MUTUAL INFORMATION JAMMER-RELAY GAMES*

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ABSTRACT

We consider a two-person zero-sum mutual information game between one jammer ($\mathcal{J}$) and one relay ($\mathcal{R}$), in a non-fading scenario. Supposing that the source ($\mathcal{S}$) and the destination ($\mathcal{D}$) are unaware of the game, we derive optimal pure or mixed strategies for $\mathcal{J}$ and $\mathcal{R}$ depending on the link qualities and whether the players are active during the $\mathcal{S} \to \mathcal{D}$ channel training. When both $\mathcal{J}$ and $\mathcal{R}$ have full knowledge of the source signal, the optimal strategies amount to linear jamming (LJ) and linear relaying (LR), respectively. When the $\mathcal{S} \to \mathcal{J}$ and $\mathcal{S} \to \mathcal{R}$ links are noisy, LJ strategies (pure or mixed) are still optimal under LR. In this case, instead of always transmitting with full power as when the $\mathcal{S} \to \mathcal{R}$ link is perfect, $\mathcal{R}$ should adjust transmit-power according to its power constraint and the reliability of the source signal it receives.

Index Terms— Two-person zero-sum games, jammer channel, relay channel, mutual information, Nash equilibrium.

1. INTRODUCTION

A jammer (or hacker) may be present to inhibit or halt the transmission of signals in a tactical (or commercial) communication system. If a jammer node can acquire fully or partially the signal transmitted by a source, it can disrupt the source-destination link severely by implementing what is referred to as correlated jamming. Because of its severity, the latter has received attention recently from both information-theoretic and game-theoretic perspectives [1, 2, 3]. Optimal source/jammer strategies are reported in [1] for an additive white Gaussian noise (AWGN) channel of a point-to-point link where a source and a jammer participate in a two-person zero-sum game with the mutual information adopted as the objective function. In [2], related strategies are pursued for a single-user multi-input multi-output (MIMO) fading channel where the jammer has full knowledge of the source signal. Recently, a non-cooperative zero-sum game has been investigated in a setup involving two sources and one-correlated jammer, in both AWGN and fading user channels [3]. For the non-fading two-user channel, the optimal strategy amounts to Gaussian signalling for the sources and linear jamming for the jammer. In fading channels, sub-games are defined per channel state and the optimal solution reduces to a set of power allocation strategies for the players.

While a jammer tries to harm, a relay node can facilitate the communication between a source and a destination. Without being necessary to pack multiple antennas per terminal as in MIMO systems, cooperation among distributed single-antenna nodes (source and relays) offers an alternative spatial diversity enabler and brings resilience to shadowing as well as enhanced link-coverage [4, 5, 6]. Different from the jamming channel, the capacity achieving strategy for the relay is generally unknown even for the Gaussian single-relay channel without fading [5]. However, upper and lower bounds on the capacity of the AWGN relay channel have been developed in various scenarios and many simple relaying strategies have been either proved to be capacity(-bound) achieving under certain conditions [4], or justified in terms of the diversity order [6]. Security issues in relay communications have been investigated in [7] when one of the two relay nodes is adversarial and tries to disrupt communications by sending garbled signals. The objective in [7] is to trace and identify the adversarial relay.

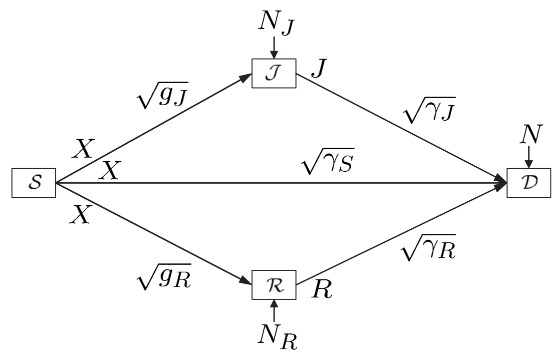

Fig. 1. Communication model with one jammer and one relay.

In this paper, we consider a non-fading AWGN channel with one jammer ($\mathcal{J}$) and one relay ($\mathcal{R}$) participating in the link between a source ($\mathcal{S}$) and a destination ($\mathcal{D}$), as depicted in Fig. 1. Each node is equipped with a single antenna. Nodes $\mathcal{J}$ and $\mathcal{R}$ have completely antithetical goals regarding the communication over the $\mathcal{S} \to \mathcal{D}$ link, whose effectiveness is assessed by the mutual information $I(X;Y)$ between the input X and the output Y. The conflicting objectives of $\mathcal{J}$ and $\mathcal{R}$ motivate well a two-person zero-sum game formulation in which the players are $\mathcal{J}$, who tries to minimize $I(X;Y)$, and $\mathcal{R}$ who aims to maximize the $I(X;Y)$. Different from existing works where the game is played between $\mathcal{S}$ and $\mathcal{J}$, in our jammer-relay game setup we suppose that $\mathcal{S}$ and $\mathcal{D}$ are unaware of $\mathcal{J}$ and $\mathcal{R}$, while $\mathcal{J}$ ($\mathcal{R}$) can eavesdrop the channel and use the information obtained to perform correlated jamming (relaying). We also differentiate between availability of perfect and noisy versions of the source signal at $\mathcal{J}$ ($\mathcal{R}$). Under reasonable assumptions on link qualities and the activity of $\mathcal{J}$ and $\mathcal{R}$ during the $\mathcal{S} \to \mathcal{D}$ channel training, we establish that jammer-relay games reach Nash equilibrium (NE) - a state where no player node has anything to gain unilaterally by chang-

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ing only its own strategy. In scenarios where the single action of a player (a.k.a. pure strategy) cannot reach NE, one resorts to what is known as mixed strategy, which comprises a set of pure strategies with assigned probabilities; in such cases the payoff is modified to the *average* mutual information. When both $\mathcal{J}$ and $\mathcal{R}$ have perfect information about the source signal, the optimal (pure or mixed) strategies turn out to be linear jamming (LJ) and linear relaying (LR), respectively. If the source signal received at $\mathcal{J}$ and $\mathcal{R}$ is only corrupted by AWGN, the optimal strategy for $\mathcal{R}$ is an open problem for decades even without $\mathcal{J}$. In this case, we prove that corresponding to LR at $\mathcal{R}$, LJ at $\mathcal{J}$ leads to NE. Furthermore, we show that instead of always transmitting with full power as when the $\mathcal{S} \to \mathcal{R}$ link is perfect, $\mathcal{R}$ at NE should adjust its transmit-power to a proper value depending on the channel quality of the $\mathcal{S} \to \mathcal{R}$ link.

Notation: $\mathcal{CN}(0, \sigma^2)$ will denote a circularly symmetric complex Gaussian distribution with zero mean and variance σ^2; for a real number x, $\operatorname{sgn}(x)$ denotes its sign and $(x)^+ := \max(x, 0)$; for random variables X and Y, $I(X;Y)$ denotes their mutual information; $h(X)$ differential entropy, and $E[X]$ expectation.

2. SYSTEM MODEL

We consider several settings categorized according to the information available at $\mathcal{J}$ and $\mathcal{R}$. In the absence of fading, all channel coefficients are taken without loss of generality (w.l.o.g.) real and positive because they are assumed available both at receiving ends through training and at the transmitting ends through feedback. As illustrated in Fig. 1, the received signal at $\mathcal{D}$ is

$$Y = \sqrt{\gamma_S}X + \sqrt{\gamma_J}J + \sqrt{\gamma_R}R + N \tag{1}$$

where X, J, and R denote signals transmitted from $\mathcal{S}$, $\mathcal{J}$ and $\mathcal{R}$, respectively; $\sqrt{\gamma_S}$, $\sqrt{\gamma_J}$ and $\sqrt{\gamma_R}$ are the gains of the $\mathcal{S} \to \mathcal{D}$, $\mathcal{J} \to \mathcal{D}$ and $\mathcal{R} \to \mathcal{D}$ channels, respectively; and $N \sim \mathcal{CN}(0, \sigma_N^2)$. For the source signal X to be capable of maximizing mutual information, it is assumed zero-mean Gaussian. Power is constrained at $\mathcal{S}$, $\mathcal{J}$ and $\mathcal{R}$ so that $E[X^2] \le P_S$, $E[J^2] \le P_J$ and $E[R^2] \le P_R$, respectively.

We will analyze both cases of perfect and imperfect information about X acquired through eavesdropping, at $\mathcal{J}$ and $\mathcal{R}$. First, we suppose that both $\mathcal{J}$ and $\mathcal{R}$ can obtain the exact X. In the second case, we assume that AWGN is present in both $\mathcal{S} \to \mathcal{J}$ and $\mathcal{S} \to \mathcal{R}$ links; i.e., $\mathcal{J}$ and $\mathcal{R}$ observe, respectively,

$$\begin{aligned} Y_J &= \sqrt{g_J}X + N_J \quad (2)\\ Y_R &= \sqrt{g_R}X + N_R \quad (3)\end{aligned}$$

where g_J (g_R) denotes the gain of the $\mathcal{S} \to \mathcal{J}$ ($\mathcal{S} \to \mathcal{R}$) channel, $N_J \sim \mathcal{CN}(0, \sigma_{N_J}^2)$, and $N_R \sim \mathcal{CN}(0, \sigma_{N_R}^2)$. Notice that the first case is subsumed by the second after setting $g_J = g_R = 1$ and $N_J = N_R = 0$ in (2) and (3).

3. PERFECT SOURCE INFORMATION AT $\mathcal{J}$ AND $\mathcal{R}$

In this section, we will find the best jamming/relaying strategies in the sense of reaching NE in the jammer-relay mutual-information based game, when both $\mathcal{J}$ and $\mathcal{R}$ know X exactly. Specifically, we will establish that if $\mathcal{R}$ employs linear relaying (LR), through an amplified version of X, the best strategy for $\mathcal{J}$ is linear jamming (LJ), a linear combination of X and Gaussian noise; and vice versa when $\mathcal{J}$ employs LJ, the best strategy for $\mathcal{R}$ is LR. Subsequently, we will analyze the NE of the mutual-information zero-sum game between $\mathcal{J}$ and $\mathcal{R}$ under various conditions.

3.1. Linear Jamming and Linear Relaying

Suppose $\mathcal{J}$ uses LJ, i.e., $J = \rho X + W_J$, where $W_J \sim \mathcal{CN}(0, \sigma_{W_J}^2)$, and $\rho, \sigma_{W_J}^2$ are chosen to satisfy the power constraint $\rho^2 P_S + \sigma_{W_J}^2 \le P_J$. The relay wishes to find the signalling strategy R which maximizes $I(X;Y)$. To evaluate the latter, introduce the variable $Z := R - XE[XR]/P_S$ which represents the error in linearly estimating R using the source signal X. Using Z, we can rewrite (1) as

$$Y = (\sqrt{\gamma_S} + \sqrt{\gamma_J}\rho + \sqrt{\gamma_R}\frac{E[XR]}{P_S})X + \sqrt{\gamma_J}W_J + \sqrt{\gamma_R}Z + N. \tag{4}$$

It is easy to verify that Z is uncorrelated with X and $E[Z^2] + (E[XR])^2/P_S \le P_R$. We can now upper bound $I(X;Y) := h(Y) - h(Y|X) = h(Y) - h(\sqrt{\gamma_J}W_J + \sqrt{\gamma_R}Z + N|X)$ as

$$\begin{aligned} I(X;Y) &\le h(Y) - h(\sqrt{\gamma_J}W_J + \sqrt{\gamma_R}Z + N|X, Z) \quad (5)\\ &= h(Y) - h(\sqrt{\gamma_J}W_J + N) \quad (6)\end{aligned}$$

where the equality in (5) holds when $Z = 0$. But for $Z = 0$, Y is Gaussian and the maximum of $h(Y)$ is attained when the upper bound on $E[Y^2]$ is maximized. Using the power constraints and the definition $A := \sqrt{\gamma_S} + \sqrt{\gamma_J}\rho$, it follows readily that

$$E[Y^2] \le P_S A^2 + 2A\sqrt{\gamma_R}E[XR] + \gamma_R P_R + \gamma_J \sigma_{W_J}^2 + \sigma_N^2. \tag{7}$$

To maximize $E[Y^2]$ and consequently $I(X;Y)$, it thus suffices to set $Z = 0$, and

$$E[XR] = \begin{cases} \sqrt{P_S P_R}, & \text{if } A \ge 0 \\ -\sqrt{P_S P_R}, & \text{if } A < 0. \end{cases} \tag{8}$$

Recalling that $Z = 0 = R - XE[XR]/P_S$, we arrive at the $I(X;Y)$ maximizing relay strategy

$$R^* = \begin{cases} \sqrt{\frac{P_R}{P_S}}X, & \text{if } A \ge 0 \\ -\sqrt{\frac{P_R}{P_S}}X, & \text{if } A < 0. \end{cases} \tag{9}$$

Eq. (9) reveals that if $\mathcal{J}$ relies on LJ, the best strategy for $\mathcal{R}$ is LR. The sign of R depends on the scalar ρ used by $\mathcal{J}$ to restrain reception of the source signal at $\mathcal{D}$.

Supposing now that R is chosen as in (9), we will prove next that the best strategy for $\mathcal{J}$ is LJ. Since $I(X;Y) = h(X) - h(X|Y)$, and $\mathcal{J}$ can only affect $h(X|Y)$, the jammer aims at minimizing $I(X;Y)$ by maximizing $h(X|Y) = h(X - aY|Y) \le h(X - aY) \le \frac{1}{2}\log(2\pi e\Lambda)$, where $\Lambda := E[(X - aY)^2]$. Although the inequalities hold for any a, we choose $a = E[XY]/E[Y^2]$. We will carry out the rest of the proof in two steps. First, we will prove that if there is an LJ signal giving rise to a certain Λ, it is optimal over all other jamming signals leading to the same Λ. Second, we will prove that any given Λ reachable by a feasible jamming signal can also be reached by a feasible LJ signal. (Feasibility here means adherence to power constraints.)

For the first step, since X is Gaussian and the jamming is linear, J is also Gaussian. Moreover, because $R = R^* = \pm\sqrt{\frac{P_R}{P_S}}X$, we infer that Y and $X - \frac{E[XY]}{E[Y^2]}Y$ are also Gaussian. Now, as $X - \frac{E[XY]}{E[Y^2]}Y$ is uncorrelated with Y and they are both Gaussian, $X - \frac{E[XY]}{E[Y^2]}Y$ is independent of Y. Thus, the upper bound on $h(X|Y)$ is achieved with equality; i.e.,

$$h(X|Y) = \frac{1}{2}\log\left\{2\pi e\left[P_S - \frac{(E[XY])^2}{E[Y^2]}\right]\right\} \tag{10}$$

which establishes the optimality of LJ given its existence.

For the second step, since $Y = (\sqrt{\gamma_S} \pm \sqrt{\gamma_R}\sqrt{\frac{P_R}{P_S}})X + \sqrt{\gamma_J}J + N$, one can verify that $\Lambda = P_S - (E[XY])^2/E[Y^2]$ is a function of $E[XJ]$. Consider now any J and define $U := J - X\frac{E[XJ]}{P_S}$ which is clearly uncorrelated with X. For the jamming signal J to be feasible, we should have $\frac{(E[XJ])^2}{P_S} \leq P_J$. Now define an LJ signal $J_l := \frac{E[XJ]}{P_S}X + W_J$, where $W_J \sim \mathcal{CN}(0, \sigma^2_{W_J})$ denotes noise uncorrelated with X, having $\sigma^2_{W_J} = E[U^2]$. Since $E[XJ_l] = E[XJ]$, this J_l results in the same Λ and has the same power as J. Thus, J_l is also feasible. Hence, for any signal in the set of feasible jamming signals, there is an equivalent LJ signal which leads to in the same upper bound (10).

3.2. Nash Equilibria

Since the optimal jamming signal is $J = \rho X + W_J$, to fully describe the NE, we should specify the slope ρ and the noise variance $\sigma^2_{W_J}$ which maximize (10), and thus minimize $I(X;Y)$. To this end, note that since a linear combination of Gaussian signals is received at $\mathcal{D}$, minimizing $I(X;Y)$ is equivalent to minimizing the output SNR at $\mathcal{D}$. The pertinent minimization problem is thus [cf. (4) with $Z = 0$]

$$\begin{aligned} \min_{\rho, \sigma^2_{W_J}} \quad & \frac{[\sqrt{\gamma_S} + \sqrt{\gamma_J}\rho + \sqrt{\gamma_R}\frac{E[XR]}{P_S}]^2 P_S}{\gamma_J \sigma^2_{W_J} + \sigma^2_N} \\ \text{s.t.} \quad & \rho^2 P_S + \sigma^2_{W_J} \leq P_J \text{ and } \sigma^2_{W_J} \geq 0. \end{aligned}$$

After solving the Karush-Kuhn-Tucker (KKT) necessary conditions

$$\begin{cases} \frac{[\sqrt{\gamma_S} + \sqrt{\gamma_J}\rho + \sqrt{\gamma_R}\frac{E[XR]}{P_S}]P_S\sqrt{\gamma_J}}{\gamma_J \sigma^2_{W_J} + \sigma^2_N} + \lambda P_S \rho = 0 \\ \frac{-[\sqrt{\gamma_S} + \sqrt{\gamma_J}\rho + \sqrt{\gamma_R}\frac{E[XR]}{P_S}]^2 P_S \gamma_J}{(\gamma_J \sigma^2_{W_J} + \sigma^2_N)^2} + \lambda - \delta = 0 \\ \lambda(\rho^2 P_S + \sigma^2_{W_J} - P_J) = 0, \ \delta(-\sigma^2_{W_J}) = 0 \\ \lambda \geq 0, \ \delta \geq 0 \end{cases} \tag{11}$$

one can obtain the optimal ρ in (12) at the bottom of this page, where the parameter ρ_m is given by

$$\rho_m = \min\left\{ \frac{P_J\gamma_J + \sigma^2_N}{|\sqrt{\gamma_S} + \frac{E[XR]\sqrt{\gamma_R}}{P_S}|P_S\sqrt{\gamma_J}}, \sqrt{\frac{P_J}{P_S}} \right\} \tag{13}$$

and the optimal noise variance is ${\sigma^2}^*_{W_J} = (P_J - [\rho^*(R)]^2 P_S)^+$.

As we will see soon, determining the J^* and R^* signals at NE depends critically on the activity of $\mathcal{J}$ and $\mathcal{R}$ during the training stage of the $\mathcal{S} \rightarrow \mathcal{D}$ link. Specifically, we will differentiate between two operational assumptions:

a1. *$\mathcal{J}$ and $\mathcal{R}$ are inactive during the training of $\mathcal{S} \rightarrow \mathcal{D}$; and*

a2. *$\mathcal{J}$ and $\mathcal{R}$ are active during the training of $\mathcal{S} \rightarrow \mathcal{D}$.*

Under a1, $\mathcal{D}$ acquires the $\mathcal{S} \rightarrow \mathcal{D}$ channel phase before the game and relies on it to coherently decode X when the game is played. This rules out the choice corresponding to the negative sign in (9), because $\mathcal{R}$ would then cancel the source signal while $\mathcal{D}$, being unaware of the $\mathcal{J} - \mathcal{R}$ game, will erroneously decode X using the channel phase it acquired during training. A consequence of a1 is that NE is reached by a pair of pure strategies, namely $R^* = \sqrt{\frac{P_R}{P_S}}X$ and $J^* = [\rho^*(\sqrt{\frac{P_R}{P_S}}X)]X + W^*_J$.

Under a2, $\mathcal{D}$ acquires the $\mathcal{S} \rightarrow \mathcal{D}$ channel phase when the $\mathcal{J} - \mathcal{R}$ game is played. In this case, even if $\mathcal{R}$ cancels X using the negative sign in (9), $\mathcal{D}$ can detect the aggregate ($\mathcal{S} \rightarrow \mathcal{D}$ plus $\mathcal{R} \rightarrow \mathcal{D}$ plus $\mathcal{J} \rightarrow \mathcal{D}$) channel via training and can coherently decode X. Now either choice in (9) is possible and this provides one more parameter for $\mathcal{J}$ and $\mathcal{R}$ to play with. As a result, the optimal strategies for $\mathcal{J}$ and $\mathcal{R}$ under a2 are not always pure. Whether pure or not depends on the following conditions:

c1. $\sqrt{P_J\gamma_J} \leq \sqrt{P_S\gamma_S}$;

c2. $\sqrt{P_S\gamma_S} + \sqrt{P_R\gamma_R} \leq \sqrt{P_J\gamma_J}$; and

c3. $\sqrt{P_S\gamma_S} < \sqrt{P_J\gamma_J} < \sqrt{P_S\gamma_S} + \sqrt{P_R\gamma_R}$.

Under c1, whether $\mathcal{J}$ chooses or not, it cannot cancel the source signal completely because of its power limitation. This in turn implies $A := \sqrt{\gamma_S} + \sqrt{\gamma_J}\rho \geq 0$, and the best LR signal is [cf. (9)] $R^* = \sqrt{\frac{P_R}{P_S}}X$. Correspondingly, the optimal LJ signal has slope $\rho^* = -\rho_m$ [cf. (12) and (13)] and ${\sigma^2}^*_{W_J} = (P_J - \rho_m^2 P_S)^+$. Notewithstanding, NE is achieved under c1 with a pure strategy.

Under c2, we find that $P_J\gamma_J \geq (\sqrt{P_S\gamma_S} + E[XR]\sqrt{\gamma_R/P_S})^2$, and thus $\rho^* = -\frac{\sqrt{\gamma_S}}{\sqrt{\gamma_J}} - \frac{E[XR]\sqrt{\gamma_R}}{P_S\sqrt{\gamma_J}}$ [cf. (12)]. In this case, $\mathcal{J}$ has enough power to cancel signals transmitted by both $\mathcal{S}$ and $\mathcal{R}$, regardless of the relaying signal. But because R can have either opposite or the same sign as X, the players $\mathcal{J}$ and $\mathcal{R}$ cannot arrive at NE with pure strategies. To demonstrate this, let us first check the relationship between $A := \sqrt{\gamma_S} + \sqrt{\gamma_J}\rho$ and $B := \frac{E[XR]\sqrt{\gamma_R}}{P_S}$. Substituting the optimal $\rho = \rho^* = -\frac{\sqrt{\gamma_S}}{\sqrt{\gamma_J}} - \frac{E[XR]\sqrt{\gamma_R}}{P_S\sqrt{\gamma_J}}$ in A and the $E[XR]$ expression from (8) in B, we find

$$A = -\frac{E[XR]\sqrt{\gamma_R}}{P_S} = -B \quad \text{and} \quad \mathrm{B} = \begin{cases} \sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{if } A \geq 0 \\ -\sqrt{\frac{P_R\gamma_R}{P_S}}, & \text{if } A < 0. \end{cases}$$

Since A (B) depends on the strategy of $\mathcal{J}$ ($\mathcal{R}$), the strategies of $\mathcal{J}$ and $\mathcal{R}$ are clearly coupled. Indeed, $\mathcal{J}$ aims to have sign opposite to $\mathcal{R}$ while at the same time $\mathcal{R}$ wishes to follow the sign of $\mathcal{J}$. This coupling implies that pure individual player actions cannot drive $\mathcal{J}$ and $\mathcal{R}$ to a stable NE. To reach a stable NE under c2, we consider the following mixed strategies:

$$A = \begin{cases} -\sqrt{\frac{P_R\gamma_R}{P_S}}, \text{w.p. } p_J \\ \sqrt{\frac{P_R\gamma_R}{P_S}}, \text{w.p. } 1 - p_J \end{cases} \text{ and } \mathrm{B} = \begin{cases} \sqrt{\frac{P_R\gamma_R}{P_S}}, \text{w.p. } p_R \\ -\sqrt{\frac{P_R\gamma_R}{P_S}}, \text{w.p. } 1 - p_R \end{cases}$$

where w.p. means with probability. The objective function is now $E[I(X;Y)]$ i.e., the capacity of the channel linking X to Y; and the players seek the optimal probability assignments (p_J and p_R). For this two-person zero-sum game, optimal strategies can be found through a minimax approach, by which each player node tries to maximize its payoff ($E[I(X;Y)]$ for $\mathcal{R}$ and $-E[I(X;Y)]$ for J) in the worst outcome determined by the opponent's strategy.

The relay $\mathcal{R}$ seeks the best probability distribution of pure strategies by solving the following max-min problem:

$$\begin{aligned} \max_{p_R}\min_{p_J} E[I(X;Y)] &= \max_{p_R}\min_{p_J} E\left\{\frac{1}{2}\log\left[1 + \frac{(A+B)^2 P_S}{\gamma_J\sigma^2_{W_J} + \sigma^2_N}\right]\right\} \\ &= \max_{p_R}\{\min\{p_R I_-, (1-p_R)I_+\}\} \end{aligned} \tag{14}$$

$$\rho^*(R) = \begin{cases} -\frac{\sqrt{\gamma_S}}{\sqrt{\gamma_J}} - \frac{E[XR]\sqrt{\gamma_R}}{P_S\sqrt{\gamma_J}}, & \text{if } (\sqrt{P_S\gamma_S} + \frac{E[XR]\sqrt{\gamma_R}}{\sqrt{P_S}})^2 \leq P_J\gamma_J \\ -\rho_m \operatorname{sgn}(\sqrt{\gamma_S} + \frac{E[XR]\sqrt{\gamma_R}}{P_S}), & \text{if } (\sqrt{P_S\gamma_S} + \frac{E[XR]\sqrt{\gamma_R}}{\sqrt{P_S}})^2 > P_J\gamma_J \end{cases} \tag{12}$$

where

$$I_- := \frac{1}{2}\log\left[1 + \frac{4P_R\gamma_R}{P_J\gamma_J + \sigma_N^2 - (\sqrt{P_R\gamma_R} - \sqrt{P_S\gamma_S})^2}\right] \quad (15)$$

$$I_+ := \frac{1}{2}\log\left[1 + \frac{4P_R\gamma_R}{P_J\gamma_J + \sigma_N^2 - (\sqrt{P_R\gamma_R} + \sqrt{P_S\gamma_S})^2}\right]. \quad (16)$$

It is easy to recognize p_R maximizing (14) is $p_R^* = I_+/(I_+ + I_-)$.

Similarly, in selecting its optimum strategy, the jammer node $\mathcal{J}$ solves the min-max problem $\max_{p_R}\min_{p_J} E[I(X;Y)]$ to obtain (by symmetry of the corresponding expressions) the optimal $p_J^* = I_-/(I_+ + I_-)$.

Results under c3 are similar to those under c1 and c2, but we omit them due to page limitations. (Proofs omitted due to lack of space can be found in [8].)

4. NOISY SOURCE INFORMATION AT $\mathcal{J}$ AND $\mathcal{R}$

Here we assume that both $\mathcal{S} \to \mathcal{J}$ and $\mathcal{S} \to \mathcal{R}$ links are modeled as AWGN channels. The received signals at $\mathcal{J}$ and $\mathcal{R}$ are given by (2) and (3), respectively.

4.1. Relaying Strategy

We first look to optimize the strategy of $\mathcal{R}$, assuming that $\mathcal{J}$ relies on an LJ signal $J = \rho Y_J + W_J$ and adheres to the power constraint

$$\rho^2 E[Y_J^2] + \sigma_{W_J}^2 = \rho^2(g_J P_S + \sigma_{N_J}^2) + \sigma_{W_J}^2 \le P_J. \quad (18)$$

In this scenario, the overall optimal strategy for $\mathcal{R}$ is difficult to find, primarily because the optimal relaying strategy even for the AWGN relay channel is still unknown [5]. But recent works on cooperative communications have suggested a number of useful relay strategies, including the popular decode-and-forward (DF), amplify-and-forward (AF) and estimate-and-forward (EF) ones [5, 6]. For a Gaussian X, the DF, AF and EF schemes perform identically as far as mutual information is concerned [9]. Under the power constraint $E[R^2] \le P_R$, we can thus consider w.l.o.g. the AF relaying strategy

$$R = \frac{\sqrt{P_R}}{\sqrt{E[Y_R^2]}}Y_R = \frac{\sqrt{P_R}}{\sqrt{g_R P_S + \sigma_{N_R}^2}}Y_R \quad (19)$$

where Y_R is given by (3). We will verify that this is not the optimal strategy even if we are restricted to the class of LR functions only.

In general, the optimal R should be a non-linear function of Y_R. But for analytical tractability, we confine the relaying strategy to linear forwarding, i.e., we assume that $R = \alpha Y_R + Z_R$, where Z_R is a signal independent of Y_R. However, the data processing inequality asserts that $I(X;Y)$ is maximized if the "interference" Z_R is 0. But then Y is Gaussian and the objective of maximizing $I(X;Y)$ is equivalent to maximizing the output SNR; i.e., the relay seeks to

$$\begin{aligned} \max_{\alpha} \quad & \frac{(\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} + \alpha\sqrt{\gamma_R g_R})^2 P_S}{\rho^2\gamma_J\sigma_{N_J}^2 + \gamma_J\sigma_{W_J}^2 + \sigma_N^2 + \alpha^2\gamma_R\sigma_{N_R}^2} \\ \text{s.t.} \quad & \alpha^2(g_R P_S + \sigma_{N_R}^2) \le P_R. \end{aligned}$$

The solution of this constrained maximization problem is

$$\alpha^* = \begin{cases} \mathrm{sgn}(\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J})\alpha_m(\rho), & \text{if } \sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} \ne 0 \\ \pm\sqrt{\frac{P_R}{g_R P_S + \sigma_{N_R}^2}}, & \text{if } \sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J} = 0 \end{cases} \quad (20)$$

where

$$\alpha_m(\rho) = \min\left\{\frac{(\rho^2\gamma_J\sigma_{N_J}^2 + \gamma_J\sigma_{W_J}^2 + \sigma_N^2)\sqrt{g_R}}{|\sqrt{\gamma_S} + \rho\sqrt{\gamma_J g_J}|\sqrt{\gamma_R}\sigma_{N_R}^2}, \sqrt{\frac{P_R}{g_R P_S + \sigma_{N_R}^2}}\right\}.$$

Interestingly, the optimal α^* suggests that $\mathcal{R}$ *should not always use its full power to linearly forward the received signal*. Fixing all other parameters, mathematically this happens when $\sigma_{N_R}^2$ is large enough, which implies that the receive-SNR at $\mathcal{R}$ is very low. This is reasonable because when Y_R at $\mathcal{R}$ is not reliable, forwarding it to $\mathcal{D}$ with too much power will only downgrade detection performance at $\mathcal{D}$ by decreasing its receive-SNR.

4.2. Jamming Strategy

Following steps similar to those in Section 3, we have also proved that when $R = \alpha Y_R = \alpha\sqrt{g_R}X + \alpha N_R$, the best strategy for $\mathcal{J}$ is LJ. And the optimal ρ for $\mathcal{J}$ can be expressed as in (17) at the bottom of this page, where

$$\rho'_m(\alpha) = \min\left\{\frac{\gamma_J P_J + \sigma_N^2 + \alpha^2\gamma_R\sigma_{N_R}^2}{|\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R}|P_S\sqrt{\gamma_J g_J}}, \sqrt{\frac{P_J}{g_J P_S + \sigma_{N_J}^2}}\right\},$$

and $\sigma_{W_J}^{2*} = (P_J - \rho^{*2}(g_J P_S + \sigma_{N_J}^2))^+$. With the expressions of ρ^* in (17) and α^* in (20), one can derive the NE under various assumptions and conditions as in Subsection 3.2.

5. REFERENCES

[1] M. Médard, "Capacity of correlated jamming channels," in *Proc. of 35th Annual Allerton Conference on Communications, Control and Computing*, Monticello, IL, Sep. 29 - Oct. 1, 1997, pp. 1043–1052.

[2] A. Kashyap, T. Basar, and R. Srikant, "Correlated jamming on MIMO Gaussian fading channels," *IEEE Transactions on Information Theory*, vol. 50, no. 9, pp. 2119–2123, Sept. 2004.

[3] S. Shafiee and S. Ulukus, "Mutual information games in multi-user channels with correlated jamming," *IEEE Trans. on Information Theory*, 2005 (submitted); downloadable from http://www.enee.umd.edu/˜ulukus/papers/journal/corj-34.pdf.

[4] T. Cover and A.E. Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572–584, Sept. 1979.

[5] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3037–3063, Sept. 2005.

[6] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. on Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[7] Y. Mao and M. Wu, "Security issues in cooperative communications: Tracing adversarial relay," in *Proc. of IEEE Internatianal Conference on Acoustics, Speech, and Signal Processing*, Toulouse, France, May 15-19, 2006, vol. 4, pp. 69–72.

[8] T. Wang and G. B. Giannakis, "Mutual information jammer-relay games," *IEEE Transactions on Information Forensics and Security*, 2006 (submitted).

[9] K. S. Gomadam and S. A. Jafar, "Optimal relay functionality for SNR maximization in memoryless relay networks," *IEEE Journal on Selected Areas in Communications*, 2007 (to appear); downlable from http://newport.eecs.uci.edu/˜syed/papers/j11.pdf.

$$\rho^* = \begin{cases} -\frac{\sqrt{\gamma_S}}{\sqrt{\gamma_J g_J}} - \frac{\alpha\sqrt{\gamma_R g_R}}{\sqrt{\gamma_J g_J}}, & \text{if } (g_J P_S + \sigma_{N_J}^2)(\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R})^2 \le \gamma_J g_J P_J \\ -\rho'_m(\alpha)\,\mathrm{sgn}(\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R}), & \text{if } (g_J P_S + \sigma_{N_J}^2)(\sqrt{\gamma_S} + \alpha\sqrt{\gamma_R g_R})^2 > \gamma_J g_J P_J \end{cases} \quad (17)$$