR. By optimally estimating the DOA of strong interference and using this information to compute the interference subspace, the performance of robust beamformers can be improved significantly. More precisely, using the MLE for interference DOA estimation, the proposed ORCB robust beamformer is optimal in the maximum SINR sense. To take multipath into account, we have proposed the modified algorithm (MORCB). To simplify presentations, only one reflected path has been considered, with the general case of several multipath reflections following similarly. The application of these results to interference mitigation for GPS has shown that the proposed algorithms outperform existing techniques.

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