THE CAPACITY REGION OF A CLASS OF DETERMINISTIC INTERFERENCE CHANNELS WITH COMMON INFORMATION

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ABSTRACT

In this paper, we establish the capacity region of a class of deterministic interference channels with common information. For such a class of channels, each sender needs to transmit not only the private information but also certain common information to the corresponding receiver. Moreover, the channel outputs are deterministic with respect to given channel inputs.

***Index Terms*—** Interference channel, deterministic channel, common information, capacity region.

1. INTRODUCTION

As one of the fundamental building blocks, the interference channel (IC) was introduced by Shannon [1], where the two-way channel was studied. Since then, many work has been done on this channel, which includes various inner bounds and outer bounds (see [2] and references therein). Capacity regions are only found for some special cases including the strong interference channel, a class of degraded additive interference channels and a class of deterministic interference channels. Most of the previous work is based on the assumption that the source messages at the senders are statistically independent. However, the assumption fails in some emerging scenarios, i.e., neighboring sensors in a dense wireless sensor network may obtain correlated data due to the short distance in between, and when the correlation can be extracted, the neighboring sensors share certain common information. The IC under this new setting is termed as the interference channel with common information (ICC).

The ICC was first studied by Maric et al. in [3], where the capacity region of the strong ICC (SICC) was reported. In [4], the authors investigated the general ICC, and obtained an achievable rate region which generalizes the capacity region for SICC as well as the Chong-Motani-Garg region (one of the best achievable rate region for IC) [5]. It is shown in this paper that our achievable rate region is tight for a class of deterministic ICCs (DICCs).

2. CHANNEL MODEL

Fig. 1 depicts the graphical model of the class of DICCs. The channel is defined by its finite channel input output alphabets $\mathcal{X}_t$, $\mathcal{Y}_t$, $t = 1, 2$, and the channel transition which is governed by the following deterministic functions:

$$V_t = k_t(X_t), \;\; t = 1, 2;$$
$$Y_1 = o_1(X_1, V_2), \text{ and } Y_2 = o_2(X_2, V_1),$$

where V_1 and V_2 represent the interference signals caused by X_1 and X_2 at the corresponding receivers. Source messages w_0, w_1, and w_2 are assumed to be independently and uniformly generated over their respective ranges. Furthermore, we require that there exist two more deterministic functions, $V_2 = h_1(Y_1, X_2)$ and $V_1 = h_2(Y_2, X_2)$.

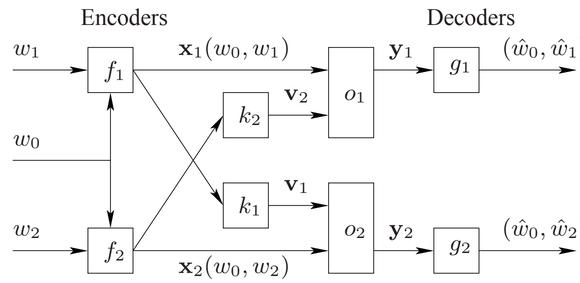

Fig. 1. The class of deterministic interference channels with common information.

Note that the channel defined above is similar to the one investigated in [6], but there is a slight difference. In [6], it is required that $H(Y_1|X_1) = H(V_2)$ and $H(Y_2|X_2) = H(V_1)$ for all product distributions of X_1X_2. It has also been pointed out in [6] that this requirement is equivalent to requiring the existence of $V_2 = h_1(Y_1, X_1)$ and $V_1 = h_2(Y_2, X_2)$. Nevertheless, we require the latter instead of the former, and in fact the former is not satisfied in our case.

We denote this class of DICCs by C_d. An (M_0, M_1, M_2, n, P_e) code exists for the channel C_d, if and only if there exist two encoding functions

$$f_1 : \mathcal{M}_0 \times \mathcal{M}_1 \to \mathcal{X}_1^n, \qquad f_2 : \mathcal{M}_0 \times \mathcal{M}_2 \to \mathcal{X}_2^n,$$

and two decoding functions

$$g_1 : \mathcal{Y}_1^n \to \mathcal{M}_0 \times \mathcal{M}_1, \qquad g_2 : \mathcal{Y}_2^n \to \mathcal{M}_0 \times \mathcal{M}_2,$$

such that $\max\{P_{e,1}^{(n)}, P_{e,2}^{(n)}\} \le P_e$, where $P_{e,t}^{(n)}$, $t = 1, 2$, denotes the average decoding error probability of decoder t, and is computed by one of the following equations:

$$P_{e,1}^{(n)} = \frac{1}{M} \sum_{w_0 w_1 w_2} p((\hat{w}_0, \hat{w}_1) \neq (w_0, w_1) | (w_0, w_1, w_2)),$$
$$P_{e,2}^{(n)} = \frac{1}{M} \sum_{w_0 w_1 w_2} p((\hat{w}_0, \hat{w}_2) \neq (w_0, w_2) | (w_0, w_1, w_2)),$$

with $M = M_0 M_1 M_2$.

A non-negative rate triple (R_0, R_1, R_2) is achievable for the channel C_d if for any given $0 < P_e < 1$, and for any sufficiently large n, there exists a $(2^{nR_0}, 2^{nR_1}, 2^{nR_2}, n, P_e)$ code.

The capacity region for the channel C_d is defined as the closure of the set of all the achievable rate triples.

3. MAIN RESULT

Let $\mathcal{P}_d$ denote the set of all joint distributions $p(\cdot)$ that factor as

$$p(v_0, x_1, x_2) = p(v_0)p(x_1|v_0)p(x_2|v_0), \quad (1)$$

where v_0 is the realization of an auxiliary random variable V_0 defined on an arbitrary finite set $\mathcal{V}_0$. Let $\mathcal{R}_d(p)$ denote the set of all rate triples (R_0, R_1, R_2) such that

$$R_0 \le H(Y_1), \quad (2)$$
$$R_0 \le H(Y_2), \quad (3)$$
$$R_1 \le H(Y_1|V_0V_2), \quad (4)$$
$$R_2 \le H(Y_2|V_0V_1), \quad (5)$$
$$R_1 + R_2 \le H(Y_1|V_0V_1) + H(Y_2|V_0V_2); \quad (6)$$
$$R_1 + R_2 \le H(Y_1|V_0) + H(Y_2|V_0V_1V_2), \quad (7)$$
$$R_0 + R_1 + R_2 \le H(Y_1) + H(Y_2|V_0V_1V_2); \quad (8)$$
$$R_1 + R_2 \le H(Y_1|V_0V_1V_2) + H(Y_2|V_0), \quad (9)$$
$$R_0 + R_1 + R_2 \le H(Y_1|V_0V_1V_2) + H(Y_2); \quad (10)$$
$$2R_1 + R_2 \le H(Y_1|V_0) + H(Y_1|V_0V_1V_2) + H(Y_2|V_0V_2), \quad (11)$$
$$R_0 + 2R_1 + R_2 \le H(Y_1) + H(Y_1|V_0V_1V_2) + H(Y_2|V_0V_2); \quad (12)$$
$$R_1 + 2R_2 \le H(Y_2|V_0) + H(Y_2|V_0V_1V_2) + H(Y_1|V_0V_1), \quad (13)$$
$$R_0 + R_1 + 2R_2 \le H(Y_2) + H(Y_2|V_0V_1V_2) + H(Y_1|V_0V_1), \quad (14)$$

for some fixed joint distribution $p(\cdot) \in \mathcal{P}_d$.

Theorem 1 The capacity region of C_d is the closure of

$$\bigcup_{p(\cdot)\in\mathcal{P}_d} \mathcal{R}_d(p).$$

Proof: 1) **Achievability***:* It suffices to show that $\mathcal{R}_d(p)$ is achievable for the channel C_d for a fixed joint distribution $p(\cdot) \in \mathcal{P}_d$. Since the joint distribution $p(\cdot) \in \mathcal{P}_d$ does not involve V_1 and V_2, it appears incurring difficulty for us to apply the cascaded superposition coding strategy developed for the general ICC to this channel, due to the lack of auxiliary random variables. Nevertheless, because the interferences V_1 and V_2 are determined by the channel inputs X_1 and X_2, we can extend the joint distribution in the form of (1) to one containing V_1 and V_2 as

$$p(v_0, x_1, x_2, v_1, v_2) = p(v_0)p(x_1|v_0)p(x_2|v_0) \cdot\delta(v_1 - k_1(x_1))\delta(v_2 - k_2(x_2)), \quad (15)$$

where $\delta(\cdot)$ is the Kronecker Delta function. Since X_1 and X_2 are conditionally independent given V_0, the interferences V_1 and V_2 also become conditionally independent given V_0. Therefore, the extended joint distribution (15) can be factored as

$$p(v_0, x_1, x_2, v_1, v_2) = p(v_0)p(v_1|v_0)p(v_2|v_0) \cdot p(x_1|v_1, v_0)p(x_2|v_2, v_0),$$

and the achievability of the region $\mathcal{R}_d(p)$ follows readily from Theorem 4 in [4].

2) **Converse***:* It suffices to show that for any $(2^{nR_0}, 2^{nR_1}, 2^{nR_2}, n, P_e)$ code with $P_e \to 0$, the rate triple (R_0, R_1, R_2) must satisfy (2)–(14) for some joint distribution that factors in the form of (1).

Consider a $(2^{nR_0}, 2^{nR_1}, 2^{nR_2}, n, P_e)$ code with $P_e \to 0$. Note that $P_e \to 0$ implies $P^n_{e,1} \to 0$ and $P^n_{e,2} \to 0$. Applying Fano-inequality [7] on decoder 1, we obtain

$$H(W_0, W_1|Y_1^n) \le n(R_0 + R_1)P^n_{e,1} + h(P^n_{e,1}) \triangleq n\epsilon_{1n}, \quad (16)$$

where $h(\cdot)$ is the binary entropy function and $\epsilon_{1n} \to 0$ as $P^n_{e,1} \to 0$. It easily follows that

$$H(W_1|Y_1^n, W_0) \le H(W_0, W_1|Y_1^n) \le n\epsilon_{1n}. \quad (17)$$

By symmetry, we can also get

$$H(W_2|Y_2^n, W_0) \le H(W_0, W_2|Y_2^n) \le n\epsilon_{2n}. \quad (18)$$

We now expand the entropy term $H(Y_1^n, V_2^n|W_0, W_1)$ as

$$\begin{aligned} H(Y_1^n, V_2^n|W_0, W_1) &\overset{(a)}{=} H(Y_1^n, V_2^n|X_1^n, W_0, W_1) \\ &\overset{(b)}{=} H(V_2^n|X_1^n, W_0, W_1) + H(Y_1^n|V_2^n, X_1^n, W_0, W_1) \\ &\overset{(c)}{=} H(Y_1^n|X_1^n, W_0, W_1) + H(V_2^n|Y_1^n, X_1^n, W_0, W_1), \end{aligned}$$

where (a) follows from the fact that $X_1^n = f_1(W_0, W_1)$ is a deterministic function of W_0 and W_1 for a given $(2^{nR_0}, 2^{nR_1}, 2^{nR_2}, n, P_e)$ code; both (b) and (c) are based on the chain rule. Since Y_1 is a deterministic function of X_1 and V_2, $H(Y_1^n|V_2^n, X_1^n, W_0, W_1) = 0$. Similarly, due to $V_2 = h_1(Y_1, X_1)$, $H(V_2^n|Y_1^n, X_1^n, W_0, W_1) = 0$. Hence, we obtain the following equality

$$H(V_2^n|X_1^n, W_0, W_1) = H(Y_1^n|X_1^n, W_0, W_1),$$

which can be further simplified as follows

$$\begin{aligned} H(V_2^n|W_0, W_1) &\overset{(a)}{=} H(Y_1^n|W_0, W_1), \\ H(V_2^n|W_0) &\overset{(b)}{=} H(Y_1^n|W_0, W_1), \end{aligned} \quad (19)$$

where (a) again follows from the deterministic relation between X_1^n and (W_0, W_1), and (b) follows from the conditional independence between V_2^n and W_1 given W_0. Analogously, we can

$$H(V_1^n|W_0) = H(Y_2^n|W_0W_2). \quad (20)$$

One more pair of crucial inequalities are to be shown before we proceed to the main part of the converse, and the two are listed as follows

$$I(W_1; Y_1^n|W_0) \le I(W_1; Y_1^nV_1^n|V_2^nW_0), \quad (21)$$
$$I(W_2; Y_2^n|W_0) \le I(W_2; Y_2^nV_2^n|V_1^nW_0). \quad (22)$$

The inequality (21) can be derived in the following:

$$\begin{aligned} I(W_1; Y_1^n|W_0) &= H(W_1|W_0) - H(W_1|Y_1^nW_0) \\ &\overset{(a)}{\le} H(W_1|V_2^nW_0) - H(W_1|Y_1^nV_2^nW_0) \\ &\overset{(b)}{\le} H(W_1|V_2^nW_0) - H(W_1|Y_1^nV_1^nV_2^nW_0) \\ &= I(W_1; Y_1^nV_1^n|V_2^nW_0), \end{aligned}$$

where (a) follows from the facts that $H(W_1|W_0) = H(W_1|V_2^nW_0)$ which is due to the conditional independence between W_1 and V_2^n given W_0, and "conditioning reduces entropy", i.e., $H(W_1|Y_1^nV_2^n$

$W_0) \le H(W_1|Y_1^n W_0)$; and (b) follows from "conditioning reduces entropy" as well. Similarly, we can obtain (22).

Now we prove each of inequalities (2)–(14) with (17)–(22). Firstly, inequalities (2) and (3) are obvious.

For (4), we have

$$\begin{aligned} nR_1 &= H(W_1) = H(W_1|W_0) \\ &\overset{(a)}{=} H(W_1|W_0V_2^n) \\ &= I(W_1;Y_1^n|W_0V_2^n) + H(W_1|Y_1^nW_0V_2^n) \\ &\overset{(b)}{\le} H(Y_1^n|W_0V_2^n) - H(Y_1^n|W_0W_1V_2^n) + n\epsilon_{1n} \\ &\overset{(c)}{=} H(Y_1^n|W_0V_2^n) + n\epsilon_{1n} \\ &\le \sum_{i=1}^n H(Y_{1i}|V_{2i}W_0) + n\epsilon_{1n}, \end{aligned} \tag{23}$$

where (a) follows from the fact that W_1 and V_2^n are conditionally independent given W_0; (b) follows from $H(W_1|Y_1^nW_0V_2^n) \le H(W_1|Y_1^nW_0) \le n\epsilon_{1n}$; (c) follows from $H(Y_1^n|W_0W_1V_2^n) = H(Y_1^n|X_1^n V_2^nW_0W_1) = 0$.

Analogously for (5), we have

$$nR_2 \le \sum_{i=1}^n H(Y_{2i}|V_{1i}W_0) + n\epsilon_{2n}. \tag{24}$$

With respect to (6), we can get

$$\begin{aligned} n(R_1+R_2) &= H(W_1) + H(W_2) \\ &= H(W_1|W_0) + H(W_2|W_0) \\ &= I(W_1;Y_1^n|W_0) + H(W_1|Y_1^nW_0) + I(W_2;Y_2^n|W_0) \\ &\quad + H(W_2|Y_2^nW_0) \\ &\overset{(a)}{\le} H(Y_1^n|W_0) - H(Y_1^n|W_0W_1) + H(Y_2^n|W_0) \\ &\quad - H(Y_2^n|W_0W_2) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\overset{(b)}{=} H(Y_1^n|W_0) - H(V_2^n|W_0) + H(Y_2^n|W_0) \\ &\quad - H(V_1^n|W_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\le H(Y_1^nV_1^n|W_0) - H(V_1^n|W_0) + H(Y_2^nV_2^n|W_0) \\ &\quad - H(V_2^n|W_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &= H(Y_1^n|V_1^nW_0) + H(Y_2^n|V_2^nW_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\le \sum_{i=1}^n H(Y_{1i}|V_{1i}W_0) + \sum_{i=1}^n H(Y_{2i}|V_{2i}W_0) \\ &\quad + n(\epsilon_{1n}+\epsilon_{2n}), \end{aligned} \tag{25}$$

where (a) follows from inequalities (17) and (18); (b) follows from equalities (19) and (20).

Regarding to (7), we have

$$\begin{aligned} n(R_1+R_2) &= H(W_1|W_0) + H(W_2|W_0) \\ &\overset{(a)}{\le} I(W_1;Y_1^n|W_0) + I(W_2;Y_2^n|W_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\overset{(b)}{\le} I(W_1;Y_1^n|W_0) + I(W_2;Y_2^nV_2^n|V_1^nW_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &= I(W_1;Y_1^n|W_0) + I(W_2;V_2^n|V_1^nW_0) \\ &\quad + I(W_2;Y_2^n|V_1^nV_2^nW_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\le H(Y_1^n|W_0) - H(Y_1^n|W_0W_1) + H(V_2^n|V_1^nW_0) \\ &\quad - H(V_2^n|V_1^nW_2W_0) + H(Y_2^n|V_1^nV_2^nW_0) \\ &\quad - H(Y_2^n|V_1^nV_2^nW_2W_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\overset{(c)}{=} H(Y_1^n|W_0) + H(Y_2^n|V_1^nV_2^nW_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\le \sum_{i=1}^n H(Y_{1i}|W_0) + \sum_{i=1}^n H(Y_{2i}|V_{1i}V_{2i}W_0) \\ &\quad + n(\epsilon_{1n}+\epsilon_{2n}), \end{aligned} \tag{26}$$

where (a) follows from inequalities (17) and (18); (b) follows from inequality (21); (c) follows from the facts that 1) $H(Y_1^n|W_0W_1) = H(V_2^n|V_1^nW_0)$, 2) $H(V_2^n|V_1^nW_2W_0) = 0$ due to that V_2^n is determined by X_2^n which is again determined by (W_0, W_2), and 3) $H(Y_2^n|V_1^nV_2^nW_2W_0) = H(Y_2^n|\, X_2^nV_1^nV_2^nW_2W_0) = 0$.

Similarly, we have

$$\begin{aligned} n(R_1+R_2) &\le \sum_{i=1}^n H(Y_{1i}|W_0) + \sum_{i=1}^n H(Y_{2i}|V_{1i}V_{2i}W_0) \\ &\quad + n(\epsilon_{1n}+\epsilon_{2n}), \end{aligned} \tag{27}$$

which corresponds to (9).

For (8), we have

$$\begin{aligned} n(R_0+R_1+R_2) &= H(W_0W_1) + H(W_2|W_0) \\ &\overset{(a)}{\le} I(W_0W_1;Y_1^n) + I(W_2;Y_2^n|W_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\overset{(b)}{\le} I(W_0W_1;Y_1^n) + I(W_2;Y_2^nV_2^n|V_1^nW_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &= I(W_0W_1;Y_1^n) + I(W_2;V_2^n|V_1^nW_0) \\ &\quad + I(W_2;Y_2^n|V_1^nV_2^nW_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\le H(Y_1^n) - H(Y_1^n|W_0W_1) + H(V_2^n|V_1^nW_0) \\ &\quad - H(V_2^n|V_1^nW_2W_0) + H(Y_2^n|V_1^nV_2^nW_0) \\ &\quad - H(Y_2^n|V_1^nV_2^nW_2W_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\overset{(c)}{=} H(Y_1^n) + H(Y_2^n|V_1^nV_2^nW_0) + n(\epsilon_{1n}+\epsilon_{2n}) \\ &\le \sum_{i=1}^n H(Y_{1i}) + \sum_{i=1}^n H(Y_{2i}|V_{1i}V_{2i}W_0) \\ &\quad + n(\epsilon_{1n}+\epsilon_{2n}), \end{aligned} \tag{28}$$

where (a), (b) and (c) follow from the same arguments for (26). Note that the proof for (28) and the one for (26) only differ in the first few steps, and the rest follows from the same set of arguments and procedures.

Instead of expressing $n(R_0+R_1+R_2)$ as $H(W_0W_1)+H(W_2|W_0)$, we set $n(R_0+R_1+R_2) = H(W_0|W_1) + H(W_0W_2)$. Following the similar steps used in deriving (28), we now obtain

$$\begin{aligned} n(R_0+R_1+R_2) &\le \sum_{i=1}^n H(Y_{2i}) + \sum_{i=1}^n H(Y_{1i}|V_{1i}V_{2i}W_0) \\ &\quad + n(\epsilon_{1n}+\epsilon_{2n}), \end{aligned} \tag{29}$$

which corresponds to (10).

Now for (11), we can get

$$\begin{aligned} &n(2R_1+R_2) \\ &= H(W_1|W_0) + H(W_1|W_0) + H(W_2|W_0) \\ &\overset{(a)}{\le} I(W_1;Y_1^n|W_0) + I(W_1;Y_1^n|W_0) + I(W_2;Y_2^n|W_0) \\ &\quad + n(2\epsilon_{1n}+\epsilon_{2n}) \\ &\overset{(b)}{\le} I(W_1;Y_1^n|W_0) + I(W_1;Y_1^nV_1^n|V_2^nW_0) + I(W_2;Y_2^n|W_0) \\ &\quad + n(2\epsilon_{1n}+\epsilon_{2n}) \end{aligned}$$

$$
\begin{aligned}
&= I(W_1;Y_1^n|W_0) + I(W_1;V_1^n|V_2^nW_0) \\
&\quad + I(W_1;Y_1^n|V_1^nV_2^nW_0) + I(W_2;Y_2^n|W_0) \\
&\quad + n(2\epsilon_{1n} + \epsilon_{2n}) \\
&= H(Y_1^n|W_0) - H(Y_1^n|W_0W_1) + H(V_1^n|V_2^nW_0) \\
&\quad - H(V_1^n|V_2^nW_0W_1) + H(Y_1^n|V_1^nV_2^nW_0) \\
&\quad - H(Y_1^n|V_1^nV_2^nW_0W_1) + H(Y_2^n|W_0) - H(Y_2^n|W_0W_2) \\
&\quad + n(2\epsilon_{1n} + \epsilon_{2n}) \\
&\overset{(c)}{=} H(Y_1^n|W_0) - H(Y_1^n|W_0W_1) + H(Y_1^n|V_1^nV_2^nW_0) \\
&\quad + H(Y_2^n|W_0) + n(2\epsilon_{1n} + \epsilon_{2n}) \\
&\overset{(d)}{=} H(Y_1^n|W_0) - H(V_2^n|W_0) + H(Y_1^n|V_1^nV_2^nW_0) \\
&\quad + H(Y_2^n|W_0) + n(2\epsilon_{1n} + \epsilon_{2n}) \\
&\leq H(Y_1^n|W_0) - H(V_2^n|W_0) + H(Y_1^n|V_1^nV_2^nW_0) \\
&\quad + H(Y_2^nV_2^n|W_0) + n(2\epsilon_{1n} + \epsilon_{2n}) \\
&= H(Y_1^n|W_0) + H(Y_1^n|V_1^nV_2^nW_0) + H(Y_2^n|V_2^nW_0) \\
&\quad + n(2\epsilon_{1n} + \epsilon_{2n}) \\
&\leq \sum_{i=1}^{n} H(Y_{1i}|W_0) + \sum_{i=1}^{n} H(Y_{1i}|V_{1i}V_{2i}W_0) \\
&\quad + \sum_{i=1}^{n} H(Y_{2i}|V_{2i}W_0) + n(2\epsilon_{1n} + \epsilon_{2n}), \qquad (30)
\end{aligned}
$$

where (a) follows from inequalities (17) and (18); (b) follows from inequality (21); (c) follows from the facts that $H(V_1^n|V_2^nW_0) = H(V_1^n|W_0) = H(Y_2^n|W_0W_2), H(V_1^n|V_2^nW_0W_1) = H(V_1^n|X_1^nV_2^n W_0W_1) = 0$, and $H(Y_1^n|V_1^nV_2^nW_0W_1) = H(Y_1^n|\ V_1^nX_1^nV_2^nW_0 W_1) = 0$; (d) follows from $H(V_2^n|W_0) = H(Y_1^n|W_0W_1)$. Following similar procedures, we can easily obtain

$$
\begin{aligned}
n(R_1 + 2R_2) &\leq \sum_{i=1}^{n} H(Y_{2i}|W_0) + \sum_{i=1}^{n} H(Y_{2i}|V_{1i}V_{2i}W_0) \\
&\quad + \sum_{i=1}^{n} H(Y_{1i}|V_{1i}W_0) + n(\epsilon_{1n} + 2\epsilon_{2n}), \qquad (31) \\
n(R_0 + 2R_1 + R_2) &\leq \sum_{i=1}^{n} H(Y_{1i}) + \sum_{i=1}^{n} H(Y_{1i}|V_{1i}V_{2i}W_0) \\
&\quad + \sum_{i=1}^{n} H(Y_{2i}|V_{2i}W_0) + n(2\epsilon_{1n} + \epsilon_{2n}), \text{ and} \qquad (32) \\
n(R_0 + R_1 + 2R_2) &\leq \sum_{i=1}^{n} H(Y_{2i}) + \sum_{i=1}^{n} H(Y_{2i}|V_{1i}V_{2i}W_0) \\
&\quad + \sum_{i=1}^{n} H(Y_{1i}|V_{1i}W_0) + n(\epsilon_{1n} + 2\epsilon_{2n}), \qquad (33)
\end{aligned}
$$

which correspond to (13), (12) and (14) respectively.

Note that we have obtained a number of inequalities (23)–(33) which, together with (2) and (3), bound the rate triple (R_0, R_1, R_2) of the given code for the DICC channel. We now apply the technique used to prove the converse for the capacity region of the MACC in [8] and [9]. Define $V_0 = W_0$, or equivalently $V_{0i} = W_0$, i.e., V_0 or V_{0i} is an auxiliary random variable uniformly distributed over the common message set $\mathcal{W}_0 = \{1, ..., M_0\}$. Since X_1 and X_2 are conditionally independent given W_0, i.e., $p(x_{1i}, x_{2i}|w_0) = p(x_{1i}|w_0)$ $p(x_{2i}|w_0)$, we write $p(x_{1i}, x_{2i}|v_{0i}) = p(x_{1i}|v_{0i})p(x_{2i}|v_{0i})$.

Note that due to the existence of V_0, the region inherits the convexity from the achievable rate region for the general ICC. We can now conclude that as $n \to \infty$ and $P_e \to 0$, we have the rate of the given code (R_0, R_1, R_2) bounded by (2)–(14) for some choice of joint distribution $p(v_0)p(x_1|v_0)p(x_2|v_0)$. This completes the proof of the converse and the theorem. ∎

Remark 1 1) As mentioned earlier, our assumption of this class of deterministic channel is slightly different from the one given in [6]. We directly require the existence of functions $V_2 = h_1(Y_1, X_1)$ and $V_1 = h_2(Y_2, X_2)$ such that we have the two equalities $H(V_2^n|W_0) = H(Y_1^n|W_0W_1)$ and $H(V_1^n|W_0) = H(Y_2^n|W_0W_2)$. As demonstrated in the above proof, the two inequalities are crucial, without which we are not able to establish the converse. Moreover, the two equalities in fact reduce to the assumptions made in [6] for the case of no common information. Therefore, we can claim that the existence of $V_2 = h_1(Y_1, X_1)$ and $V_1 = h_2(Y_2, X_2)$ is the more general condition for this class of deterministic interference channels. 2) The capacity region of the class of DICCs which we derive above generalizes the one given in [6].

4. CONCLUSIONS

In summary, it is shown in this paper that the achievable rate region obtained in [4] is indeed the capacity region for the class of deterministic channels investigated. It is also demonstrated that the assumption of the existence of the two crucial deterministic functions is the key to establish the converse, and this is a generalized assumption compared with the one raised in [6].

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