MULTIUSER DETECTION IN OVERLOADED SYNCHRONOUS CDMA USING MODIFIED PROBABILISTIC DATA ASSOCIATION

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ABSTRACT

In synchronous CDMA, having more users than the signature length results in a singular correlation matrix. However, with a careful design of the correlations and with the help of the binary feature of user signals, good performance in a slightly overloaded system can still be achieved [6, 7]. However, the singularity of the correlation matrix does make the direct implementation of many multiuser detectors impossible. With the help of successive cancellation, taking inverse of a singular matrix is avoided.

Index Terms— Synchronous CDMA, probabilistic data association, multiuser detection, overloaded system, singular correlation matrix.

1. INTRODUCTION

In a CDMA system, several users simultaneously transmit information over a common channel using pre-assigned codes. The conventional single user detector consists of a bank of filters matched to the spreading codes. The Multiuser Detection (MUD) problem in CDMA communication systems has been widely studied in the past decade. Since the computation of an optimal Maximum Likelihood (ML) detector is exponential in the number of users, suboptimal solutions are proposed to provide reliable decisions with relatively low computational cost. The ML detector evaluates a log-likelihood function over the set of all possible information sequences. It achieves low error probability at the expense of high computational complexity that increases exponentially with the number of users. Therefore, this method is extremely complex for a realistic number of users. Consequently, there has been considerable research into suboptimal detectors. These detectors achieve significant performance gains over the conventional detector without the exponential increase in the receiver complexity. The well known and easily implemented suboptimal detectors are the conventional decorrelator, the Decision Feedback Detector (DFD), the multistage detector and the group detector [3]. The PDA filter [1] is a highly successful approach to tracking in the case that measurements are unlabelled and may be spurious. Its key feature is a repeated conversion of a multimodal Gaussian mixture probability structure to a single Gaussian with matched mean and covariance. Now, in the CDMA case the true probability function is also a Gaussian mixture, and complexity is also the issue [2]. This paper is organized as follows, in section 2, the traditional PDA detector in nonoverloaded CDMA system is discussed. In section 3, we discuss the PDA for MUD in overloaded synchronous CDMA, and the new modified PDA detector is introduced in section 4, section 5 contains the simulations and results, and some concluding remarks are provided at the end.

2. MULTISTAGE PDA DETECTOR IN NONOVERLOADED CDMA

A discrete-time equivalent model for the matched-filter output at the receiver of K-user synchronous CDMA channel is given by the K-length vector [3]

$$\boldsymbol{x} = \boldsymbol{S}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{z}, \quad \boldsymbol{x} = [x_1, x_2, \cdots, x_K]^T, \tag{1}$$
$$\boldsymbol{b} = [b_1, b_2, \cdots, b_K]^T,\ \boldsymbol{W} = diag\{W_1, W_2, \cdots, W_K\},$$

Where $\boldsymbol{S}$ is a $N \times K$ matrix whose k-th column, is the normalized signature of the k-th user. Since the symbol matched-filter output satisfies $\boldsymbol{y} = \boldsymbol{S}\boldsymbol{x}$, we obtain $\boldsymbol{R} = \boldsymbol{S}^T\boldsymbol{S}$, $\boldsymbol{n} = \boldsymbol{S}^T\boldsymbol{z}$, then,

$$\boldsymbol{y} = \boldsymbol{R}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{R}$ is the normalized signature correlation matrix; $\boldsymbol{W}$ is a diagonal matrix whose i-th diagonal element, w, is the square root of the received signal energy per bit of the i-th user; $b \in \{-1, 1\}^K$ denotes the vector of bits transmitted by the K active users, and $\boldsymbol{n}$ is a zero-mean Gaussian noise vector with a covariance matrix $E\{\boldsymbol{n}\boldsymbol{n}^T\} = \sigma^2 I$. When all the user signals are equally probable, the optimal solution of (2) is the output of an ML detector [3]. Assume that the chip matched-filter output is given. Then the optimal decision for system model (2) is given by

$$\begin{aligned}\hat{\boldsymbol{b}} &= \arg \min_{\boldsymbol{b} \in \{-1,1\}^K} \left(\boldsymbol{b}^T\boldsymbol{W}\boldsymbol{S}^T\boldsymbol{S}\boldsymbol{W}\boldsymbol{b} - 2\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{W}\boldsymbol{b}\right) \\ &= \arg \min_{\boldsymbol{b} \in \{-1,1\}^K} \left(\boldsymbol{b}^T\boldsymbol{W}\boldsymbol{R}\boldsymbol{W}\boldsymbol{b} - 2\boldsymbol{y}^T\boldsymbol{W}\boldsymbol{b}\right). \end{aligned} \tag{3}$$

It is known that obtaining the ML solution is generally NP-hard [3], unless the signature correlation matrix has a special structure [4, 5]. Multiplying by $\boldsymbol{W}^{-1}\boldsymbol{R}^{-1}$ on both sides of (2) from the left, the system model can be reformulated as

$$\tilde{\boldsymbol{y}} = \boldsymbol{b} + \tilde{\boldsymbol{n}} = b_k\boldsymbol{e}_k + \sum_{j \neq k} b_j\boldsymbol{e}_j + \tilde{\boldsymbol{n}}, \tag{4}$$

where $\tilde{\boldsymbol{y}} = \boldsymbol{W}^{-1}\boldsymbol{R}^{-1}\boldsymbol{y}$. The variable b_i represents the i-th element of vector $\boldsymbol{b}$, and e_i is a column vector whose i-th component is 1 and all other components are 0. We call (3) the decorrelated model, since $\tilde{\boldsymbol{y}}$ is in fact a normalized version of the decorrelator output before the hard decision.

2.1. The basic algorithm

In the CDMA system model (3), we treat the decision variables $\boldsymbol{b}$ as binary random variables. For any user i, we associate a probability $P_b(i)$ with user signal b_i to express the current belief on its value, i.e., $P_b(i)$ is the current estimate of the probability that $b_i = 1$, and $1 - P_b(i)$ is the corresponding estimates for $b_i = -1$. Based on the decorrelated model, the basic form of the multistage PDA detector is as follows.

1. $\forall i$, initialize the probabilities as $P_b(i) = 0.5$, and initialize the probabilities as $k = 1$.
2. Initialize the user counter $i = 1$.
3. Based on the current value of $P_b(j), (j \neq i)$, for user i, update $P_b(i)$ by

$$P_b(i) = P\left\{ b_i = 1 |\, \tilde{y}, \{P_b(j)\}_{j\neq i} \right\} \tag{5}$$

4. If $i < K$, let $i = i + 1$ and go to step 1.
5. If $\forall i$, $P_b(i)$ has converged, go to next step, Otherwise, let $k = k + 1$ and return to step 2.
6. $\forall i$, make a decision on user signal i via

$$b_i = \begin{cases} +1, & P_b(i) \geqslant 0.5 \\ -1, & P_b(i) < 0.5 \end{cases} \tag{6}$$

Next, the PDA detector in nonoverloaded CDMA is extended to synchronous overloaded system.

3. MODIFIED PDA DETECTOR FOR SYNCHRONOUS OVERLOADED SYSTEM

In synchronous CDMA, having more users than the signature length results in a singular correlation matrix. However, with a careful design of the correlations and with the help of the binary feature of user signals, good performance in a slightly-overloaded system can still be achieved [6, 7]. However, the singularity of the correlation matrix does make the direct implementation of many multiuser detectors impossible. Define the length of the signature sequence as N, when the system is overloaded ($K > N$), $\boldsymbol{R}$ becomes singular. The optimal solution of (3) may not be unique even when the noise is not present. This evidently results in an unavoidably high probability of error in multiuser detection. Nevertheless, with a careful design of the signature sequences and the correlations, in [6, 7], it is shown that it is possible to avoid multi solutions, and still, one achieves good performance in slightly overloaded systems. We first consider the optimal solution. Suppose the chip matched-filter is available at the receiver side, the system model of the chip matched-filer output can be represented by

$$\boldsymbol{x} = \boldsymbol{S}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{z}, \tag{7}$$

where $\boldsymbol{S}$ is a $N \times K$ matrix whose k-th column, $\boldsymbol{S}_k$ is the normalized signature of the k-th user. Since the symbol matched-filter output satisfies $\boldsymbol{y} = \boldsymbol{S}^T\boldsymbol{x}$ we obtain $\boldsymbol{R} = \boldsymbol{S}^T\boldsymbol{S}$, $\boldsymbol{n} = \boldsymbol{S}^T\boldsymbol{z}$. Assume that the chip matched-filter output is given. Then the optimal decision for system model (7) is given by (3). Furthermore, since $\forall k$, $b_k^2 = 1$, the ML detector can be equivalently written as,

$$\hat{\boldsymbol{b}} = \arg \min_{\boldsymbol{b}\in\{-1,1\}^K} \left(\boldsymbol{b}^T\boldsymbol{W}\left(\boldsymbol{R} + \boldsymbol{\Lambda}\right)\boldsymbol{W}\boldsymbol{b} - 2\boldsymbol{y}^T\boldsymbol{W}\boldsymbol{b} \right), \tag{8}$$

where $\boldsymbol{\Lambda}$ is an arbitrary diagonal matrix with positive diagonal components. Evidently, $\boldsymbol{R}+\boldsymbol{\Lambda}$ is positive definite. Therefore the branch-and-bound [3]. Next, we introduce the modified PDA detector.

4. MODIFIED PDA

The optimal algorithm can be applied with minor modifications. For PDA detector, two issues need to be addressed. The first one is the user ordering in (step 1). Since $\boldsymbol{R}^{-1}$ does not exist, the original white noise model is no longer valid. However, since PDA works with soft MAI cancellations, the performance is less sensitive to user ordering than DFD. Therefore, as a small modification, we use $\boldsymbol{R} + \sigma^2\boldsymbol{I}$ instead of $\boldsymbol{R}$ in the user ordering algorithm. Although the actual values of the elements in $\boldsymbol{R} + \sigma^2\boldsymbol{I}$ may not be reliable when σ^2 is small, the resulting user order is good enough for PDA to achieve near optimal performance. The other issue is the probability up date in $P_b(i) = P\{b_i = 1|\tilde{y}, \{P_b(j)\}_{j\neq i}\}$ (step 4). Again, since $\boldsymbol{R}$ does not exist, the decorrelator model is no longer valid. Therefore, the original PDA method must be modified to avoid taking the inverse of a singular matrix. Similar to the analysis of the ML detector, assume that the chip matched-filter outputs are available. Rewrite (7) as

$$\boldsymbol{x} = \boldsymbol{S}_k w_{kk} b_k + \sum_{j\neq k} \boldsymbol{S}_j w_{jj} b_j + \boldsymbol{z}. \tag{9}$$

Define the effective noise for user k as

$$\boldsymbol{N}_k = \sum_{j\neq k} \boldsymbol{S}_j w_{jj} b_j + \boldsymbol{z} \tag{10}$$

The mean and covariance matrix of $\boldsymbol{N}_k$ are

$$\begin{array}{rcl} E(N_k) & = & \sum_{j\neq k} \boldsymbol{S}_j w_{jj}(2P_{bj} - 1) \\ \mathrm{Cov}(N_k) & = & \sum_{j\neq k} 4P_{bj}(1 - P_{bj}) w_{jj}^2 \boldsymbol{S}_j\boldsymbol{S}_j^T + \sigma^2\boldsymbol{I}. \end{array} \tag{11}$$

Similarly, define $\boldsymbol{\theta}_k = E(N_k)$, and $\boldsymbol{\Omega}_k = \mathrm{Cov}(N_k)$. The updated probability $P_b(k)$ is given by

$$\frac{P_{b_k}}{1 - P_{b_k}} = \exp\left\{ -2\boldsymbol{\theta}_k^T \boldsymbol{\Omega}_k^{-1} \boldsymbol{S}_k w_{kk} \right\} \tag{12}$$

Now, define auxiliary variables

$$\begin{array}{rcl} \boldsymbol{\mu} & = & [w_{11}(2P_{b_1} - 1), \cdots, w_{KK}(2P_{b_K} - 1)]^T \\ \boldsymbol{\Sigma} & = & diag\left(4P_{b_1}(1 - P_{b_1})w_{11}^2, \cdots, 4P_{b_K}(1 - P_{b_K})w_{KK}^2\right) \\ \boldsymbol{\theta} & = & \boldsymbol{S}\mu - \boldsymbol{x} \\ \boldsymbol{\Omega} & = & \boldsymbol{S}\boldsymbol{\Sigma}\boldsymbol{S}^T + \sigma^2\boldsymbol{I} \end{array}$$

Define G to be the group of user such that $\forall j \in G$, $P_{b_j}(1 - P_{b_j}) \neq 0$, also assume that user $k \in G$. Define

$$G_k = G \backslash \{\text{user } k\}. \tag{13}$$

Since for any $j \ni G$, $P_{b_j}(1 - P_{b_j}) = 0$, we have

$$\boldsymbol{\Omega}_k = \boldsymbol{S}_{G_k} D_{G_k G_k} \boldsymbol{S}_{G_k}^T + \sigma^2\boldsymbol{I}, \tag{14}$$

where D is defined in [3], $\boldsymbol{S}_{G_k}$ denotes the $\boldsymbol{S}$ matrix that only contains the columns corresponding to users in G_k; and $\boldsymbol{\Sigma}_{G_kG_k}$ represents the $\boldsymbol{\Sigma}$ matrix that only contains the columns and rows corresponding to users in G_k. Using the Sherman-Morrison matrix inverse lemma, we have

$$\boldsymbol{\Omega}_k^{-1} = \frac{1}{\sigma^2}\boldsymbol{I} - \frac{1}{\sigma^4}\boldsymbol{S}_{G_k}\left(\frac{1}{\sigma^2}\boldsymbol{S}_{G_k}^T\boldsymbol{S}_{G_k} + \boldsymbol{\Sigma}_{G_kG_k}^{-1}\right)^{-1}\boldsymbol{S}_{G_k}^T \tag{15}$$

It is easy to see from $\boldsymbol{R} = \boldsymbol{S}^T\boldsymbol{S}$, and $\boldsymbol{n} = \boldsymbol{S}^T\boldsymbol{z}$ that

$$\begin{array}{rcl} \boldsymbol{S}_{G_k}^T\boldsymbol{S}_{G_k} & = & R_{G_kG_k} \\ \theta_k^T\boldsymbol{S}_k & = & r_{\{k\}G_k}^T\boldsymbol{\mu}_{G_k} - y_k \\ \theta_k^T\boldsymbol{S}_{G_k} & = & \boldsymbol{\mu}_{G_k}^T\boldsymbol{R}_{G_kG_k} - \boldsymbol{y}_{G_k}^T \\ \boldsymbol{S}_{G_k}^T\boldsymbol{S}_k & = & \boldsymbol{\mu}_{\{k\}G_k} \end{array} \tag{16}$$

G_k is the group that k-th user belongs to it, S is defined at the first of the right column of previous page. Here R_{GG} denotes the sub-block matrix of R that only contains the columns and rows corresponding to users in G, [8]. Therefore,

$$\boldsymbol{\theta}_k^T \boldsymbol{\Omega}_k^{-1} \boldsymbol{S}_k = \frac{1}{\sigma^2}\left(r_{\{k\}G_k}^T \boldsymbol{\mu}_{G_k} - \boldsymbol{y}_k\right) - \frac{1}{\sigma^4} r_{\{k\}G_k}^T \left(\frac{1}{\sigma^2}\boldsymbol{R}_{G_kG_k} + \boldsymbol{\Sigma}_{G_kG_k}^{-1}\right)^{-1} \left(\boldsymbol{R}_{G_kG_k}\boldsymbol{\mu}_{G_k} - \boldsymbol{y}_{G_k}\right) \quad (17)$$

Similar to the optimal detector case, chip matched-filter outputs do not appear in the final result. Therefore, only the symbol matched-filters are required. Furthermore, as shown in [8] for the original PDA detector, the complexity of computing (17) can also be reduced to $O(K^2)$ per user. Define,

$$\boldsymbol{\Xi}_{GG} = \frac{1}{\sigma^2}\boldsymbol{R}_{GG} + \boldsymbol{\Sigma}_{GG}^{-1} \quad (18)$$

Since user $k \in G$, we have

$$\boldsymbol{\Xi}_{GG}^{-1} = \begin{bmatrix} \boldsymbol{\Xi}_{G_kG_k} & \frac{1}{\sigma^2} r_{G_k\{k\}} \\ \frac{1}{\sigma^2} r_{G_k\{k\}}^T & \frac{1}{\sigma^2} + \boldsymbol{\Sigma}_{kk}^{-1} \end{bmatrix}^{-1} \quad (19)$$

$$= \begin{bmatrix} \left(\boldsymbol{\Xi}_{G_kG_k} - \frac{1}{\sigma^4} r_{G_k\{k\}} r_{G_k\{k\}}^T\right)^{-1} & -\boldsymbol{\Xi}_{G_kG_k}^{-1} r_{G_k\{k\}} \boldsymbol{\Delta}^{-1} \\ -\boldsymbol{\Delta}^{-1} r_{G_k\{k\}}^T \boldsymbol{\Xi}_{G_kG_k}^{-1} & \boldsymbol{\Delta}^{-1} \end{bmatrix},$$

where,

$$\boldsymbol{\Delta} = \frac{1}{\sigma^2} + \boldsymbol{\Sigma}_{kk}^{-1} - \frac{1}{\sigma^4} r_{G_k\{k\}}^T \boldsymbol{\Xi}_{G_kG_k}^{-1} r_{G_k\{k\}}$$

Evidently, if we always keep the updated version of $\boldsymbol{\Xi}_{GG}^{-1}$, (17) as well as $P_b(k)$ can be obtained with $O(K^2)$ computations, where $|G|$ denotes the number of users in G. If the updated $P_b(k)$ satisfies $P_{b_k}(1 - P_{b_k}) \neq 0$ we can update $\boldsymbol{\Xi}_{GG}^{-1}$ using Sherman-Morrison formula. We can invoke the successive cancellation idea, make decision on b_k immediately. Consequently, only $\boldsymbol{\Xi}_{GG}^{-1}$, which can also be obtained from $\boldsymbol{\Xi}_{GG}^{-1}$ in $O(K^2)$ computations, is needed in further updates. Although successive cancellation is not necessary for non-overloaded systems and is introduced to reduce the computational complexity [2], it is required for overloaded system to avoid numerical error. The modified PDA detector for overloaded system can be summarized as follows:

1. Sort users according to the user ordering criterion proposed for the decision-feedback detector in [9] (substitute $\boldsymbol{R}$ by $\boldsymbol{R} + \sigma^2\boldsymbol{I}$.
2. $\forall k$, initialize the probabilities as $P_b(k) = 0.5$, initialize G to be the set of all K users; and initialize threshold γ with a small positive number.
3. Initialize
$$\boldsymbol{\Xi}_{GG} = \frac{1}{\sigma^2}\boldsymbol{R}_{GG} + \boldsymbol{\Sigma}_{GG}^{-1} \quad (20)$$
4. Initialize $k = 1$
5. If user $k \in G$, obtain $r_{G_k\{k\}}^T \boldsymbol{\Xi}_{G_kG_k}^{-1}$ from (17)), and then obtain the updated probability $\hat{P}_b(k)$ by
$$\hat{P}_b(k) = \frac{\exp\left(-2\boldsymbol{\theta}_k^T \boldsymbol{\Omega}_k^{-1} \boldsymbol{S}_k w_{kk}\right)}{1 + \exp\left(-2\boldsymbol{\theta}_k^T \boldsymbol{\Omega}_k^{-1} \boldsymbol{S}_k w_{kk}\right)}. \quad (21)$$
If user $k \ni G$, go to step (8).
6. If $\hat{P}_{b_k}(1 - \hat{P}_{b_k}) > \gamma$, update $\boldsymbol{\Xi}_{GG}^{-1}$ by
$$\boldsymbol{\Xi}_{GG}^{-1} = \boldsymbol{\Xi}_{GG}^{-1} - \frac{\delta_k w_{kk}^2 \left[\boldsymbol{\Xi}_{GG}^{-1}\right]_k \left[\boldsymbol{\Xi}_{GG}^{-1}\right]_k^T}{1 + \delta_k w_{kk}^2 \left[\boldsymbol{\Xi}_{GG}^{-1}\right]_{kk}}, \quad (22)$$
where $\delta_k = 4\hat{P}_{bk}\left(1 - \hat{P}_{bk}\right) - 4P_{bk}\left(1 - P_{bk}\right)$. And set $P_b(k) = \hat{P}_b(k)$.
7. If $\hat{P}_{b_k}(1 - \hat{P}_{b_k}) \leqslant \gamma$, make decision on user k via
$$b_k = \begin{cases} +1, & \hat{P}_b(k) \geqslant 0.5 \\ -1, & \hat{P}_b(k) < 0.5. \end{cases} \quad (23)$$
Subtract the interference of user k from the matched-filter output by updating
$$\boldsymbol{y} = \boldsymbol{y} - r_k b_k w_{kk} \quad (24)$$
From (19), define $A = [\boldsymbol{\Xi}_{GG}^{-1}]_{G_kG_k}$, let $G = G_k$ and update
$$\boldsymbol{\Xi}_{GG}^{-1} = A - \frac{A r_{\{k\}G} r_{\{k\}G}^T A}{\sigma^4 + r_{\{k\}G}^T A r_{\{k\}G}}. \quad (25)$$
8. $k = k + 1$. If $k \leqslant K$, go to step (5). Otherwise, perform a coordinate descent search as proposed in [10], output final decisions and stop.

In the next section, we perform some simulations to validate the new PDA algorithm for MUD.

5. SIMULATIONS AND RESULTS

In this section, we compare the performances of the PDA detector, the MMSE detector, the MMSE-based DFD [9] and the optimal detector. The decorrelator and the decorrelator-based DFD are not included since they require $\boldsymbol{R}^{-1}$, which does not exist for an overloaded system. The first example is similar to example 1 in [6] but of a smaller size. Suppose we have 5 users and the signature length is 4. The first 4 users use orthogonal signature sequences generated from Walsh-Hadamard codes. The 5-th user is a TDMA user, whose signature sequence is $s_5 = [1\,0\,0\,0\,0]^T$. The user signal powers are set to 4. Noting that the user signature sequences are normalized to have unit two-norm, the correlation matrix is then given by

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}$$

Figure 1 gives the performance comparisons of different multiuser detectors. The curve labelled "iterative detector" refers to the performance of the iterative detection method proposed in [6]. We can see that the performance of PDA method is close to that of the optimal algorithm and is also significantly better than other methods. In the second example, we have 8 users with a signature length of 5. The signatures are Welch Bound Equality (WBE) sequences generated from the iterative procedure introduced in [11]. The user signal powers are again set to 4. The performance comparisons are given in Figure 2. Although PDA achieves near-optimal performance in most of the cases, in this example, the performance of the PDA detector is significantly worse than the optimal detector. However, it is still better than the MMSE detector and the MMSE-based DFD. Figure 3 demonstrates another scenario in which a different (WBE) sequence is used. We note that in all cases the modified PDA detector introduced in this paper is closer to the optimal detector.

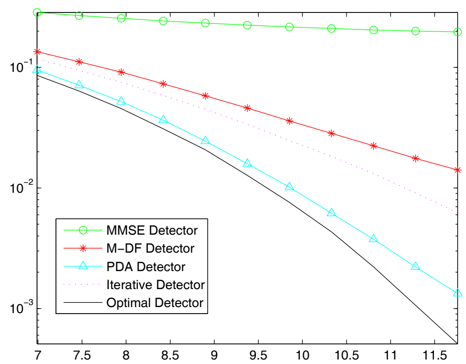

Fig. 1. Performance comparison, 5 users, spreading factor = 4.

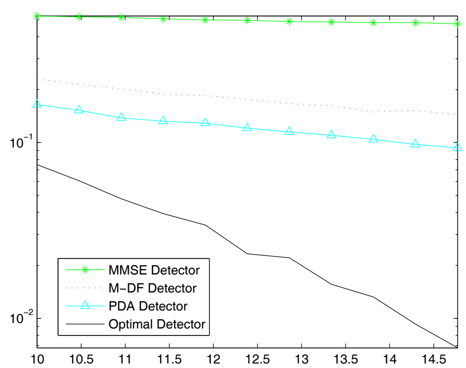

Fig. 2. Performance comparison, 8 users, length 5 WBE signature sequences.

6. CONCLUSION

Modified PDA detector has been extended to synchronous overloaded system. With the help of successive cancellation, taking inverse of a singular matrix is avoided. Simulation results show that PDA outperforms the MMSE detector and the MMSE-based DFD, the performance is also close to optimal in many situations. The performance comparisons demonstrate the superiority of the modified PDA detector.

7. REFERENCES

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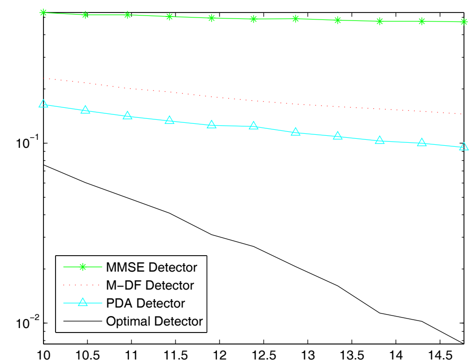

Fig. 3. Performance comparison, 7 users, length 5 WBE signature sequences.

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