JOINTLY OPTIMAL SIGNATURE SEQUENCES AND POWER ALLOCATION FOR CDMA

Duy T. Ngo, H. D. Tuan

School of Electrical Engineering and Telecommunications
University of New South Wales
Sydney, NSW 2052, AUSTRALIA
trong.ngo@student.unsw.edu.au, h.d.tuan@unsw.edu.au

Ha H. Nguyen

Department of Electrical and Computer Engineering
University of Saskatchewan
57 Campus Drive, Saskatoon, SK, CANADA S7N 5A9
ha.nguyen@usask.ca

ABSTRACT

The problems of designing signature sequences and power allocation policy for code-division multiple access (CDMA) are important and have been the subject of intensive research in recent years. Two different criteria adopted in such design problems are the user capacity and the information-theoretic capacity. Regarding the maximization of the information-theoretic capacity, most of the previous works only consider the optimizations of signature sequences and power allocation separately. In contrast, this paper presents a jointly optimal design of signature sequences and power allocation under the sum power constraint. The proposed design is of closed-form and applicable for the general case of correlated signals and colored noise. Numerical results verify the superiority of the proposed design over the existing ones.

***Index Terms*—** CDMA, information-theoretic capacity, signature sequences, power allocation, colored noise.

1. INTRODUCTION

A central problem in designing a CDMA system is how to allocate the system's resources in order to maximize the overall system capacity. One approach is to maximize the user capacity, which is defined as the maximum number of users per unit processing gain that can be admitted in the system such that each user is guaranteed its quality-of-service (QoS) requirements. The user's QoS is typically expressed as the signal-to-interference ratio at the output of the multiuser detector [1]. Another approach, which is more fundamental, is to maximize the information-theoretic sum capacity of the system [2–4]. The information-theoretic sum capacity of a CDMA system is defined as the total number of bits per channel use that can be reliably transmitted over the CDMA channel (see, e.g., [3,4]).

Two important resources in CDMA systems are the available transmission bandwidth over which all the users' signals can simultaneously occupy, and the transmitted power. The transmission bandwidth in CDMA is shared by means of spectrum spreading, where each user is assigned a unique spreading sequence of length N (also known as the processing gain). Therefore, the bandwidth (or spectrum) resource of a CDMA system is directly related to the processing gain N, and how to use this resource is equivalent to how to design the set of users' signature sequences.

The above discussion naturally leads to three resource allocation solutions: (i) Design the optimal signature sequences for some fixed power allocation scheme, (ii) Design the optimal power allocation policy for some fixed set of signature sequences, and (iii) Jointly design the optimal signature sequences and power allocation.

Regarding the design of optimal signature sequences for additive white Gaussian noise (AWGN) channels under a fixed power allocation, the study in [3] shows that if the number of users is not greater than the processing gain and each user has an average power constraint, then the sum capacity of a CDMA system is maximized by assigning orthogonal signature sequences to all the users. More significantly, [3] proves that if the number of users is greater than the processing gain and if all the users have the same average power constraint, the optimal solution is to use the so-called Welch-bound-equality (WBE) sequences. The work in [4] extends the results in [3] to the general case of unequal power constraints. In the important case where the number of users is greater than the processing gain, if one user has a relatively large power constraint than the others, and is called the "oversized" user, then this user is allocated an orthogonal signature sequence. Other users, called "non-oversized" users, are allocated the so-called generalized Welch-bound-equality (GWBE) sequences. Even more general than [4], reference [5] studies a similar problem but one that involves colored additive noise.

As for the optimal power allocation over a fading channel and under a fixed set of signature sequences, the study in [6] shows that the optimal power allocation scheme, which maximizes the ergodic capacity of a single-user system under an average power constraint, is a water-filling of powers over the inverse of the fade levels. In essence, with the assumption of channel side information availability, this scheme allocates more power to strong channel states and no power at all to channel states below some particular threshold. In the case of multiuser systems, the result obtained in [7] provides a water-filling scheme for users with respect to their fading states. In this scheme, each user transmits only when its channel state is not less than that of all the other users.

A joint design of optimal signature sequence and power allocation for CDMA systems, in the presence of fading, is recently examined in [8, 9], where an optimal signature sequence and power allocation is obtained via an iterative method. It is shown that the number of active users at any channel state cannot be greater than the processing gain, and that the active users should be allocated orthogonal signature sequences. The power allocation scheme, together with the chosen signature sequences, is basically a single-user water-filling one over sets of channel states that are favorable to each of those active users. The algorithm presented in [9] iteratively fixes the power and finds the signature sequences, then fixes the signature sequences to find the power and so on. It is also possible that the algorithm fixes the signature sequences and iteratively finds the power. Once the power allocation scheme is found, the users are then allocated orthogonal signature sequences.

This paper considers non-fading CDMA system where the channel gains of all users are fixed but unequal. The objective is to find the jointly optimal signature sequences and power allocation policy to maximize the sum capacity under the total power constraint. Although the solution assumes fixed channel gains, it might also be applied to slow-varying fading channels, where the channel gains do not vary too significantly over time, therein making the "instantaneous" total power constraints an acceptable one. Our solution is also given in a closed-form expression and hence there is no convergence issue, which is a typical concern arising from iterative methods. Another clear advantage of the closed-form solution is that it is very fast to compute. More importantly, the derived solution is valid for the very general case of correlated signals and colored noise.

Notation: We denote by the superscript T the transposition operator, $\mathbf{I}_N$ the $N \times N$ identity matrix, $\mathbf{0}_{n\times m}$ the $n \times m$ zero matrix, $E\{\cdot\}$ the expectation, $\langle\cdot\rangle$ the trace of a matrix. Given two matrices $\mathbf{A}$ and $\mathbf{B}$, diag $\begin{bmatrix}\mathbf{A} & \mathbf{B}\end{bmatrix} = \begin{bmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}\end{bmatrix}$. The notation $\mathbf{A} > 0$ means that $\mathbf{A}$ is a positive definite matrix. We denote by $\mathcal{M}_n$ the set of all orthogonal matrices of dimension $n \times n$, i.e., $\mathbf{U} \in \mathcal{M}_n$ if and only if $\mathbf{U} \in \mathcal{R}^{n\times n}$ and $\mathbf{U}\mathbf{U}^T = \mathbf{I}_n$. The set of all diagonal matrices of dimension $n \times n$ is denoted by $\mathcal{D}_n$.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider the uplink of a single-cell synchronous CDMA system with K users and processing gain N. Let s_i, p_i, and $\mathbf{c}_i$ denote the information symbol, the transmitted power, and the unit-energy *real* signature sequence of length N, respectively, of the ith user. Let h_i be the channel gain of the ith user. All the users' channel gains are assumed to be fixed but unequal. In the presence of additive Gaussian noise, the equivalent received signal vector in one symbol interval is given by [2]

$$\mathbf{r} = \sum_{i=1}^{K} h_i\sqrt{p_i}s_i\mathbf{c}_i + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is the length-N vector of additive Gaussian noise samples. For convenience, define $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K]$, $\mathbf{s} = [s_1, s_2, \ldots, s_K]$, $\mathbf{P} = \mathrm{diag}[p_1, p_2, \ldots, p_K]$, and $\mathbf{H} = \mathrm{diag}[h_1, h_2, \ldots, h_K]$. Then (1) can be written in the following form

$$\mathbf{r} = \mathbf{C}\mathbf{P}^{\frac{1}{2}}\mathbf{H}\mathbf{s} + \mathbf{n}. \tag{2}$$

Assume that the signals are correlated with $E\{\mathbf{s}\mathbf{s}^T\} = \mathbf{R}_s > 0$ and the noise is colored Gaussian with zero-mean and covariance $E\{\mathbf{n}\mathbf{n}^T\} = \mathbf{R}_n > 0$. Moreover, it is reasonable to assume that signal and noise are independent, i.e., $E\{\mathbf{s}\mathbf{n}^T\} = \mathbf{0}$. It is further assumed that channel state information is known at the transmitters (i.e. the users). Since the case $K < N$ is of neither practical nor theoretical interest, we only focus on the case $K \geq N$, i.e., the system is overloaded. This is because it is of practical interest to support more and more users for a given amount of transmission bandwidth dictated by the processing gain N.

Let $\mathbf{F} = \mathbf{C}\mathbf{P}^{\frac{1}{2}} \in \mathcal{R}^{N\times K}$, then (2) becomes

$$\mathbf{r} = \mathbf{F}\mathbf{H}\mathbf{s} + \mathbf{n}. \tag{3}$$

The design goal is to find $\mathbf{F}$ (hence $\mathbf{C}$ and $\mathbf{P}$) such that the mutual information $I(\mathbf{s};\mathbf{r})$ between the transmitted signal $\mathbf{s}$ and the received signal $\mathbf{r}$ is maximized, subject to the total power constraint $P = p_1 + p_2 +, \ldots, +p_K$. It is well-known that when $\mathbf{F}$ and $\mathbf{H}$ are fixed, the maximization of $I(\mathbf{s};\mathbf{r})$ is achieved when the joint distribution of all the input variables is Gaussian. Furthermore, with the jointly Gaussian distribution of the input signals, it is not difficult to show that the maximum mutual information, also known as the information-theoretic sum capacity, can be expressed as

$$C_{\mathrm{sum}}(\mathbf{F}, \mathbf{R}_s, \mathbf{R}_n) = \frac{1}{2}\log\frac{\det[(\mathbf{F}\mathbf{H})\mathbf{R}_s(\mathbf{F}\mathbf{H})^T + \mathbf{R}_n]}{\det[\mathbf{R}_n]}. \tag{4}$$

Now, let $\mathbf{x} = \mathbf{H}\mathbf{s} \in \mathcal{R}^K$. Then $\mathbf{R}_x = E\{\mathbf{x}\mathbf{x}^T\} = \mathbf{H}\mathbf{R}_s\mathbf{H}^T$. Since $\langle\mathbf{F}\mathbf{F}^T\rangle = \langle\mathbf{P}\rangle = P$, the design problem is formulated as follows:

$$\max_{\mathbf{F}} \log\det(\mathbf{I}_N + \mathbf{F}\mathbf{R}_x\mathbf{F}^T\mathbf{R}_n^{-1}) \quad \text{s.t. } \langle\mathbf{F}\mathbf{F}^T\rangle \leq P. \tag{5}$$

The above matrix optimization problem is far more complex than the seemingly similar problems of optimal design of precoder, which appear in the literature (see, e.g., [10]). In the precoder design situation, either noise $\mathbf{n}$ is assumed to be white, or power is constrained as $\langle\mathbf{F}\mathbf{R}_x\mathbf{F}^T\rangle \leq P$. In such cases, by changing the variable $\mathbf{X} = \mathbf{F}^T\mathbf{F}$ or $\mathbf{X} = \mathbf{F}\mathbf{R}_x\mathbf{F}^T$, and applying some commutative properties of trace and determinant operations, the following optimization problem is formulated: $\max_{\mathbf{X}} \log\det(\mathbf{I}_N + \mathbf{X}\mathbf{Q})$, s.t. $\langle\mathbf{X}\rangle \leq \mathbf{P}$. The Hadamard-inequality-based approach can be used to find the solution for this problem. However, for the optimization problem of interest in (5), any such legal permutation does not simplify the problem, leaving both the objective and constraint of (5) highly nonlinear in the variable $\mathbf{F}$. In the next section, the approach of variational inequalities and matrix partition [11] is employed to derive the closed-form solution to Problem (5).

3. JOINTLY OPTIMAL SIGNATURE SEQUENCES AND POWER ALLOCATION

This section derives the closed-form solution for the jointly optimal signature sequences and power allocation scheme. We begin by making the following singular value decompositions (SVDs):

$$\mathbf{R}_x = \mathbf{U}_x^T\mathbf{\Sigma}_K\mathbf{U}_x \quad \text{and} \quad \mathbf{R}_n = \mathbf{U}_n^T\mathbf{\Sigma}_N\mathbf{U}_n, \tag{6}$$

with orthogonal matrices $\mathbf{U}_x, \mathbf{U}_n$ and diagonal matrices $\mathbf{\Sigma}_K, \mathbf{\Sigma}_N$. Also, $\{\mathbf{\Sigma}_K(i,i)\}$ are arranged in decreasing order while $\{\mathbf{\Sigma}_N(i,i)\}$ are in increasing order. By making the variable change

$$\mathbf{F} \leftarrow \mathbf{U}_n\mathbf{F}\mathbf{U}_x^T, \tag{7}$$

the problem in (5) reduces to

$$\max_{\mathbf{F}} \log\det(\mathbf{I}_N + \mathbf{F}\mathbf{\Sigma}_K\mathbf{F}^T\mathbf{\Sigma}_N^{-1}) \quad \text{s.t. } \langle\mathbf{F}\mathbf{F}^T\rangle \leq P. \tag{8}$$

Moreover, a further variable change

$$\mathbf{F} \leftarrow \mathbf{F}\mathbf{\Sigma}_K^{1/2} \tag{9}$$

leads to

$$\max_{\mathbf{F}} \log\det(\mathbf{I}_N + \mathbf{F}\mathbf{F}^T\mathbf{\Sigma}_N^{-1}) \quad \text{s.t. } \langle\mathbf{\Sigma}_K^{-1}\mathbf{F}^T\mathbf{F}\rangle \leq P. \tag{10}$$

Observe that

$$\begin{aligned}\log\det(\mathbf{I}_N + \mathbf{F}\mathbf{F}^T\mathbf{\Sigma}_N^{-1}) &= \\ -\log\det(\mathbf{\Sigma}_N) &+ \log\det(\mathbf{\Sigma}_N + \mathbf{F}\mathbf{F}^T).\end{aligned} \tag{11}$$

Thus, by ignoring the constant term $\{-\log\det(\mathbf{\Sigma}_N)\}$, Problem (10) becomes

$$\max_{\mathbf{F}} \log\det(\mathbf{\Sigma}_N + \mathbf{F}\mathbf{F}^T) \quad \text{s.t. } \langle\mathbf{\Sigma}_K^{-1}\mathbf{F}^T\mathbf{F}\rangle \leq P. \tag{12}$$

Since $K \geq N$, it is known from [12, Th.7.3.5, p.414] that the SVD

$$\mathbf{F} = \mathbf{U}_N\begin{bmatrix}\sqrt{\mathbf{D}_N} & \mathbf{0}_{N\times(K-N)}\end{bmatrix}\mathbf{V}_K \tag{13}$$

is possible, where $\mathbf{U}_N \in \mathcal{M}_N$, $\mathbf{V}_K \in \mathcal{M}_K$ and $\mathbf{D}_N \in \mathcal{D}_N$. It follows that

$$\mathbf{F}\mathbf{F}^T = \mathbf{U}_N\mathbf{D}_N\mathbf{U}_N^T \text{ and } \mathbf{F}^T\mathbf{F} = \mathbf{V}_K^T\,\mathrm{diag}\begin{bmatrix}\mathbf{D}_N & \mathbf{0}_{K-N}\end{bmatrix}\mathbf{V}_K.$$

Therefore, Problem (12) is rewritten as

$$\max_{\mathbf{U}_N \in \mathcal{M}_N, \mathbf{V}_K \in \mathcal{M}_K, \mathbf{D}_N \in \mathcal{D}_N} \log\det(\mathbf{\Sigma}_N + \mathbf{U}_N \mathbf{D}_N \mathbf{U}_N^T) \quad (14)$$
$$\text{s.t.} \quad \langle \mathbf{\Sigma}_K^{-1} \mathbf{V}_K^T \text{diag}\begin{bmatrix} \mathbf{D}_N & \mathbf{0}_{K-N} \end{bmatrix} \mathbf{V}_K \rangle \leq P.$$

For the optimal solution $(\mathbf{U}_N, \mathbf{V}_K, \mathbf{D}_N)$, it must be true that

$$\langle \mathbf{\Sigma}_K^{-1} \mathbf{V}_K^T \text{diag}\begin{bmatrix} \mathbf{D}_N & \mathbf{0}_{K-N} \end{bmatrix} \mathbf{V}_K \rangle = \min_{\mathbf{V} \in \mathcal{M}_K} \langle \mathbf{\Sigma}_K^{-1} \mathbf{V}^T \text{diag}\begin{bmatrix} \mathbf{D}_N & \mathbf{0}_{K-N} \end{bmatrix} \mathbf{V} \rangle. \quad (15)$$

Proof: Suppose that (15) does not hold. Then there exists $\lambda > 1$ and $\mathbf{V} \in \mathcal{M}_K$ such that

$$\langle \mathbf{\Sigma}_K^{-1} \mathbf{V}^T \text{diag}\begin{bmatrix} \lambda\mathbf{D}_N & \mathbf{0}_{K-N} \end{bmatrix} \mathbf{V} \rangle \leq P.$$

The above implies that

$$\log\det(\mathbf{\Sigma}_N + \mathbf{U}_N \lambda \mathbf{D}_N \mathbf{U}_N^T) > \log\det(\mathbf{\Sigma}_N + \mathbf{U}_N \mathbf{D}_N \mathbf{U}_N^T),$$

which contradicts the optimality of $\mathbf{D}_N$. □

Now, it follows from [11, Proposition 1] that

$$\min_{\mathbf{V} \in \mathcal{M}_K} \langle \mathbf{\Sigma}_K^{-1} \mathbf{V}^T \text{diag}\begin{bmatrix} \mathbf{D}_N & \mathbf{0}_{K-N} \end{bmatrix} \mathbf{V} \rangle = \sum_{i=1}^{N} \mathbf{\Sigma}_K^{-1}(i,i) \mathbf{D}_N(i,i), \quad (16)$$

where $\{\mathbf{\Sigma}_K^{-1}(i,i)\}$ are already in increasing order (since $\{\mathbf{\Sigma}_K(i,i)\}$ are in decreasing order) while $\{\mathbf{D}_N(i,i)\}$ are to be arranged in decreasing order.

From (16), the problem in (14) can be simplified to

$$\max_{\mathbf{U}_N \in \mathcal{M}_N, \mathbf{D}_N \in \mathcal{D}_N} \log\det(\mathbf{\Sigma}_N + \mathbf{U}_N \mathbf{D}_N \mathbf{U}_N^T) \quad (17)$$
$$\text{s.t.} \quad \langle \mathbf{\Sigma}_K^{-1}(1:N) \mathbf{D}_N \rangle \leq P.$$

By changing the variable $\mathbf{X} \leftarrow \mathbf{U}_N \mathbf{D}_N \mathbf{U}_N^T$, Problem (17) becomes

$$\max_{\mathbf{U}_N \in \mathcal{M}_N} \log\det(\mathbf{\Sigma}_N + \mathbf{X}) \quad (18)$$
$$\text{s.t.} \quad \mathbf{X} \geq 0, \langle \mathbf{\Sigma}_K^{-1}(1:N) \mathbf{U}_N^T \mathbf{X} \mathbf{U}_N \rangle \leq P.$$

Lemma 1 *The Problems (17) and (18) are equivalent.*

Proof: Any feasible $\mathbf{U}_N, \mathbf{D}_N$ of (17) will result in the feasible $\mathbf{X} = \mathbf{U}_N \mathbf{D}_N \mathbf{U}_N^T$ of (18), so $\max$ of (17) $\leq \max$ of (18). On the other hand, at optimality, $\mathbf{U}_N^T \mathbf{X} \mathbf{U}_N$ of (18) must be diagonal since $\langle \mathbf{\Sigma}_K^{-1}(1:N) \mathbf{U}_N^T \mathbf{X} \mathbf{U}_N \rangle$ must attain the minimum in $\mathbf{U}_N$ when $\mathbf{X}$ is fixed. This results in the feasible $\mathbf{D}_N$ of (17). It follows that $\max$ of (18) $\leq \max$ of (17). □

Next, by the variable change $\mathbf{X} \leftarrow \mathbf{U}_N^T \mathbf{X} \mathbf{U}_N$, Problem (18) is

$$\max_{\mathbf{U}_N \in \mathcal{M}_N} \log\det(\mathbf{U}_N^T \mathbf{\Sigma}_N \mathbf{U}_N + \mathbf{X}) \quad (19)$$
$$\text{s.t.} \quad \mathbf{X} \geq 0, \langle \mathbf{\Sigma}_K^{-1}(1:N) \mathbf{X} \rangle \leq P.$$

For now, relax the constraint $\mathbf{X} \geq 0$ by $\mathbf{X}(i,i) \geq 0$. Later, when the optimal solution of the relaxed problem is shown to be diagonal, then the original and relaxed problems are equivalent. Employing the Lagrangian multiplier, it can be shown that

$$\mathbf{U}_N^T \mathbf{\Sigma}_N \mathbf{U}_N + \mathbf{X} = \mathbf{D}_x, \text{ where } \mathbf{D}_x \text{ is diagonal.} \quad (20)$$

Thus, the relaxed problem is actually

$$\max_{\mathbf{U}_N \in \mathcal{M}_N} \log\det(\mathbf{D}_x) \quad (21)$$
$$\text{s.t.} \quad \mathbf{D}_x \geq 0, \langle \mathbf{\Sigma}_K^{-1}(1:N)(\mathbf{D}_x - \mathbf{U}_N^T \mathbf{\Sigma}_N \mathbf{U}_N) \rangle \leq P.$$

Now, at the optimality of (21), it must be true that

$$-\langle \mathbf{\Sigma}_K^{-1}(1:N) \mathbf{U}_N^T \mathbf{\Sigma}_N \mathbf{U}_N \rangle \quad (22)$$

attains its minimum, therefore, $\mathbf{U}_N = \mathbf{I}_N$.

Consequently, $\mathbf{X}$ in (20), (19) and (18) is diagonal. The optimal solution $\mathbf{D}_N$ is also diagonal and has the water-filling structure:

$$\mathbf{D}_N = \mathbf{X} = \text{diag}[(\mu^{-1}\mathbf{\Sigma}_K(i,i) - \mathbf{\Sigma}_N(i,i))^+]_{i=1,2,\ldots,N}, \quad (23)$$

where $x^+ = \max(x, 0)$ and μ is chosen such that

$$P = \sum_{i=1}^{N} \mathbf{\Sigma}_K^{-1}(i,i) \mathbf{D}_N(i,i) = \sum_{i=1}^{N} \left(\mu^{-1} - \frac{\mathbf{\Sigma}_N(i,i)}{\mathbf{\Sigma}_K(i,i)} \right)^+. \quad (24)$$

As $\mathbf{D}_N(i,i)$ in (23) are already in increasing order, the orthogonal $\mathbf{V}_K$ in (15) is simply $\mathbf{I}_K$. Hence it follows from (7) and (9) that the optimal solution $\mathbf{F}$ of (5) is

$$\mathbf{F} = \mathbf{F}_{\text{opt}} = \mathbf{U}_n^T \begin{bmatrix} \mathbf{D}_N^{1/2} & \mathbf{0}_{N \times (K-N)} \end{bmatrix} \mathbf{\Sigma}_K^{-1/2} \mathbf{U}_x, \quad (25)$$

and the maximum value of the sum capacity computed as in (5) is

$$C_{\text{opt}} = \frac{1}{2} \sum_{i=1}^{N} \log\left(1 + \mathbf{D}_N(i,i) \mathbf{\Sigma}_N^{-1}(i,i)\right). \quad (26)$$

4. NUMERICAL EXAMPLES

This section provides numerical results to confirm the merit and superiority of our solution. Consider a CDMA system with processing gain $N = 16$ and $K = 24$ users. The total transmitted power is constrained to $P = K = 24$. Since most of the previous works consider white input signals, here we assume $E\{\mathbf{s}\mathbf{s}^T\} = \mathbf{R}_s = \mathbf{I}_K$ for a fair comparison with other solutions. Although the channel gains are assumed to be fixed and unequal in the computation of the sum capacity, to have meaningful interpretation of the results, we randomly generate 100 sets of independent channel gains using Rayleigh distribution. The sum capacity is then obtained for each set of channel gains, and the results are then averaged for plotting.

First, we compare the sum capacity attained by our jointly optimal solution with that achieved by designing the optimal signature sequences for a fixed set of transmitted powers. Specifically, we assign a power of 1 to each of 12 users, a power of 0.5 to each of the other 6 users while each of the remaining 6 users has a power of 1.5. These power assignments constitute a total transmitted power of $P = 24$. Once the power assignments are fixed, the optimal signature sequences are the GWBE sequences. Reference [4, Sec.4] describes how to construct such signature sequences for the case of white Gaussian noise, whereas [5, Sec.II-C & Sec.II-F] details the construction for the case of colored Gaussian noise.

Second, we compare the sum capacity provided by our method with that obtained by using the optimal power allocation (also under the sum power constraint of $P = 24$) for a fixed set of signature sequences. Specifically, we select the WBE sequences as the fixed signature sequences, while the power allocation scheme is the modified version of the water-filling algorithm derived in [13, Sec.2, Alg.1]. It should be noted that the original iterative algorithm in [13] is to provide the power allocation policy to maximize the sum capacity of the Gaussian vector multiple access channel (VMAC) under the sum power constraint. Nonetheless, it can be readily modified to find the optimal power allocation to maximize the sum capacity of a CDMA system with fixed signature sequences and subject to a total power constraint.

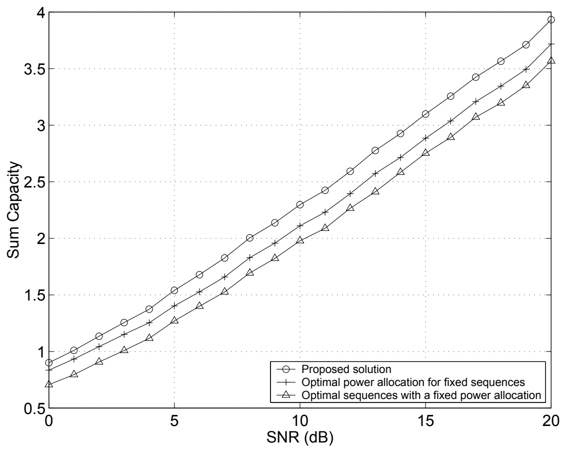

Fig. 1. Sum capacity results under white Gaussian noise.

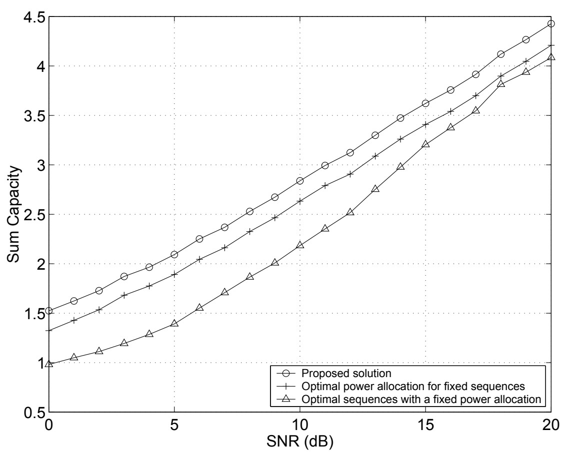

Fig. 2. Sum capacity results under colored Gaussian noise.

The comparisons of the capacity results are shown in Fig. 1 for the case of white noise, and Fig. 2 for the case of colored noise. Here the colored noise is obtained by filtering the $N \times 1$ white Gaussian noise vector $\mathbf{w} \sim \mathcal{N}(0, \sigma^2 \mathbf{I}_N)$ with an $N \times N$ matrix $\mathbf{G}$. If $\mathbf{G}$ is such that its rows have unit norms and not orthogonal, then the average total noise power in unchanged, but the covariance of $\mathbf{n} = \mathbf{G}\mathbf{w}$ becomes $\mathbf{R}_n = \sigma^2 \mathbf{G}\mathbf{G}^T > 0$. In our computations of capacity, the elements of $\mathbf{G}$ are randomly generated according to a zero-mean Gaussian distribution of unit variance. The rows of $\mathbf{G}$ are then normalized to have unit norms.

It can be clearly seen from both figures that our jointly optimal solution always yields the largest sum capacity. This observation confirms the superior performance of the proposed solution in comparison to the other approaches, namely "to find the optimal signature sequences for a fixed power allocation" and "to find the optimal power allocation for fixed signature sequences". The results in both Figures 1 and 2 also clearly show that optimizing the power allocation (with fixed signature sequences) is more effective than optimizing the signature sequences (with fixed power allocation) in both cases of additive noise.

5. CONCLUSIONS

The jointly optimal solution of signature sequences and power allocation was derived to maximize the sum capacity of a CDMA system under the total power constraint. Numerical results were provided to illustrate the performance and superiority of the derived solution over the existing ones. Our closed-form solution is applicable to a very general case of correlated signals and colored Gaussian noise.

6. REFERENCES

[1] P. Viswanath, V. Anantharam, and D. Tse, "Optimal sequences, power control, and user capacity of synchronous CDMA systems with linear MMSE multiuser receivers," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1968–1983, 1999.

[2] S. Verdu, "Capacity region of gaussian CDMA channels: the symbol-synchronous case," in *Proc. Allerton Conference on Commmunication, Control and Computing*, 1986, pp. 1025–1034.

[3] M. Rupf and J. Massey, "Optimal sequence multisets for synchronous code-division multiple access channels," *IEEE Trans. Inf. Theory*, vol. 40, pp. 1261–1266, 1994.

[4] P. Viswanath and V. Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1984–1991, 1999.

[5] P. Viswanath and V. Anantharam, "Optimal sequences for CDMA under colored noise: a Schur-saddle function property," *IEEE Trans. Inf. Theory*, vol. 48, pp. 1295–1318, 2002.

[6] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, pp. 1986–1992, 1994.

[7] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE ICC*, 1995.

[8] O. Kaya and S. Ulukus, "Jointly optimal power and signature sequence allocation for fading CDMA," in *Proc. IEEE GLOBECOM*, 2003, pp. 1872–1876.

[9] O. Kaya and S. Ulukus, "Ergodic sum capacity maximization for CDMA: Optimum resource allocation," *IEEE Trans. Inf. Theory*, vol. 51, pp. 1831–1836, 2005.

[10] A. Scaglione, P. Stoica, S. Barbarossa, G.B. Giannakis, and H. Sampath, "Optimal design for space-time linear precoder and decoders," *IEEE Trans. Signal Process.*, vol. 50, pp. 1051–1064, 2002.

[11] D.H. Pham, H.D. Tuan, Ba-Ngu Vo, and T.Q. Nguyen, "Jointly optimal precoding/postcoding for colored MIMO systems," in *Proc. IEEE ICASSP*, 2006.

[12] R.A. Horn and C.R. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.

[13] W. Yu, "A dual decomposition approach to the sum power Gaussian vector multiple access channel sum capacity problem," in *Proc. 37th Annual Conference on Information Sciences and Systems*, 2003.