BLIND DIVERSITY RECEPTION AND INTERFERENCE CANCELLATION USING ICA

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ABSTRACT

In this paper, we consider *blind* diversity reception and interference rejection in multi-antenna communications context, in terms of maximizing the output signal-to-interference-and-noise ratio (SINR). More specifically, we demonstrate that independent component analysis (ICA), although originally designed for noise-free linear models, is able to provide essentially the best possible output SINR among all linear transformations of received data in noisy linear models. In particular, our experiments indicate that one of the most widely applied ICA algorithms, equivariant adaptive source identification (EASI) algorithm, is, in practice, identical with SINR maximizing generalized eigenfilter in terms of SINR, even though it does not use explicit knowledge of the channel states and noise statistics. We also show that, in a special case of interference-free (that is, noise only) system, the EASI algorithm attains the greatest diversity gain blindly, i.e., performs as a blind maximal ratio combiner (MRC).

***Index Terms*—** Blind diversity reception, blind interference cancellation, independent component analysis, multi-antenna communications

1. INTRODUCTION

Multiple transmit and/or receiver antennas are generally seen as one of the key elements in future wireless communications system developments [1]. In terms of the overall link quality, multiple antennas can be used for diversity purposes to mitigate the fading characteristics of the individual links (diminish the effects of noise), and also for removing other interfering signal components falling on top of the desired signal. This is also the central theme in this paper. In general, we consider the previous challenging system scenario in which both additive noise and interference are present in the received signals. Assuming multiple receiver antennas, the purpose is then to push down the noise and interference as much as possible using linear signal processing techniques. Furthermore, the focus here in general is on *blind* signal processing and reception in the sense that the noise statistics and channel state information are assumed unknown.

One relatively new idea in interference rejection is to employ blind source separation (BSS) techniques [2]. What makes BSS techniques attractive is their ability to separate signals from a mixture of original source signals in a completely blind manner, i.e., without an explicit knowledge of waveform structure (modulation) or mixing coefficients. In addition, typical BSS methods rely solely on higher-order statistical properties of data in the temporal domain, which makes the methods also very robust against possible spectral distortions. Communications related applications of BSS have been found, e.g., in MIMO systems [3], I/Q processing receivers [4], DS-CDMA blind multi-user detection [5] and DS-CDMA out-of-cell interference cancellation [6].

Independent component analysis (ICA) [2] is a fairly new statistical technique by which BSS can be performed. In ICA a set of observed signals or random variables are basically expressed as linear combinations of statistically independent components, which are often called sources or source signals. The ICA problem is blind, because not only the source signals but also the mixing coefficients are unknown. Many different methods have been proposed to solve the ICA problem [2]. Most of these are proper ICA methods exploiting the statistical independence of the sources, but there exist also other approaches which utilize temporal correlations or nonstationarity of the sources. In general, the mutual performance of these methods depends largely on the validity of the above assumptions.

Typically, a *noise-free* linear mixing model is assumed in derivation of ICA algorithms in the literature. Needless to say, the noise-free model is unrealistic in most of the applications, especially, in telecommunications. Consequently, applications of ICA often assume a noisy linear model, but exploit one of the ICA algorithms developed for noise-free models. Thus, a presence of reasonable level of additive noise is thought to cause "only" some feasible distortion due to the model mismatch. In this paper, we demonstrate that, although noise can never be suppressed completely by any linear technique, the performance gain (in terms of input-output SINR) obtained using ICA is practically identical to that of the optimum linear receiver utilizing known channel and noise statistics. In other words, ICA performs blind SINR maximization. We also bear out that ICA provides the greatest diversity gain, i.e., maximizes output SNR, among linear receivers assuming an interference-free model. That is to say, ICA acts as a *blind* maximal ratio combiner (MRC). In numerical experiments, we have selected a popular equivariant adaptive source identification (EASI) algorithm to represent ICA.

2. SYSTEM MODEL AND LINEAR RECEIVERS

The basic system model used in the following assumes one transmit antenna and $M(\geq 2)$ receiver antennas used for diversity reception and interference rejection. Thus the received signal at the m-th antenna is of the form

$$x_m(t) = h_{m,u}u(t) + h_{m,v}v(t) + n_m(t) \qquad (1)$$

in which $u(t)$ and $v(t)$ denote the desired and interfering signals, respectively, $n_m(t)$ models additive channel noise, and $h_{m,u}$ and

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$h_{m,v}$ are the corresponding channel coefficients. Assuming that a) the source components, $u(t)$ and $v(t)$ are mutually uncorrelated, and b) that the noise components, $n_m(t),\ m = 1 \ldots M$, are temporally white, Gaussian and mutually uncorrelated, this yields an $M \times 2$ linear model,

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

in which $\mathbf{s}(t) = [u(t)\ v(t)]^T$ is a source signal vector with zero mean and $\mathbb{E}\{\mathbf{s}(t)\mathbf{s}(t)^H\} = \mathbf{I}$, $\mathbf{n}(t) = [n_1\ n_2 \ldots n_M]^T$ is a zero mean Gaussian noise vector with $\mathbb{E}\{\mathbf{n}(t)\mathbf{n}(t)^H\} = \sigma^2\mathbf{I}$ ($\sigma^2 > 0$) and $\mathbf{A} = [\mathbf{h}_u\ \mathbf{h}_v] \in \mathbb{C}^{M\times 2}$ (with $\mathbf{h}_u = [h_{1,u}\ h_{2,u} \ldots h_{M,u}]^T$ and $\mathbf{h}_v = [h_{1,v}\ h_{2,v} \ldots h_{M,v}]^T$) is a full-rank mixing (or channel) matrix, which is assumed to stay constant during one processing block of data. Provided that it is assumed, in addition to a) and b), for all t that source component processes, $u(t)$ and $v(t)$, are c) non-Gaussian (or at least that one of them is) and d) mutually statistically independent and, also, that e) $\mathbf{n}(t)$ is independent from $\mathbf{s}(t)$, then (2) equals also a noisy ICA model [2]. The purpose in the following is to blindly estimate $u(t)$, i.e., suppress noise and interference as much as possible under the assumptions a) – e). Notice also that the assumption that $u(t)$ is the desired signal and $v(t)$ is the interfering one is completely formal and any distinctive assumptions between the source components are not made.

Let $\mathbf{w} \in \mathbb{C}^M$ be an arbitrary linear filter and $y_\mathbf{w} = \mathbf{w}^H\mathbf{x}$ the corresponding output of a linear receiver. Signal-to-interference-and-noise ratio (SINR) of the output $y_\mathbf{w}$ is then defined as

$$\begin{aligned}\eta(\mathbf{w}) &= \frac{\mathbb{E}\{\mathbf{w}^H\mathbf{h}_u|u|^2\mathbf{h}_u^H\mathbf{w}\}}{\mathbb{E}\{\mathbf{w}^H(\mathbf{h}_v v + \mathbf{n})(\mathbf{h}_v^H v^* + \mathbf{n}^H)\mathbf{w}\}} \\ &= \frac{\mathbf{w}^H\mathbf{R}_u\mathbf{w}}{\mathbf{w}^H(\mathbf{R}_v + \sigma^2\mathbf{I})\mathbf{w}},\end{aligned} \tag{3}$$

in which $\mathbf{R}_u = \mathbf{h}_u\mathbf{h}_u^H$ and $\mathbf{R}_v = \mathbf{h}_v\mathbf{h}_v^H$. Now, as seen in (3), maximizing SINR among all linear transformations of received data, i.e., maximizing $\eta(\mathbf{w})$, equals to solving the generalized eigenvalue problem [7] associated with matrix pair $(\mathbf{R}_u, \mathbf{R}_v + \sigma^2\mathbf{I})$. Hence,

$$\max_{\mathbf{w}\in\mathbb{C}^M} \eta(\mathbf{w}) = \lambda \quad \text{and} \tag{4}$$

$$\arg\max_{\mathbf{w}\in\mathbb{C}^M} \eta(\mathbf{w}) = \mathbf{e}, \tag{5}$$

in which λ stands for the greatest eigenvalue of hermitian matrix $(\mathbf{R_v} + \sigma^2\mathbf{I})^{-1}\mathbf{R_u}$ and $\mathbf{e}$ for any (eigen)vector in the corresponding eigensubspace. The vector $\mathbf{e}$ is called SINR maximizing generalized eigenfilter (M-GEF) wrt. the source component u (i.e., the desired signal here). This solution assumes the knowledge of the channel coefficients and noise variance, and forms a natural reference technique for the forthcoming blind ICA developments.

It is also interesting to note that a linear minimum mean square estimator (LMMSE) of a source can be shown to maximize SINR among linear transformations. This is basically stated in [8] and in references therein. Some earlier works have used the LMMSE reception as the reference method in their numerical evaluations (see, e.g., [9]). However, the generalized eigenfilter based approach shown in this paper gives certain benefits, especially, in analytical studies and comparisons of methods.

While the above eigenfilter based solution gives the optimum reference technique, the other two natural reference receivers are obtained by simply considering (i) only noise or (ii) only interference. A conventional diversity reception, or maximal ratio combining (MRC) as often referred to, is actually closely related to M-GEF reception. Nevertheless, in MRC, the main objective is to maximize the signal-to-(AWG)-noise ratio (SNR) and, consequently, MRC ignores possible interfering signal components which results in a sub-optimum SINR performance, in general. However, M-GEF reception is consistent with MRC in the sense that, without interference, v, these two methods coincide. This is easy to see by setting $v = 0$ and constraining $\mathbf{w}$ to have, say, a unit norm in which case (3) reduces to ordinary eigenvalue problem, which, for one, is well known to yield the MRC solution [10]. In a noise-free model, as the other extreme, infinite SINR is naturally obtained by inverting $\mathbf{A}$. However in general, the inversion is not equal to M-GEF solution in the noisy model, due to arbitrary noise amplification. These two methods (conventional MRC and pure system inversion) are used also in the following as reference. Notice that channel knowledge is also needed in these reference techniques.

3. ICA AND BLIND SINR MAXIMIZATION

In basic ICA, the goal is essentially to invert the model (2) blindly, that is, to find an unmixing matrix $\mathbf{W}$ such that $\mathbf{WA}$ is as close to identity as possible by using only the observations $\mathbf{x}(t)$. Because of the blindness, a solution of the ICA problem, $\mathbf{W}$, can be unique only up to left multiplication by arbitrary permutation and diagonal matrices. Identifiability of such a $\mathbf{W}$ is guaranteed in theory only for *noise-free* linear models and, consequently, basic ICA algorithms can not produce exactly an inverse of matrix $\mathbf{A}$ (not even up to the indeterminacies) if additive noise is present. Nevertheless, inverting $\mathbf{A}$ does not lead to the best SINR gain in a noisy system anyway, as stated above. For this reason, it is well-advised to compare the performance of noise-free ICA algorithms to the above-mentioned M-GEF bound if noisy model (2) is used.

Some of the resent studies have also proposed so called noisy ICA algorithms [2] that assume a presence of additive noise and tries to take it into account. However, finding an inverse of $\mathbf{A}$ is usually a main objective also in these algorithms instead of, e.g., maximizing SINR. Other ICA related blind algorithms, that assume the noisy model, try to de-noise received data either before or after the ICA separation. These approaches, nevertheless, lead necessarily to non-linear (affine, at least) processing of data.

In the following numerical experiments, we assume $M = 2$ receiver antennas and use the widely applied EASI algorithm [11], which is originally intended to perform noise-free ICA. EASI is an online algorithm which operates on individual samples of received data. One step of the EASI algorithm is given as

$$\mathbf{B}(t+1) = \mathbf{B}(t) - \mu\mathbf{U}(t)\mathbf{B}(t), \tag{6}$$

in which μ is a scalar step size and the update matrix, $\mathbf{U}(t)$, is defined as

$$\mathbf{U}(t) = \mathbf{y}(t)\mathbf{y}(t)^H - \mathbf{I} + \mathbf{g}(\mathbf{y}(t))\mathbf{y}(t)^H - \mathbf{y}(t)\mathbf{g}(\mathbf{y}(t))^H. \tag{7}$$

Here $\mathbf{y}(t) = \mathbf{B}(t)\mathbf{x}(t)$, $\mathbf{I}$ stands for identity matrix and $\mathbf{g} : \mathbb{C}^2 \to \mathbb{C}^2$ is an arbitrary nonlinear function. Since only the current sample is used in each step of the algorithm, the update matrix (7) does not vanish asymptotically. Instead, a stability point of the algorithm is defined *stochastically* to be a matrix $\mathbf{B}'$ for which the *mean* of the update term (7) is zero (i.e., $\mathbb{E}\{\mathbf{U}(t)\} = \mathbf{0}$).

Fig. 1 depicts an example behavior of elements of matrix B under a significant noise level (SNR=5 dB). In this 2×2-example, coefficients converge after twenty thousand EASI updates.

Results in the next section show that the output SINR of the EASI algorithm wrt. to the desired source is almost identical to M-GEF bound. In theory, a small difference exists [12], but SINR figures are in practice almost indistinguishable according to the results.

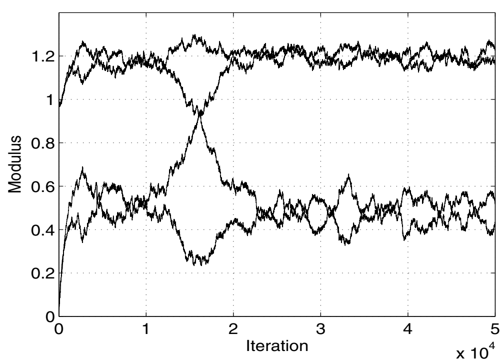

Fig. 1. Convergence of EASI update coefficients (elements of matrix $\mathbf{B}$). Received SNR and SIR are fixed to 5 dB and 0 dB, respectively. Sample size is 50000.

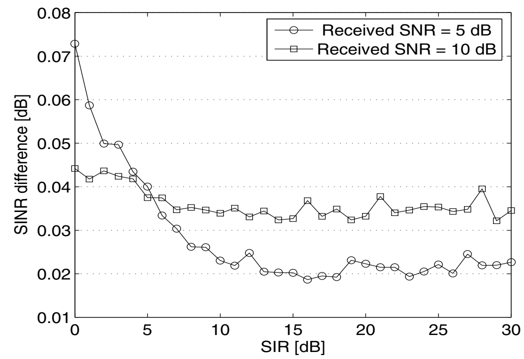

Fig. 2. Difference in output SINR between M-GEF and EASI. Received SNR is 5 dB and sample size is 50000. 1000 independent channel realization is simulated.

Especially, SINR gain of EASI is significant compared to SINR of plain inversion of $\mathbf{A}$ and, on the other hand, to MRC bound when both noise and interference are present.

Here, as also in the continuation, signal-to-noise ratio (SNR) is defined as the average ratio of the *received* desired signal power and additive noise power. The signal-to-interference ratio (SIR), in turn, is the average ratio of the *received* desired signal and interference powers. Given the power normalization of the formal sources stated below (2), the SIR values other than 0 dB are implemented by corresponding scaling of the interference channel coefficients.

An important particular case is a system with finite SNR, but in which interference is absent, thus, the case in which M-GEF and MRC bounds coincide. Interestingly enough, also the EASI algorithm provides exactly the same output SINR in this case. More precisely, assuming an interference-free 2×2-model, $\mathbf{x} = \mathbf{h}_u u + \mathbf{n}$ and a vector $\widehat{\mathbf{h}}_u \in \mathbb{C}^2$ such that $\widehat{\mathbf{h}}_u^H \mathbf{h}_u = 0$, a matrix $\mathbf{B}' = [\alpha \mathbf{h}_u \; \beta \widehat{\mathbf{h}}_u]^H \in \mathbb{C}^{2\times 2}$ is a stability point of the EASI algorithm, i.e.,

$$\mathbb{E}\left\{\mathbf{y}\mathbf{y}^H - \mathbf{I} + \mathbf{g}(\mathbf{y})\mathbf{y} - \mathbf{y}\mathbf{g}(\mathbf{y})^H\right\} = 0, \tag{8}$$

for $\mathbf{y} = \mathbf{B}'\mathbf{x}$ and for appropriate complex scalars α and β. Recall, that $\mathbf{h}_u$ is now the MRC filter [10]. We prove the tenability of (8) rigorously in [12]. Here, simulation results support the claim above. Fig. 2 plots the difference between simulated SINR of EASI and M-GEF bound as a function of SIR. In the figure, the difference decreases to negligible level (of roughly 0.01 dB-unit) when SIR increases (i.e., when interference goes down). A difference in the order of 0.01 dB-unit is basically explained by the finite sample statistics.

4. FURTHER NUMERICAL EXPERIMENTS

Numerical results in this section set against the performance of EASI algorithm and SINR maximizing M-GEF approach under noisy environment. Also SINR performances of plain maximal ratio combining (MRC) and inversion of the channel matrix, the matrix $\mathbf{A}$ in model (2), are simulated in the experiments. Both of the latter methods are, thus, suboptimum since both an interfering source component and additive noise are present.

In the experiments, two receiver antennas are used. Desired signals are QPSK signals where as interfering signals are 16-QAM signals. The selection of source constellations is more or less arbitrary, and it should not affect the general validity of the results. Channel coefficients, i.e., elements of the matrix $\mathbf{A}$, are drawn randomly from zero mean Gaussian distributions (one distribution for each source component) for each processing block of $N = 50000$ symbols of data. Variances of these distributions are selected such that received SNR and SIR values correspond to given values on average. M-GEF bound is evaluated directly from the data model for each block. Hence, the bounds are not affected by finite sample statistics and, more importantly, they are the absolute upper bounds among all linear transformations of received data in case of each realization. Also output SINR's of the MRC and inversion of $\mathbf{A}$ are evaluated from the model.

A simple third-order nonlinearity [11]

$$\mathbf{g} = [g_1 \; g_2]^T : \mathbb{C}^2 \to \mathbb{C}^2; \; g_i(\mathbf{z}) = |z_i|^2 z_i, \; i = 1, 2, \tag{9}$$

is used in the EASI algorithm in the experiments. A permutation ambiguity of EASI outputs is circumvented by, first, evaluating the output SINR wrt. the desired source component for both outputs and, then, selecting the maximum one. Practical ways to select the desired output component are not concerned in this paper. All the gains plotted are wrt. the received SINR.

Figs. 3 and 4 show SINR gains as a function of SIR with fixed received SNR. The figures illustrate that the gains of EASI algorithm are, in practice, undistinguishable from M-GEF bounds. A difference is less than 0.1 dB-unit in whole SIR range plotted in the both figures. Figs. 5 and 6 give two examples of SINR gain vs. received SNR with fixed received SIR. Again, performances of EASI and M-GEF bound are essentially identical.

5. CONCLUSIONS

In this paper, we illustrated that basic independent component analysis (ICA) designed for noise-free linear models is able to provide essentially the best possible output SINR among all linear transformations of received data, in the challenging case of having both additive noise and interference disturbing the desired signal observation in a multi-antenna receiver context. Thus in effect, the ICA is able to do joint diversity reception and interference cancellation in a blind manner, such that the output SINR is maximized. In particular, our experiments indicated that one of the most widely applied ICA algorithms, EASI algorithm, is, in practice, identical with SINR maximizing generalized eigenfilter (M-GEF) in terms of SINR. In theory, EASI can not attain exactly the M-GEF bound when both noise

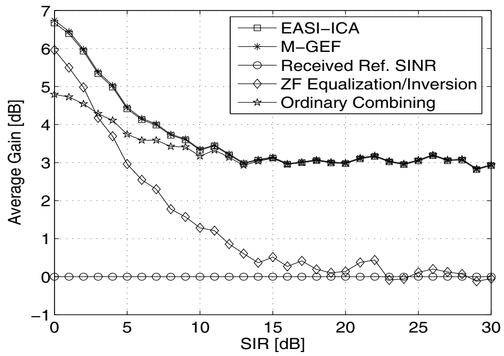

Fig. 3. Output SINR gains as a function of SIR. Received SNR is fixed to 5 dB and number of channel realizations is 1000.

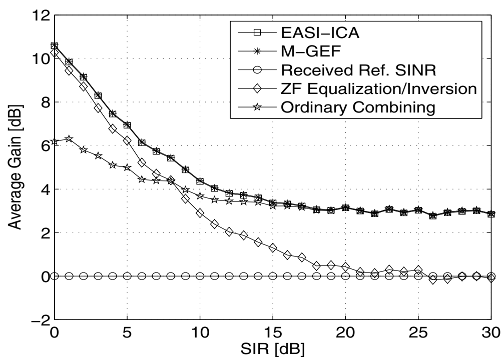

Fig. 4. Output SINR gains as a function of SIR. Received SNR is fixed to 10 dB and number of channel realizations is 1000.

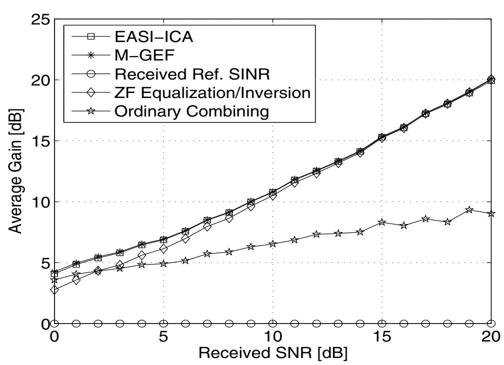

Fig. 5. Output SINR gains as a function of received SNR. Received SIR is fixed to 0 dB and number of channel realizations is 1000.

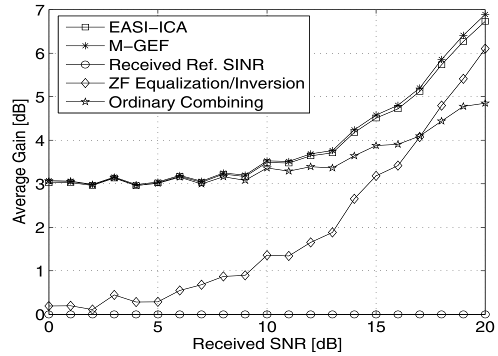

Fig. 6. Output SINR gains as a function of received SNR. Received SIR is fixed to 15 dB and number of channel realizations is 1000.

and interference are present, but difference was negligible (< 0.1 dB-unit) in all of our experiments. We also showed that, in an important special case of interference-free (i.e., noise only) system, the EASI algorithm provides precisely the greatest linear diversity gain blindly, i.e., performs as a blind maximal ratio combiner (MRC).

The observed output SINR behavior almost identical to the theoretical upper bound rises a question whether the EASI algorithm, or ICA in general, could be further fine-tuned at the algorithm level to attain exactly the best linear SINR blindly also in theory. Such an algorithm would readily be a generalization of both conventional blind diversity reception and blind interference cancellation. In this paper, we left this question open to be dealth with in future studies.

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