JOINT ADMISSION CONTROL AND POWER ALLOCATION FOR COGNITIVE RADIO NETWORKS

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ABSTRACT

In this paper, we study the problem of joint admission control and power allocation for cognitive radio networks. In such a scenario, the quality of service for primary and secondary users is needed to be guaranteed, which can be translated into the following two constraints: the inference temperature constraint for primary users and the minimum signal-to-interference-plus-noise ratio (SINR) constraint for secondary users. Due to the high density or the mobility of the secondary users, not all the secondary users are supportable. The problem of our interest is to select the maximum subset of secondary users given that the above constraints are satisfied. Moreover, because different secondary users have different revenue outputs, the problem becomes how we can find a subset of the secondary users such that the total revenue output of the networks is maximized. It can be shown that finding the optimal removal set is a NP hard problem. Therefore, we transform the original problem into a smooth optimization problem, and solve it by using a gradient descent based algorithm. This algorithm solves the power allocation and admission control jointly, and its superior performance over the existing algorithms is demonstrated through simulations.

Index Terms— Admission control, cognitive radio networks, power control, signal to interference-plus-noise ratio.

1. INTRODUCTION

With the rapid emergence of wireless device and services, the limited radio spectrum becomes increasingly crowded. Therefore, the issue of how to efficiently utilize radio spectrum has attracted considerable attentions. Cognitive radio (CR), which allows secondary users to share the spectrum with the primary users, shows great promise to enhance the spectrum utilization efficiency [2]. One constraint on the secondary users is that the interference from the secondary users to the primary user should be less than acceptable value. As a result, power control schemes need to be developed for cognitive radio networks.

Power control problem has been well studied in conventional cellular systems where the channel reuse is an important measure to improve the capacity of the systems. Various schemes for power control, centralized or distributed, have been proposed [5], [6]. These methods, however, assume that there exists a solution that all the users in the networks can coexist with each other subject to the signal to interference-plus-noise ratio (SINR) constraints. Due to the mobility of the transmitters and random effect of signal propagations, there are situations where not all transmissions can be supported at the same time. Consequently, some of the transmitters have to be removed from the networks. This problem is called joint admission control and power allocation problem, which can be stated as whether we could find a power vector for the left users in the networks, such that the quality of the transmission can be guaranteed. The paper [3] proposed single or multiple accumulative removals technique (SMART), [9] proposed stepwise removal algorithm (SRA). Both of these two algorithms involve two stages, power control and user removal. When power control reaches steady state, user removal algorithm is invoked, i.e., one or more transmitters are removed until all the remaining users in the networks are feasible.

In cognitive radio networks, because of the presence of the primary users, more constraints have to be introduced into the problem. When the secondary users in the cognitive radio networks are infeasible, the problem becomes to find a maximum subset of secondary users such that all the users in the subset are provisioned with the quality of service (QoS) in the sense of a guaranteed minimum achievable SINR. The methods proposed for conventional cellular networks cannot be directly applied to cognitive radio networks. Moreover, conventional algorithms in general handle all the users with the same weight, and thus they cannot solve the networks where each user has a different weight. In [4], a game-theoretic method is proposed to solve such a problem. In this spectrum sharing game, each user turns on and switches off the radio sequentially while other users are fixed; subsequently one compares the revenue output of the networks between the two actions of turning on and switching off, and choose the action with high output. Typically, this method converges to a local optimal point, and its convergence behavior is quite random.

In this paper, we propose an algorithm to solve the problem of finding the optimal subset of the secondary users where the total secondary users are not feasible, such that the total output revenue of the networks is maximized. We first define the penalty functions for all the constraints, and then transform the constrained integer programming problem into a smooth optimization problem. A gradient descent based algorithm is introduced to solve this problem.

2. INFEASIBLE SECONDARY USER SHARING MODEL

Consider a cognitive radio network model with one primary user and multiple secondary users. In such a model, the primary and secondary users often have different QoS requirements, which are normally described in terms of interference constraints. For the primary user, we quantify the QoS by adopting *interference temperature*, which was suggested by the FCC Spectrum Policy Task Force in [1]. Specifically, written in form of interference temperature, the total received power at the primary user's receiver need satisfy

$$\sum_i g_{io} p_i \leq B, \tag{1}$$

where g_{io} is the channel gain from the transmitter of secondary user i to the receiver of the primary user denoted by o[1], p_i denotes the transmit power of secondary user i, and B represents a preselected threshold. On the other hand, the QoS for secondary users is measured by their SINRs. For a prescribed power allocation of *active* secondary users, the SINR at a secondary user's receiver is given by:

$$\gamma_i = \frac{g_{ii}p_i}{\sum_{j\neq i} g_{ij}p_j + n_i}, \tag{2}$$

where g_{ij} is the channel gain from user i's transmitter to the receiver of user j, and n_i is the background noise power that is assumed to be the same for all users. We further require that a secondary user is allowed to share the resource with the primary user only if its SINR exceeds a certain threshold. This is a reasonable restriction for the admission of a secondary user to the network. Suppose that if there is no such requirement for secondary users, then certain secondary users with very low received SINRs could be admitted to the network. In such a scenario, these secondary users not only can not accomplish their transmissions properly but also at the same time cause strong interference to other users. Thus, we introduce the notion of the *infeasible* cognitive radio networks as follows:

Definition: A cognitive radio network is infeasible if and only if there does not exist power allocation for all the secondary users in the network such that both the interference temperature for the primary user (1) and the SINR constraints for secondary users (2) are satisfied.

In an infeasible network, there might be more than one feasible subnetworks. Our objective is to choose a subset of secondary users from the network to optimize our objective function. Certainly, the choice of the objective function depends upon applications, which will be addressed in the next section.

3. PROBLEM FORMULATION

In this paper, we select the objective function as the total revenue output since maximizing revenue output subject to certain interference constraints is of practical interests in a commercial network. Mathematically, the problem is formulated as follows:

$$\max \sum_i w_i x_i \tag{3}$$

subject to

$$\sum_i g_{io}p_i \leq B \text{ and } \mathrm{SINR}_i \geq \gamma \cdot x_i, \tag{4}$$

where w_i is the revenue output of secondary user i if user i is active in the network, and x_i denotes an indicator function, i.e., $x_i = 1$ indicates that user i is active, and $x_i = 0$ indicates that user i is not allowed to be admitted to the system. The optimization problem (3) is an integer programming problem, which can be shown to be a NP hard problem. Hence, we transform the original problem (3) into the following equivalent optimization problem :

$$\min\{\sum_{i=1}^{N} w_i f_{\mathrm{s}}(v_i) + w_o f_{\mathrm{s}}(v_o)\} \tag{5}$$

[1]Since the uplink and downlink channels are symmetric, the channel gain can be obtained from the pilot signal from the primary receiver.

where

$$v_o = B - \sum_i g_{io}p_i, \tag{6}$$
$$v_i = \frac{g_{ii}p_i}{\sum_{j\neq i} g_{ij}p_j + n_i} - \gamma,$$

$$f_{\mathrm{s}}(v) = \begin{cases} 1, v_i < 0 \\ 0, v_i \geq 0 \end{cases},$$

where w_o is the weight of the primary user, which is assumed to satisfy that $w_o \geq \sum_{i=1}^{N} w_{\mathrm{i}}$, and N is the number of possible secondary users. However, the problem is still difficult to handle due to the nondifferentiability of $f_{\mathrm{s}}(v)$. We therefore introduce a differentiable logistic function :

$$f(v) = \frac{1}{1 + e^{a(v+b)}} \tag{7}$$

to substitute $f_{\mathrm{s}}(v)$ in (5), which is depicted in Fig. 1. We divide

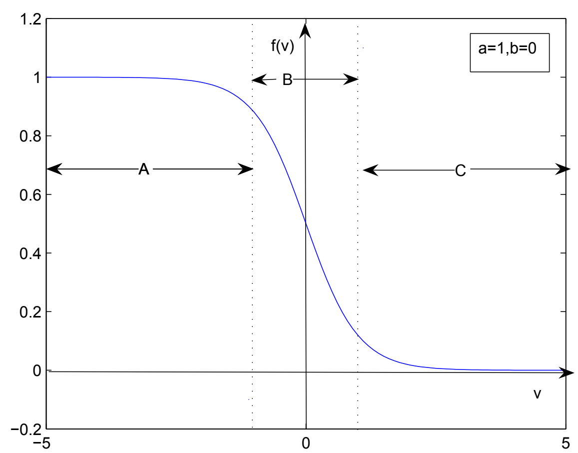

Fig. 1. The shape of the logistic function.

the domain of the logistic function into three different regions, A: $(-\infty, -1]$, B:$(-1, 1)$, and C:$[1, \infty)$. We call region B as active region, in which the slope of the function is largest; we call regions A and C as saturate region in which the slopes of the function are much flatter compare with the one active region. The desirable feature of the logistic function is its differentiability, which converts the original problem into the following a smooth optimization problem.

$$\min\{\sum_{i=1}^{N} w_i f(v_i) + w_o f(v_o)\} \tag{8}$$

where

$$v_o := B - \sum_i g_{io}p_i,$$
$$v_i := (g_{ii}p_i)/(\sum_{j\neq i} g_{ij}p_j + n_i) - \gamma.$$

If we select the parameters of logistic function properly, this smooth optimization is approximately equivalent to the original problem.

4. AN ADMISSION CONTROL AND POWER ALLOCATION ALGORITHM

In this section, we present a gradient descent based algorithm to solve this admission control and power allocation problem. Determining an optimal subset of secondary users and its corresponding power allocation jointly involves large complexity, which is practically not affordable. Alternatively, this problem can be approached by applying the following two algorithms, which essentially decouple the original jointly optimization problem into two subproblems and solved them in an iterative manner. In the algorithm adopted in [4], one first selects a subset of users according to some criterion, and then uses a power control algorithm to check whether this selection is feasible. If the selection is feasible, the revenue output of the networks is computed, otherwise, a different subset of secondary users will be selected. The algorithm repeats until a local optimal point is reached. The other algorithm is to apply power control algorithm for the infeasible network until the algorithm reaches a steady state. If not all the constraints are satisfied simultaneously, certain secondary users will be removed according to some criterion. Such a process is repeated until a feasible subset of users is found. In the paper, we adopted the second algorithm. Since the logistic function is not convex, the power control algorithm only converges to a local optimal stationary point. In the algorithm, when the algorithm converges to a stationary point but certain interference constraints are not satisfied, we will evoke a user removal step, in which one secondary user will be removed. The algorithm ends only when the interference constraints for the primary user and the remaining secondary users in the network are satisfied. In what follows, we detailed our algorithm that is used to solve the problem (8).

Minimal SINR Removal Algorithm(MSRA)

Minimal SINR Removal Algorithm

1. Initialize the parameters k, a, b, η, and $\mathbf{P}^{(0)}$
2. **loop**
3. calculate the $F^{(k)}$ and its gradient $\nabla F^{(k)}$
4. **if** all the constraints are satisfied, stop;
   elseif stationary state reaches, evoke user remove process, go to 2.
   else using backtracking line search algorithm to select a step size η, and update the $\mathbf{P}$, according to
   $\mathbf{P}^{(k+1)} = \min\{\max\{\mathbf{P}^{(k)} - \eta \bullet \nabla F^{(k)}, \mathbf{0}\}, \mathbf{P}_{\max}\}$,
   go to 2.
5. **end loop**

where $F(\mathbf{P}) := \sum_{i=1}^{N} w_i f_i(v_i) + w_o f_o(v_o)$, $F^{(k)}$ denotes the value of $F(\mathbf{P^{(k)}})$ at the kth iteration step, and $\mathbf{P}^{(k)}$ denotes the power allocation vector at kth iteration step, where $k = 0, 1, \cdots$, and $\mathbf{P}_{\max}$ denotes the maximum possible transmit power vector for seconder users.

4.1. Iteration step size

In computing the step size η, we would like to choose η to give a substantial reduction of F, but at the same time, we do not want to spend too much time making the choice. The ideal choice would be the global minimizer of the univariate function $\phi(\cdot)$ defined by

$$\phi(\eta) = F(\mathbf{P^{(k)}} - \eta \nabla F),\ \eta > 0, \tag{9}$$

but in general, it is too expensive to find the exact value of η. Moreover, to guarantee the convergence of the algorithm, Armigo condition [8] must be satisfied. As a result, we adopted the backtracking line search algorithm [8].

4.2. Convergence analysis

We summarize the convergence analysis in the following theorem:

Theorem 1 *The MSRA converges to a stationary point in the range* $[0, P_{\max}]^n$.

$Proof$: According to the backtracking line search algorithm:

$$F^{(k+1)} < F^{(k)} - c\eta ||\nabla F||, \tag{10}$$

where $\eta > 0$, and $c \in (0, 1)$. Thus, Armijo's condition is satisfied. Moreover, in the iterative process, $F^{(k+1)} < F^{(k)}$, thus, the strong descent conditions [7] is satisfied. As a result, according to the Theorem 3.2 in [7], $\lim_{k\to\infty} ||\mathbf{P}^{(k)}|| = +\infty$, or there exists a single point $\mathbf{P}^* \in \mathbf{R}^n$ such that $\lim_{k\to\infty} ||\mathbf{P}^{(k)}|| = \mathbf{P}^*$. Because the $\mathbf{P}^{(k)}$ are in the range of $[0, P_{\max}]^n$, it is impossible that $\lim_{k\to\infty} ||\mathbf{P}^{(k)}|| = +\infty$. Therefore, our algorithm converges to stationary point $\mathbf{P}^*$.

4.3. User removal process

If some interference constraints are not satisfied when the iterative process reaches a steady point, then the whole network is infeasible and at least one secondary user needs be removed. In the proposed algorithm, the criteria is to remove the user with least SINR in the steady state. Intuitively speaking, if a less SINR a user has, the larger impacts it has on the other users. Therefore, it is highly likely that the network becomes feasible after the user is removed. Once a user is removed from the networks, we re-initialize the algorithm for the remaining users in the network, and repeat the process.

4.4. Stop criteria

As mentioned earlier, the algorithm ends until all the interference constraints for the primary user and remaining secondary users are satisfied. In the following, we provide a condition to test whether given power allocation can support all the users in the network.

Condition: For $\mathbf{P}^{(k)}$, if the related $v_i > 0, i \in [0, 1, \ldots, |N_o|]$, then all the transmitters in network N_o are supportable under $\mathbf{P}^{(k)}$, where N_o is the subnetwork with $|N_o|$ denoting number of users in it.

5. SIMULATION RESULTS

In this section, we provide several simulation examples to demonstrate the effectiveness of our proposed algorithm. In all simulation examples, we model the channels between different users as a path loss model with pass loss exponent being 4, i.e., $g_{ij} = K/d_{ij}^4$, where d_{ij} is the distance between user j and user i and K denotes a constant.

Example 1 *In this example, we consider a network with one primary user and five secondary users. The target SINRs for secondary users are selected to be* $\gamma = 4$*, the noise power denoted by* σ^2 *is chosen to be* 5×10^{-4}*, the ratio between the preselected threshold B and the noise power is* $B/\sigma^2 = 40$*, and the weight of the primary user is* 10*. Fig. 2 shows the weights of the secondary users. It is easy to check that this network is infeasible for supporting all five secondary user. As shown in Fig. 2, the MSRA removes two secondary users from the original network and yields an optimal revenue output. This shows that the MSRA can reach a global optimal point in certain scenarios.*

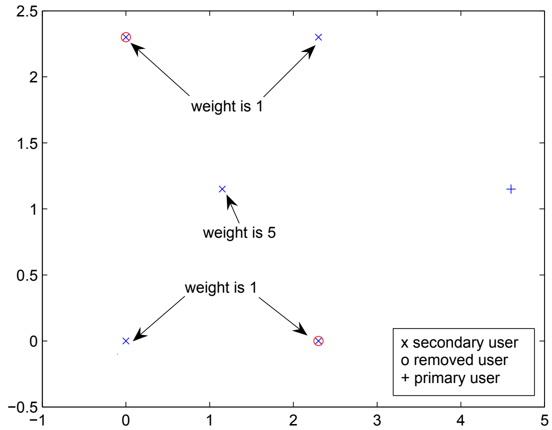

Fig. 2. Optimality of the MSRA in a particular network.

Example 2 : *Fig. 3 compares MSRA with SMART [3], SRA [9] and Game Theory Based Algorithm (GTBA) [4]. In this example, forty equal-weight secondary users are randomly generated and the primary user is assumed to be far from all the secondary users. Each point in the curve represents an averaging over 100 trials. As can be seen from Fig.3, the MSRA yields a feasible network with the maximal number of the secondary users.*

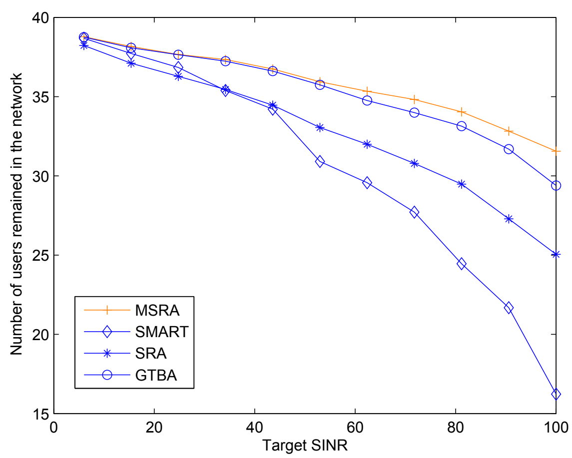

Fig. 3. Simulation comparisons between MSRA, SRA, SMART and GTBA.

Example 3 *In Fig.4, we compare the MSRA with the GTBA in a cognitive radio network scenario, where a primary user is located in the center. It can be observed from Fig.4 that the MSRA outperforms the GTBA.*

6. CONCLUSION

In this paper, we investigated the problem of maximizing the revenue of a cognitive radio network under the assumption that the QoS of the

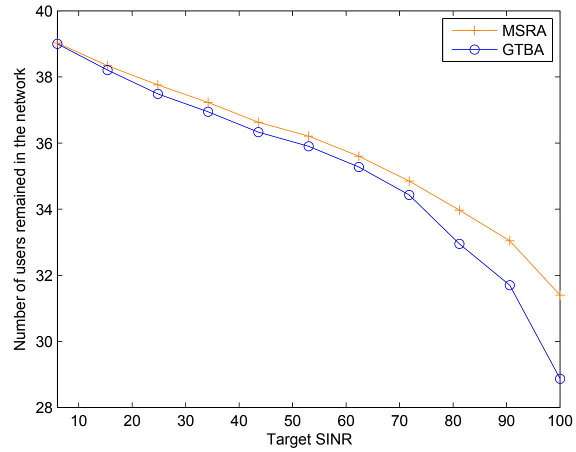

Fig. 4. Simulation comparisons between MSRA and GTBA.

primary user is guaranteed. We modeled the problem as an integer programming problem, and transformed it into a smooth optimization problem. We introduced a gradient descent based algorithm to solve this problem. Numerical simulation results show that the algorithm could effectively reach a local optimal point of this problem.

7. REFERENCES

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