THE SUPPORTABLE QOS REGION OF A MULTIUSER SYSTEM WITH LOG-CONVEX INTERFERENCE FUNCTIONS

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ABSTRACT

We address the problem of interference coupling and achievability of SIR targets in a multiuser system. Interference is modeled by an axiomatic framework, with log-convex interference functions. There are efficient algorithms which perform optimization over the boundary of the SIR region, but they typically require that the boundary is achievable. We show that achievability is closely linked to the interference coupling in the system. This effect is already known from power control theory, where achievability is commonly ensured by assuming an irreducible coupling matrix. In this paper, we consider a more general interference model which is based on an axiomatic framework. In order to describe the interference coupling in the system, the concept of a dependency matrix is introduced. It is shown that the achievability of the boundary only depends on the combinatorial structure of this matrix. Necessary and sufficient conditions are derived.

***Index Terms*— resource allocation, QoS provision, power control**

1 INTRODUCTION

Signal processing for wireless communications often deals with the problem of reducing or filtering interference in a multiuser network. Often, it is not sufficient to model the system as a collection of point-to-point radio links. In this case, mutual interference causes decencies which can affect many functionalities of the system. This motivates a system-wide optimization, where interference mitigation and resource allocation go hand in hand. The philosophy behind this cross-layer approach has already influenced current 3G standards, like HSDPA or HSUPA.

A thorough understanding of the achievable (or supportable) quality-of-service (QoS) region provides the basis for the development of algorithms for adaptive resource allocation and interference management. Assuming a one-to-one mapping QoS $= \phi(\text{SIR})$, we can focus our attention on the achievable SIR region, which is defined as the set of SIR vectors $\boldsymbol{\gamma} \in \mathbb{R}_+^K$, which can be simultaneously supported by all K communication links. This set is generally limited by the impact of mutual interference.

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The kth user has a signal-to-interference ratio

$$\text{SIR}_k(\boldsymbol{p}) = p_k / \mathcal{I}_k(\boldsymbol{p}) \ , \tag{1}$$

where $\mathcal{I}_k(\boldsymbol{p})$ is the interference (and possibly noise) caused by other users in the network, and $\boldsymbol{p} \in \mathbb{R}_+^K$ is the *power allocation* vector.

Definition 1. An SIR vector $\boldsymbol{\gamma} > 0$ (component-wise greater) is *achievable* if there exists a power allocation $\boldsymbol{p} > 0$ such that

$$\text{SIR}_k(\boldsymbol{p}) \geq \gamma_k, \quad k = 1, 2, \ldots, K \ . \tag{2}$$

The *achievable SIR region* is

$$\mathcal{S} = \{[\text{SIR}_1(\boldsymbol{p}), \ldots, \text{SIR}_K(\boldsymbol{p})] : \boldsymbol{p} > 0\} \ . \tag{3}$$

First work on the achievable SIR region $\mathcal{S}$ appeared in the context of satellite communications and CDMA (see e.g. [4] for an overview). This work was mainly focused on power control under the assumption of a constant link gain matrix. Later, the results motivated researchers to combine power allocation with signal processing techniques, e.g. in the context of beamforming [5]–[7]. Another line of research is Yates' axiomatic framework [8], which defines an interference function $\mathcal{I}(\boldsymbol{p})$ by only requiring certain monotonicity and scalability properties. This abstract approach has been proven to be a useful tool, which includes many existing problems as special cases (see e.g. [8], [9]). The axiomatic approach was further generalized in [10], where fundamental properties of the SIR region were studied.

In this paper, we focus on the general class of log-convex interference functions. Log-convex interference functions appear naturally in many problems related to resource allocation (see the examples in Section 2). An important aspect of log-convex interference functions is the convexity of the associated SIR region for certain types of log-convex QoS-to-SIR mappings, as studied in [11], [12]. This property plays an important role for the development of algorithms for multiuser resource allocation, which optimize over the boundary of the SIR region. Examples are max-min fairness and weighted utility optimization. Iterative optimization strategies for both problems were proposed in [1], [10] for example.

However, these algorithms depend on the achievability of the boundary of the QoS (resp. SIR) region. If the boundary is not achievable, then this typically means that algorithms, which are

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designed to perform optimization over the boundary, will not converge. So the question of achievability, as defined by (2), is fundamental for the development of numerical optimization strategies.

In this paper, we follow previous work (see e.g. [4]), where the SIR region is defined as a sub-level set depending on an inf-max optimum. This definition is also consistent with previous work on the Perron root of an irreducible coupling matrix (see e.g. [1], [4]), where the boundary is always achievable due to the common assumption of irreducibility.

Since our approach is based on the more general axiomatic framework, we cannot use the conventional framework based on a coupling matrix. A different approach is required instead. The boundary is only achievable if the inf-max optimum exists. In other words, it is possible that certain targets $\boldsymbol{\gamma}$ can only be achieved asymptotically, which prevents convergence of the algorithms.

General conditions for the achievability of boundary points were derived in [10], but this analysis turned out rather complicated. For instance, achievability of one boundary point does not imply achievability of the entire boundary. In general, there is no single criterion for characterizing achievability of the complete region.

One main contribution of this paper is to show that the SIR region associated with log-convex interference functions has a different behavior. It will turn out that achievability of one boundary point implies achievability of the entire SIR region. We completely characterize the interference coupling and provide necessary and sufficient conditions for achievability. The analysis is based on the special structure of log-convex interference functions [13]. In particular, we exploit that every log-convex interference function can be decomposed in a product of elementary functions.

2 Log-Convex Interference Functions

We start by introducing and analyzing basic properties.

2.1 Axiomatic Framework and Examples

A function $\mathcal{I}(\boldsymbol{p})$ is called “interference function” if it fulfills the following axioms:

A1 $\quad \mathcal{I}(\boldsymbol{p}) \geq 0, \quad \boldsymbol{p} \in \mathbb{R}^K_+$

A2 $\quad \mathcal{I}(\alpha\boldsymbol{p}) = \alpha\mathcal{I}(\boldsymbol{p}) \quad \alpha \in \mathbb{R}_+$

A3 $\quad \mathcal{I}(\boldsymbol{p}^{(1)}) \leq \mathcal{I}(\boldsymbol{p}^{(2)})$ if $\boldsymbol{p}^{(1)} \geq \boldsymbol{p}^{(2)}$

A4 $\quad \mathcal{I}(\exp\{\boldsymbol{s}\})$ is log-convex on $\mathbb{R}^K$.

Here, we use the substitution $\boldsymbol{p} = \exp\{\boldsymbol{s}\}$ (component-wise exponential). The function $\mathcal{I} : \mathbb{R}^K \mapsto \mathbb{R}_+$ is log-convex on $\mathbb{R}^K$ if and only if the logarithm of the function is convex, or equivalently

$$\mathcal{I}\big((1-\mu)\hat{x}+\mu\check{x}\big) \leq \mathcal{I}(\hat{x})^{1-\mu} \cdot \mathcal{I}(\check{x})^{\mu}, \quad \forall \mu \in [0,1],\ \hat{x}, \check{x} \in \mathbb{R}^K .$$

It should be noted that Yates' model [8] is an important special case of the generic model A1-A4. We can introduce a $(K+1)$-dimensional extended power allocation, with a fixed normalized noise power $p_{K+1} = 1$. For the model A1-A3, it is irrelevant whether the power $\mathcal{I}(\boldsymbol{p})$ is caused by noise or an interfering user, so it is general enough to incorporate a possible noise component. If constant noise is present, and if $\mathcal{I}(\boldsymbol{p})$ is strictly increasing in p_{K+1}, then joint power allocation can be performed with a fixed-point iteration [8].

But the following results hold with or without the assumption of such an additional monotonicity property. In this paper, we focus on the special property A4 (log-convexity). In the following we will give some motivating examples for log-convex interference functions.

1) Linear interference function: Consider the linear model

$$\mathcal{I}_k(\boldsymbol{p}) = [\boldsymbol{V}\boldsymbol{p}]_k, \quad \forall k \ , \tag{4}$$

where $\boldsymbol{V} \geq 0$ is a fixed *link gain matrix*, which contains the interference coupling coefficients. This is the classical model, which is used as a basis for many results in power control theory and related areas (see e.g. [4]). Under certain conditions, the log-convexity of $\mathcal{I}_k(\exp\{\boldsymbol{s}\})$ can be exploited to show convexity of the resulting log-SIR region (see e.g. [10] and the references therein). The model (4) is a special case of the more general axiomatic framework A1-A4.

2) Spectral radius: It is known [4] that the SIR region under the linear model (4) can be characterized by the spectral radius $\rho(\boldsymbol{q}) := \rho\big(\mathrm{diag}\{\mathrm{e}^{\boldsymbol{q}}\}\boldsymbol{V}\big)$. The function $\rho(\boldsymbol{q})$ fulfills A1-A4 so, it is itself a log-convex interference function. This provides an additional motivation for analyzing log-convex interference functions and shows that the theory is not limited to interference in a strict sense, but can also be applied to related areas.

3) Robustness: Another example for a log-convex interference function is

$$\mathcal{I}_k(\boldsymbol{p}) = \max_{z_k \in \mathcal{Z}_k} [\boldsymbol{V}(z)\boldsymbol{p}]_k, \quad \forall k \ , \tag{5}$$

where the parameter z_k, chosen from a closed bounded set $\mathcal{Z}_k$, can stand for the impact of error effects. So (5) can be seen as a model for the *worst-case interference*. Performing power allocation with respect to (5) guarantees a certain robustness. In the literature there exist many examples for robust power allocation strategies (see e.g. [14], [15]).

4) Elementary log-convex interference function: A further example, which will play a fundamental role in this paper, is the function

$$\mathcal{I}(\boldsymbol{p}) = C \cdot \prod_l (p_l)^{w_l} \ , \tag{6}$$

with coefficients $C > 0$ and $w_l \geq 0$, $\|\boldsymbol{w}\|_1 = 1$. Using the substitution $\boldsymbol{p} = \mathrm{e}^{\boldsymbol{s}}$, it can be verified that $\mathcal{I}(\mathrm{e}^{\boldsymbol{s}})$ is log-convex on $\mathbb{R}^K$. In addition, $\mathcal{I}(\boldsymbol{p})$ fulfills A1-A4, so it is a log-convex interference function. Note, that the normalization $\|\boldsymbol{w}\|_1 = 1$ is important since it ensures A2.

In the following section it will be shown that (6) can be regarded as a basic building block, into which every log-convex interference function can be decomposed.

2.2 Representation Theorem

For *every* log-convex interference function $\mathcal{I}(\boldsymbol{p})$ defined by A1-A4, there exists a product decomposition of the form (6). However, the coefficient C generally depends on the power allocation. To this end, we introduce a function

$$f_{\mathcal{I}}(\boldsymbol{w}) = \inf_{\boldsymbol{p}>0} \frac{\mathcal{I}(\boldsymbol{p})}{\prod_{l=1}^K (p_l)^{w_l}} \ . \tag{7}$$

It can be shown that $\log f_{\mathcal{I}}(\boldsymbol{w})$ is the conjugate of the convex function $\log \mathcal{I}(\mathrm{e}^{\boldsymbol{s}})$. With A1, $f_{\mathcal{I}}(\boldsymbol{w}) \geq 0$ is always fulfilled. But we are interested in the non-trivial case where $f_{\mathcal{I}}(\boldsymbol{w})$ is strictly positive. All coefficients for which this is fulfilled are contained in the set

$$\mathcal{L}(\mathcal{I}) = \big\{\boldsymbol{w} \in \mathbb{R}^K_+ : f_{\mathcal{I}}(\boldsymbol{w}) > 0\big\} \ . \tag{8}$$

We have the following result.

Lemma 1. *Let $\mathcal{I}(\boldsymbol{w})$ be an interference function, and $\boldsymbol{w} \in \mathbb{R}^K_+$. If $f_\mathcal{I}(\boldsymbol{w}) > 0$ then $\|\boldsymbol{w}\|_1 = 1$.*

Lemma 1 shows that all coefficients $\boldsymbol{w} \in \mathcal{L}(\mathcal{I})$ fulfill the condition $\|\boldsymbol{w}\|_1 = 1$. This property can be exploited in order to show the following result.

Theorem 1. *Every log-convex interference function $\mathcal{I}(\boldsymbol{p})$, characterized by A1-A4, with an arbitrary $\boldsymbol{p} > 0$, can be represented as*

$$\mathcal{I}(\boldsymbol{p}) = \max_{\boldsymbol{w}\in\mathcal{L}(\mathcal{I})} \Big(f_\mathcal{I}(\boldsymbol{w}) \cdot \prod_{l=1}^{K} (p_l)^{w_l} \Big) . \tag{9}$$

Representation (9) provides the basis for our analysis. It will be shown show that the coefficients $\boldsymbol{w}$ can be used to characterize achievability.

3 Achievability

We start by characterizing the QoS region by means of a min-max balancing problem. To this end, assume that $\boldsymbol{q} = [q_1, \dots, q_K]^T > 0$ is a vector containing target QoS of all K users. Let γ_k be the inverse function of ϕ_k, then $\gamma_k := \gamma_k(q_k)$ is the minimum SIR level needed to achieve the target q_k. In the following we will also collect the targets in a diagonal matrix $\boldsymbol{\Gamma} := \boldsymbol{\Gamma}(\boldsymbol{q}) = \mathrm{diag}\{[\gamma_1(q_1), \dots, \gamma_K(q_K)]\}$.

An important indicator for achievability is the function $C(\boldsymbol{\gamma})$, which is the min-max optimum

$$C(\boldsymbol{\gamma}) = \inf_{\boldsymbol{p}>0} \Big(\max_{1\le k\le K} \frac{\gamma_k \cdot \mathcal{I}_k(\boldsymbol{p})}{p_k} \Big) . \tag{10}$$

The optimum $C(\boldsymbol{\gamma})$ is a single measure for the quality of a multiuser channel. If there exists a point $\boldsymbol{p} > 0$ such that $\mathrm{SIR}_k(\boldsymbol{p}) \ge \gamma_k$, $\forall k$, then it can be observed from (10) that $C(\boldsymbol{\gamma}) \le 1$. Conversely, $C(\boldsymbol{\gamma}) \le 1$ implies that $\boldsymbol{\gamma}$ can be approached arbitrarily close. Note, that sometimes $\boldsymbol{\gamma}$ is only achieved in an asymptotic sense. In this case, $\boldsymbol{\gamma}$ is not achievable according to Definition 1 given in the introduction.

Definition 2. The SIR region with no power constraints is defined as the sub-level set

$$\mathcal{F} = \{\boldsymbol{\gamma} > 0 : C(\boldsymbol{\gamma}) \le 1\} . \tag{11}$$

For the linear model (4), the min-max optimum $C(\boldsymbol{\gamma})$ is simply the spectral radius $\rho\big(\mathrm{diag}\{\boldsymbol{\gamma}\}\boldsymbol{V}\big)$, so the definition is consistent with the classical definition of the SIR region (see e.g. [4]). But the axiomatic model is much more general, and also incorporates non-linear strategies. The price of generality is that achievability of the boundary of $\mathcal{F}$ cannot be guaranteed. We have $\mathcal{S} \subseteq \mathcal{F}$, where $\mathcal{S}$ is the achievable region, as defined by (3).

In the following we exploit log-convexity (property A4) in order to derive conditions under which $\boldsymbol{\gamma} \in \mathcal{F}$ is achievable.

3.1 Characterization of Interference Coupling

Achievability depends on the coupling between the users. For the linear model (4), this was ensured by requiring an irreducible coupling matrix $\boldsymbol{V}$.

For the general case of log-convex interference functions considered here, we can use an asymptotic characterization to describe the mutual coupling of log-convex interference functions.

Let $\boldsymbol{e}_l$ be the all-zero vector with the l-th component set to one, i.e.,

$$[\boldsymbol{e}_l]_n = \begin{cases} 1 & n = l \\ 0 & n \ne l . \end{cases}$$

We have the following result.

Lemma 2. *Assume there exists a $\hat{\boldsymbol{p}} > 0$ such that $\lim_{\delta\to\infty} \mathcal{I}_k(\hat{\boldsymbol{p}} + \delta\boldsymbol{e}_l) = +\infty$, then*

$$\lim_{\delta\to\infty} \mathcal{I}_k(\boldsymbol{p} + \delta\boldsymbol{e}_l) = +\infty \quad \textit{for all } \boldsymbol{p} > 0. \tag{12}$$

Here we have exploited A2 and the fact that $\mathcal{I}_k(\boldsymbol{p} + \delta\boldsymbol{e}_l)$ is monotonically increasing in δ. We are interested in the behavior when the increase is unbounded. This asymptotic behavior is independent of $\boldsymbol{p}$, thus it is a suitable way of characterizing the interference coupling for arbitrary interference functions satisfying A1-A3..

Definition 3. We refer to $\boldsymbol{A}_\mathcal{I}$ as the *asymptotic matrix* of $\mathcal{I}$.

$$[\boldsymbol{A}_\mathcal{I}]_{kl} = \begin{cases} 1 & \text{if there exists a } \boldsymbol{p} > 0 \text{ such that} \\ & \lim_{\delta\to\infty} \mathcal{I}_k(\boldsymbol{p} + \delta\boldsymbol{e}_l) = +\infty \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Notice that because of Lemma 2, the condition in (13) does not depend on the choice of $\boldsymbol{p}$.

The characterization provided by the following matrix $\boldsymbol{D}_\mathcal{I}$ is generally weaker.

Definition 4. $\boldsymbol{D}_\mathcal{I}$ is called dependency matrix. We have

$$[\boldsymbol{D}_\mathcal{I}]_{kl} = \begin{cases} 1 & \text{if there exists a } \boldsymbol{p} > 0 \text{ such that } \mathcal{I}_k(\boldsymbol{p} + \delta\boldsymbol{e}_l) \\ & \text{is not constant for all } \delta > 0 \\ 0 & \text{otherwise.} \end{cases}$$

That is, $[\boldsymbol{A}_\mathcal{I}]_{kl} = 1$ implies $[\boldsymbol{D}_\mathcal{I}]_{kl} = 1$, but the converse need not be true in general. But with property A4, both characterizations are indeed equivalent.

Theorem 2. *Let $\mathcal{I}_1, \dots, \mathcal{I}_K$ be log-convex interference functions, then $\boldsymbol{A}_\mathcal{I} = \boldsymbol{D}_\mathcal{I}$.*

3.2 Achievability of the QoS Region

The next theorem shows that if an arbitrary point $\boldsymbol{\gamma}$ on the boundary of $\mathcal{F}$ is achievable, then all $\boldsymbol{\gamma} \in \mathcal{F}$ are achievable. This special behavior is caused by log-convexity and cannot be generalized to arbitrary interference functions characterized by A1-A3 (examples are provided in [10]).

Achievability depends on the existence of a lower bound on the interference function $\mathcal{I}(\boldsymbol{p})$, which is directly linked to the structure of the dependency matrix. By exploiting the representation (9), the following result can be shown.

Theorem 3. *Let $\boldsymbol{\mathcal{I}} = [\mathcal{I}_1, \dots, \mathcal{I}_K]^T$ be a vector of log-convex interference functions, such that there exists a representation (9) with an irreducible matrix $\hat{\boldsymbol{W}} = [\hat{\boldsymbol{w}}_1, \dots, \hat{\boldsymbol{w}}_K]$, with $\hat{\boldsymbol{w}}_k \in \mathcal{L}(\mathcal{I}_k)$. Then for all $\boldsymbol{\gamma} > 0$ there exists a fixed-point $\boldsymbol{p}^* > 0$ such that*

$$\boldsymbol{\Gamma}\boldsymbol{\mathcal{I}}(\boldsymbol{p}^*) = C(\boldsymbol{\gamma})\boldsymbol{p}^* . \tag{14}$$

If (14) is fulfilled, then $\boldsymbol{p}^*$ achieves the infimum (10). With $\boldsymbol{\gamma} \in \mathcal{F}$, it follows that $\boldsymbol{\gamma}$ is achievable, i.e., $p^*_k/\mathcal{I}_k(\boldsymbol{p}^*) \ge \gamma_k$, $\forall k$.

Next, we characterize the existence of a non-achievable point. Assume that the dependency matrix $\boldsymbol{D}_\mathcal{I}$ is reducible. Without loss of generality, we may assume that after simultaneous permutations of rows and columns, $\boldsymbol{D}$ is reduced to canonical form (see e.g. [16, p. 75]), with irreducible blocks along the diagonal. We have $\boldsymbol{D}_{\mathcal{I}} =$

$$\left[\begin{array}{ccc|ccc} \boldsymbol{D}^{(1,1)} & & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ & \ddots & & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & & \boldsymbol{D}^{(r,r)} & \mathbf{0} & \cdots & \mathbf{0} \\ \hline \boldsymbol{D}^{(r+1,1)} & \cdots & \boldsymbol{D}^{(r+1,r)} & \boldsymbol{D}^{(r+1,r+1)} & \mathbf{0} & \mathbf{0} \\ \vdots & \cdots & \vdots & \vdots & \ddots & \mathbf{0} \\ \boldsymbol{D}^{(N,1)} & \cdots & \boldsymbol{D}^{(N,r)} & \boldsymbol{D}^{(N,2)} & \cdots & \boldsymbol{D}^{(N,N)} \end{array}\right]$$

The diagonal square blocks $\boldsymbol{D}^{(n)} := \boldsymbol{D}^{(n,n)}$ have a minimum dimension of two because of A1. We have r isolated blocks with indices $1, \ldots, r$. Let $1, , 2, \ldots, L_r$ be the indices associated with the isolated blocks. If

$$\inf_{\boldsymbol{p}>0} \max_{k>L_r} \frac{\gamma_k \mathcal{I}_k(\boldsymbol{p})}{p_k} = \underline{C}_1(\boldsymbol{\gamma}) > 0 \,, \tag{15}$$

then it can be shown that there exists a $\boldsymbol{\gamma} > 0$ such that the min-max problem has no optimizer $\boldsymbol{p}^* > 0$ (no fixed point exists).

Together with Theorem 3, this can be used in order to show the following result.

Theorem 4. *Let $C(\boldsymbol{\gamma}) > 0$ for all $\boldsymbol{\gamma} > 0$. Then all $\boldsymbol{\gamma} > 0$ are achievable if and only if $\boldsymbol{D}_{\mathcal{I}}$ (resp. $\boldsymbol{A}_{\mathcal{I}}$) is irreducible.*

Finally, it can be shown that an alternative characterization exists in terms of the coefficient matrix $\boldsymbol{W} = [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K]$, where $\boldsymbol{w}_k$ is the coefficient vector for the interference function of the kth user, as introduced in Section 2.2:

Let $\mathcal{I}$ be such that $C(\boldsymbol{\gamma}) > 0$ for all $\boldsymbol{\gamma} > 0$. Then all $\boldsymbol{\gamma} > 0$ are achievable if and only if there exists an irreducible matrix $\hat{\boldsymbol{W}}$ and constants $C_1, \ldots, C_K > 0$, such that

$$\mathcal{I}_k(\boldsymbol{p}) \geq C_k \prod_{l=1}^{K} (p_l)^{\hat{w}_{kl}}, \quad 1 \leq k \leq K \,. \tag{16}$$

for all $\boldsymbol{p} > 0$. This provides a link to the representation result Theorem 1.

4 Algorithm

If a boundary point $\boldsymbol{\gamma} > 0$ is achievable, then the min-max optimal power allocation solving (10) can be computed by an iterative technique [10]. Since the interference functions $\mathcal{I}_k(\boldsymbol{p})$ are defined in an abstract way, a particular realization is required. To this end, we consider the robust interference functions (5), where the parameter-dependent matrix $\boldsymbol{V}(z)$ models the interference coupling. Individual path gains of the K users can easily be incorporated by scaling the rows of $\boldsymbol{V}(z)$. The algorithm is summarized as follows:

$$\boldsymbol{p}^{(n+1)} = \mathrm{pev}(\boldsymbol{\Gamma}\boldsymbol{V}(z^{(n)})) \quad \text{(principal eigenvector)} \tag{17}$$

$$\text{with} \quad z_k^{(n)} = \arg\max_{z_k \in \mathcal{Z}_k} [\boldsymbol{V}(z)\boldsymbol{p}^{(n)}]_k, \;\; k \in \{1, \ldots, K\} \,. \tag{18}$$

As long as the chosen $\boldsymbol{\gamma}$ are achievable, the iteration will converge to the global optimum of the min-max balancing problem (10).

5 Conclusions

Future cross-layer strategies will require a thorough understanding of the QoS achievability region. In this paper we study an axiomatic interference model with log-convex interference functions. This class of interference functions not only incorporates the conventional linear model as a special case, but can also be applied for the analysis of robust transmission schemes.

There are efficient algorithms which perform optimization over the boundary of the region. However they all require that the boundary is achievable. Thus, the characterization of achievability for axiomatic log-convex interference functions is a problem with high practical relevance.

We show that achievability is closely linked to interference coupling. This principle is well known from power control theory, where achievability is commonly ensured by assuming an irreducible coupling matrix. However, a different approach is required for the general axiomatic framework considered in this paper. We introduce the concept of a dependency matrix in order to provide necessary and sufficient conditions for achievability.

The focus of this paper is not so much on the algorithmic aspects, but rather at providing a theoretical framework, which may prove useful for the future development of algorithms for QoS balancing and resource allocation.

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