HIGH SNR PERFORMANCE ANALYSIS OF BLIND MINIMUM OUTPUT ENERGY RECEIVERS IN LARGE DS-CDMA SYSTEMS

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ABSTRACT

High signal-to-noise ratio (SNR) performance of the blind minimum output energy (MOE) receiver and the Capon channel estimation technique are analyzed in the large code division multiple access (CDMA) network where both the spreading factor and number of users go to infinity with the same rate. Upper and lower bounds on the signal-to-interference-plus-noise ratio (SINR), efficiency and asymptotic efficiency of the MOE receiver are derived and compared with those for the optimum minimum mean squared error (MMSE) receiver.

***Index Terms*—** Asymptotic performance analysis, blind minimum output energy receiver, Capon channel estimation technique, code division multiple access.

1. INTRODUCTION

Blind multiuser receivers for code division multiple access (CDMA) systems have gained a considerable attention due to their spectral efficiency and robustness. Among various blind multiuser receiver design approaches, the minimum output energy (MOE) approach constitutes a promising trend [1]-[3]. In this approach, the receiver output energy in minimized subject to a set of constraints which guarantee that the energy of the user of interest at the receiver output is held constant. In this paper, we consider the large CDMA system in which the number of users go to infinity, while the system load, i.e., the ratio of the number of users to the spreading factor, remains constant and analyze the high signal-to-noise ratio (SNR) performance of the blind MOE receiver operating in such system. Note that, in [4] we have analyzed the MOE receiver in the large CDMA system where our main focus was on the study of the channel effects on the performance of the MOE receiver relative to that of the MMSE receiver.

To be able to use the results from the reach literature of random matrix theory, it is a common convention in large CDMA system analysis to model the user spreading codes as random vectors [5]. We follow the same convention, and, further, to simplify the derivations to a representable level, we confine ourselves to the case when the spreading vectors are drawn from i.i.d. circular Gaussian distribution. Note that the Gaussian spreading vectors have been frequently used in the literature to analyze the asymptotic performance of the MMSE receiver [6]-[8]. Moreover, simulations show that our results are still valid for more general i.i.d. distributions.

We show that if the system load is less than one, then, as the noise power converges to zero, the signal-to-interference-plus-noise ratio (SINR) of the blind MOE receiver goes to infinity irrespective to the interference powers. This near-far resistance property is quite interesting due to the fact that the multipath channel information of the user of interest is not available to the MOE receiver. For the case when the system load is larger than one, we obtain lower and upper bounds on the SINR of the MOE receiver that are independent from the realizations of the spreading codes and are determined by the system load, the constraints set used, and the user channels. We also obtain bounds on some other important performance measures of the MOE receiver such as efficiency and asymptotic efficiency and compare our results with the known results for the MMSE receiver.

The MOE approach can also be used to estimate the user channel vector. It has been shown that [1] if the set of constraints are chosen to maximize the so-obtained minimum output energy, then, under a certain identifiability condition, the constraint vector approaches the channel vector of the user of interest at high SNRs. We apply our analytical results to show that, in the large system of our concern, the channel identifiability condition holds as long as the system load is less than one.

2. SIGNAL MODEL

Consider a K-user synchronous DS-CDMA system with periodic spreading codes. The received baseband signal can be modelled as [1], [3]

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=1}^{K} b_k(m) w_k(t - mT_s) + v(t). \tag{1}$$

where T_s is the symbol period, $b_k(m)$ and $w_k(t)$ denote the mth unit-variance data symbol and the signature waveform of the kth user, respectively, and $v(t)$ is the zero-mean additive white random noise process with the variance σ^2. Let $h_k(t)$ denote the multipath channel impulse response of the kth user that is fixed during the observation period and its support is in the interval $[0, LT_c)$ where L is a positive integer and T_c is the chip period [1], [7]. Let $N \gg L$ be the spreading factor. In this paper, we assume that the users spreading vectors $\mathbf{c}_k = \left[c_k[0], c_k[1], \ldots, c_k[N-1]\right]^T$ are augmented at the transmitter by the cyclic prefixes of length $L-1$ to eliminate the effect of inter-symbol-interference induced by the multipath channel. Sampling (1) in the interval corresponding to the nth transmitted symbols and removing the cyclic prefixes, the received sampled data vector is given by [1], [7]

$$\mathbf{x}(n) = \sum_{k=1}^{K} b_k(n) \mathbf{w}_k + \mathbf{v}(n). \tag{2}$$

As $w_k(t)$ is the convolution of the spreading code $\mathbf{c}_k$ with the multipath channel $h_k(t)$, its sampled version $\mathbf{w}_k$ can be written as $\mathbf{w}_k =$

$\mathbf{C}_k\mathbf{h}_k$ [1] where the $N\times L$ matrix $\mathbf{C}_k$ consists of the first L columns of an $N\times N$ circulant matrix whose first column is $\mathbf{c}_k$ and $\mathbf{h}_k = [h_{k,0}, h_{k,1}, \ldots, h_{k,L-1}]^T$ with $h_{k,i} \triangleq h_k(iT_c)$. Throughout the manuscript, we assume that $\mathbf{h}_k$ has a bounded norm [7]. Denoting $h_k(z) = \sum_{q=0}^{L-1} h_{k,q} z^{-q}$, it directly follows from the latter assumption that $|h_k(e^{j\frac{2\pi}{N}(n-1)})|^2 \leq B$ where B is a constant and $n = 1, \ldots, N$. Furthermore, let us assume without any loss of generality that the first user is the user of interest and $\mathbf{h}_1$ (and, hence, $\mathbf{w}_1$) are unknown to the receiver. The linear blind MOE approach to design the receiver vector $\mathbf{f}$ is to minimize the output energy $\mathbf{f}^H\mathrm{E}\{\mathbf{x}(n)\mathbf{x}(n)^H\}\mathbf{f} \triangleq \mathbf{f}^H\mathbf{R}\mathbf{f}$ subject to some constraints which ensure that the energy of the first user at the receiver output is held constant. A popular set of such constraints is $\mathbf{C}_1^H\mathbf{f} = \bar{\mathbf{h}}$ where $\bar{\mathbf{h}}$ is the design parameter vector [1], [2]. The MOE receiver resulting from the latter constraints is given by [1], [2]

$$\mathbf{f}_{\mathrm{m}}(\bar{\mathbf{h}}) = \mathbf{R}^{-1}\mathbf{C}_1(\mathbf{C}_1^H\mathbf{R}^{-1}\mathbf{C}_1)^{-1}\bar{\mathbf{h}}. \tag{3}$$

Note that the above constraints guarantee that for any arbitrary $\bar{\mathbf{h}}$, the energy component of the first user at the receiver output is $\mathcal{E}_1 = |\mathbf{f}_{\mathrm{m}}(\bar{\mathbf{h}})^H\mathbf{w}_1|^2 = |\bar{\mathbf{h}}^H\mathbf{h}_1|^2$. Using the idea of Capon estimation technique to maximize the energy of the user of interest after interference suppression at the receiver output, one can select $\bar{\mathbf{h}}$ as [1]

$$\mathbf{h}_{\mathrm{c}} = \arg\max_{\bar{\mathbf{h}}} \mathbf{f}_{\mathrm{m}}(\bar{\mathbf{h}})^H\mathbf{R}\mathbf{f}_{\mathrm{m}}(\bar{\mathbf{h}}) \qquad \text{s.t.} \qquad \|\bar{\mathbf{h}}\| = 1. \tag{4}$$

Using (3) in (4), the solution to the latter problem is [1]

$$\mathbf{h}_{\mathrm{c}} = \Omega\left((\mathbf{C}_1^H\mathbf{R}^{-1}\mathbf{C}_1)^{-1}\right). \tag{5}$$

It can be obtained from (2) that

$$\mathbf{R} = \sum_{k=1}^{K}\mathbf{w}_k\mathbf{w}_k^H + \sigma^2\mathbf{I} = \sum_{k=1}^{K}\mathbf{C}_k\mathbf{h}_k\mathbf{h}_k^H\mathbf{C}_k^H + \sigma^2\mathbf{I}. \tag{6}$$

From (6) and the matrix inversion lemma, we have

$$(\mathbf{C}_1^H\mathbf{R}^{-1}\mathbf{C}_1)^{-1} = (\mathbf{C}_1^H\mathbf{A}^{-1}\mathbf{C}_1)^{-1} + \mathbf{h}_1\mathbf{h}_1^H \tag{7}$$

where $\mathbf{A} \triangleq \mathbf{R} - \mathbf{w}_1\mathbf{w}_1^H$. Applying (7) to (5), we obtain

$$\mathbf{h}_{\mathrm{c}} = \Omega\left((\mathbf{C}_1^H\mathbf{A}^{-1}\mathbf{C}_1)^{-1} + \mathbf{h}_1\mathbf{h}_1^H\right). \tag{8}$$

Moreover, it can be shown that [9]

$$\mathrm{SINR}(\mathbf{f}_{\mathrm{m}}(\bar{\mathbf{h}})) = \frac{|\bar{\mathbf{h}}^H\mathbf{h}_1|^2}{\bar{\mathbf{h}}^H(\mathbf{C}_1^H\mathbf{A}^{-1}\mathbf{C}_1)^{-1}\bar{\mathbf{h}}}. \tag{9}$$

3. ASYMPTOTIC ANALYSIS

In this section, we analyze the asymptotic properties of the MOE receiver (3) and the Capon channel estimate (5) as N and K go to infinity with the same rate and σ^2 tends to zero. As it can be observed from (8) and (9), these asymptotic properties are closely related to the asymptotic behavior of $\mathbf{C}_1^H\mathbf{A}^{-1}\mathbf{C}_1$. The following theorem holds.

Theorem 1: Assume that the entries of $\mathbf{c}_k$ are zero-mean i.i.d. circular Gaussian random variables with variance $\frac{1}{N}$ and that N and K go to $+\infty$ with $\frac{K}{N} \to \alpha$. Then,

$$\left(\mathbf{C}_1^H\mathbf{A}^{-1}\mathbf{C}_1\right)^{-1} - \mathbf{T}^{-1} \xrightarrow{e.i.p.} \mathbf{0} \tag{10}$$

where $\xrightarrow{e.i.p.}$ denotes elementwise convergence in probability and $\mathbf{T}$ is a positive definite Hermitian Toeplitz matrix with

$$[\mathbf{T}]_{1m} = \frac{1}{N}\sum_{l=1}^{N}\frac{e^{-j\frac{2\pi}{N}(m-1)(l-1)}}{\phi_l}, \qquad m = 1, \ldots, L \tag{11}$$

where $[\mathbf{T}]_{1m}$ is the mth entry of the first row of $\mathbf{T}$, and ϕ_l is the unique real and positive solution to

$$\phi_l = \sigma^2 + \frac{1}{N}\sum_{k=2}^{K}\frac{\left|h_k(e^{j\frac{2\pi}{N}(l-1)})\right|^2}{1 + \frac{1}{N}\sum_{n=1}^{N}\frac{\left|h_k(e^{j\frac{2\pi}{N}(n-1)})\right|^2}{\phi_n}}. \tag{12}$$

Moreover, assuming that for all k and n there exists a constant $b > 0$ such that

$$|h_k(e^{j\frac{2\pi}{N}(n-1)})|^2 \geq b \tag{13}$$

it holds for $\alpha < 1$ that

$$\lim_{\sigma^2\to 0}\sigma^2\mathbf{T} - \mathbf{\Omega} \quad \xrightarrow{e.i.p.} \quad \mathbf{0} \tag{14}$$

$$\lim_{\sigma^2\to 0}\mathbf{T}^{-1} \quad \xrightarrow{e.i.p.} \quad \mathbf{0} \tag{15}$$

and for $\alpha > 1$ that

$$\lim_{\sigma^2\to 0}\mathbf{T} - \mathbf{\Theta} \xrightarrow{e.i.p.} \mathbf{0}. \tag{16}$$

Matrices $\mathbf{\Omega}$ and $\mathbf{\Theta}$ are positive definite and

$$\lambda_{\min}(\mathbf{\Omega}) \geq \frac{b(1-\alpha)}{B} \tag{17}$$

$$\lambda_{\max}(\mathbf{\Omega}) \leq \min\left\{1, \frac{B(1-\alpha)}{b}\right\} \tag{18}$$

$$\lambda_{\min}(\mathbf{\Theta}) \geq \frac{b}{B^2(\alpha-1)} \tag{19}$$

$$\lambda_{\max}(\mathbf{\Theta}) \leq \frac{B}{b^2(\alpha-1)} \tag{20}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ stand for the smallest and the largest eigenvalues of a matrix, respectively.

Proof of Theorem 1 will be presented in [9]. Note that (13) is a reasonable assumption due to fact that the sporadic (k,n)s for which $|h_k(e^{j\frac{2\pi}{N}(n-1)})|^2 = 0$ do not have any impact on the value of ϕ_l and, hence, they can be discarded from (12). The following results can be directly obtained from Theorem 1.

Corollary 1: Assume that $\alpha < 1$. Then, as $\sigma^2 \to 0$,

$$\mathbf{h}_{\mathrm{c}} \xrightarrow{e.i.p.} e^{j\theta}\frac{\mathbf{h}_1}{\|\mathbf{h}_1\|} \tag{21}$$

where $\theta \in [0, 2\pi)$. Moreover, for any arbitrary $\bar{\mathbf{h}}$ such that $\bar{\mathbf{h}}^H\mathbf{h}_1 \neq 0$, $\mathrm{SINR}(\mathbf{f}_{\mathrm{m}}(\bar{\mathbf{h}}))$ converges to infinity.

Note that (21) directly follows from using (15) in (8). Convergence (15) can also be used in (9) to verify that $\mathrm{SINR}(\mathbf{f}_{\mathrm{m}}(\bar{\mathbf{h}})) \to +\infty$. The above properties of the Capon channel estimate (5) and the MOE receiver (3) worth further elaboration as follows.

- It has been shown in [1] that if

$$[\mathbf{C}_1 \;\; \mathbf{W}_1] \;\; \text{is a full column-rank matrix} \tag{22}$$

where $\mathbf{W}_1 = [\mathbf{w}_2, \ldots, \mathbf{w}_K]$, then, as $\sigma^2 \to 0$, $\mathbf{h}_c$ converges to a scaled version of $\mathbf{h}_1$. Corollary 1 proves that as $N \to \infty$ with $\frac{K}{N} \to \alpha$, the identifiability condition (22) can be substituted by the much simpler condition of $\alpha < 1$. That is certainly a desirable property since verification of (22) is a prohibitively difficult task especially when N and K are large.

- As $\lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\bar{\mathbf{h}})) = +\infty$ for $\alpha < 1$, it follows that in the absence of noise the blind MOE receiver (3) is able to completely suppress the effect of the multiuser interference regardless of the interferer powers. It is an interesting result which shows that even without knowing the signature of the user of interest $\mathbf{w}_1$ or having an estimate of the channel vector $\mathbf{h}_1$, the receiver (3) still has the near-far resistance property. This property is due to the fact that $\mathbf{w}_1$ is equal to $\mathbf{C}_1\mathbf{h}_1$ where $\mathbf{C}_1$ is a known matrix. The MOE receiver (3) effectively uses this known structure of $\mathbf{w}_1$ to suppress the interfering signals having the signatures $\mathbf{w}_i = \mathbf{C}_i\mathbf{h}_i$ for $i = 2, \ldots, K$.

Note also that if the actual channel order is $\tilde{L} < L$, i.e., $\mathbf{h}_1 = [\tilde{\mathbf{h}}_1^T \;\; \mathbf{0}^T]^T$ where $\tilde{\mathbf{h}}_1$ is a vector of length $\tilde{L}$, then, still (15) can be used in (8) to obtain (21). The latter observation shows that if $\alpha < 1$, then the Capon channel estimation technique is insensitive to the channel order overestimation in the high SNR regime.

It is known that if the number of active users exceeds the signature length, i.e., $\alpha > 1$, then, even in the absence of noise, the SINR of the MMSE receiver $\mathbf{f}_{\mathrm{mmse}} = \mathbf{R}^{-1}\mathbf{w}_1$ (that uses the knowledge of the channel and achieves the maximum SINR) does not converge to infinity [5]. It is intuitive that the similar property should also hold for the MOE receiver (3). The following corollary verifies such a property by finding an upper bound on $\lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\bar{\mathbf{h}}))$.

Corollary 2: Assuming that $\alpha > 1$ we have

$$\frac{b|\bar{\mathbf{h}}^H\mathbf{h}_1|^2}{B^2(\alpha-1)\|\bar{\mathbf{h}}\|^2} \le \lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\bar{\mathbf{h}})) \le \frac{B|\bar{\mathbf{h}}^H\mathbf{h}_1|^2}{b^2(\alpha-1)\|\bar{\mathbf{h}}\|^2}. \tag{23}$$

Moreover, if $b = B$ we have

$$\lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\bar{\mathbf{h}})) = \frac{|\bar{\mathbf{h}}^H\mathbf{h}_1|^2}{B(\alpha-1)\|\bar{\mathbf{h}}\|^2} \tag{24}$$

and

$$\lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\mathbf{h}_c)) = \frac{1}{\alpha-1}. \tag{25}$$

Inequalities (23) are directly obtained from using (19) and (20) in (9). Moreover, if $b = B$, or, equivalently, $\left|h_k(e^{j\frac{2\pi}{N}(l-1)})\right|$ is constant for all $k = 1, \ldots, K$ and $l = 1, \ldots, N$, then (24) follows from the equality of the upper and lower bounds in (23). Note that $\left|h_k(e^{j\frac{2\pi}{N}(l-1)})\right|$ does not change with k and l if the received users powers are equal and the channels are single path, that is, $\mathbf{h}_k = h_{k,q_k}\mathbf{e}_{q_k}$ where $q_k \in \{0, 1, \ldots, L-1\}$, $\mathbf{e}_{q_k}$ is the vector whose q_kth entry is one and the rest are zero, and $|h_{k,q_k}|^2 = B$. It can be shown in such case that $\mathbf{h}_c$ converges to a scaled version of $\mathbf{h}_1 = h_{1,q_1}\mathbf{e}_{q_1}$ [9]. Using the latter result in (24), equation (25) follows. Note that if $N \to \infty$ with $\frac{K}{N} \to \alpha$, and, moreover, if the received users powers are equal and the channels are single-path, then the SINR of the MMSE receiver also converges to $\frac{1}{\alpha-1}$ at high SNRs [5].

It should be mentioned that if B is much larger than b, then the derived bounds in (23) may become loose. However, for instance, in the downlink transmission scheme where the channel tap corresponding to the line-of-sight is much larger than the other taps, it can be shown that B and b are close, and, hence, (23) offers tight bounds on the asymptotic value of $\lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\bar{\mathbf{h}}))$.

We can also use Theorem 1 to analyze other performance measures of the MOE receiver such as efficiency and asymptotic efficiency. Note that the efficiency of the receiver vector $\mathbf{f}$, denoted here by $\mathrm{C}(\mathbf{f})$, is the ratio of the achieved SINR to the SINR when there is no interference [5]. Moreover, the asymptotic efficiency of the receiver vector $\mathbf{f}$ is $\eta(\mathbf{f}) = \lim_{\sigma^2 \to 0} \mathrm{C}(\mathbf{f})$. We have

Corollary 3: As $N \to \infty$ with $\frac{K}{N} \to \alpha$, the efficiency of the general blind MOE receiver (3) satisfies

$$\mathrm{C}(\mathbf{f}_m(\bar{\mathbf{h}})) - \frac{\sigma^2\|\bar{\mathbf{h}}\|^2}{\bar{\mathbf{h}}^H\mathbf{T}^{-1}\bar{\mathbf{h}}} \xrightarrow{i.p.} 0 \tag{26}$$

and is bounded by

$$\frac{\sigma^2}{\sigma^2 + \alpha B} \le \mathrm{C}(\mathbf{f}_m(\bar{\mathbf{h}})) \le 1. \tag{27}$$

Moreover, it holds that

$$\frac{b(1-\alpha)}{B} \le \eta(\mathbf{f}_m(\bar{\mathbf{h}})) \le \min\left\{1, \frac{B(1-\alpha)}{b}\right\}, \qquad \alpha < 1 \tag{28}$$

and

$$\eta(\mathbf{f}_m(\bar{\mathbf{h}})) = 0 \qquad \alpha > 1. \tag{29}$$

From (27) it follows that if either α or B tends to zero, $\mathrm{C}(\mathbf{f}_m(\bar{\mathbf{h}}))$ converges to unity. It is an expected fact since either of the above cases implies that the effect of interference is negligible. Moreover, it follows from (28) that if $\alpha < 1$ and $b = B$, then $\eta(\mathbf{f}_m(\bar{\mathbf{h}})) = 1 - \alpha$ which, under the similar conditions, is equal to the asymptotic efficiency of the MMSE receiver [5]. Finally, similar to the MMSE receiver [5], $\eta(\mathbf{f}_m(\bar{\mathbf{h}})) = 0$ for $\alpha > 1$.

4. SIMULATIONS

Numerical examples have been carried out to validate our analytical results. In Fig. 1, 200 sets of users spreading codes have been randomly generated from $\pm\frac{1}{\sqrt{N}}$ for $N = 128$ and the resulting experimental downlink $\mathrm{SINR}(\mathbf{f}_m(\mathbf{e}_1))$ versus the system load $\alpha > 1$ is shown for all sets of spreading codes. A random vector of length $L = 5$ is drawn from a zero-mean complex Gaussian distribution with the covariance matrix $\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & \mathbf{0} \\ \mathbf{0} & \gamma_2\mathbf{I}_4 \end{bmatrix}$ and is used as the channel vector between the base station and the user of interest. To simulate the line-of-sight at the first channel tap, in the left and the right subplots $\gamma_1/\gamma_2 = 10$ dB and 30 dB are chosen, respectively. Moreover, the upper, the middle, and the lower subplots correspond to the scenarios in which $\mathrm{tr}(\mathbf{\Gamma})/\sigma^2$ is equal to 20, 40, and $+\infty$ dB, respectively (for the latter case, $\sigma^2 = 0$ is used). In each subplot, the lower and upper bounds of (23) are also depicted. It can be observed from the subplots corresponding to $\gamma_1/\gamma_2 = 10$ dB that if the line-of-sight component is not strong enough, due to the fact that B is much larger than b, the lower and upper bounds of (23) are loose. On the contrary, when $\gamma_1/\gamma_2 = 30$ dB, the resulting lower and upper bounds are very tight and are able to predict $\lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\mathbf{e}_1))$ with a very good precision. It can also be noticed from the figure that the lower and upper bounds of (23) are in fact relevant for the high SNR regime, i.e., when $\mathrm{tr}(\mathbf{\Gamma})/\sigma^2 \to +\infty$. Finally, as the lower subplots show, if $\alpha > 1$, then $\lim_{\sigma^2 \to 0} \mathrm{SINR}(\mathbf{f}_m(\mathbf{e}_1))$ is upper-bounded, verifying our discussion in Section 3.

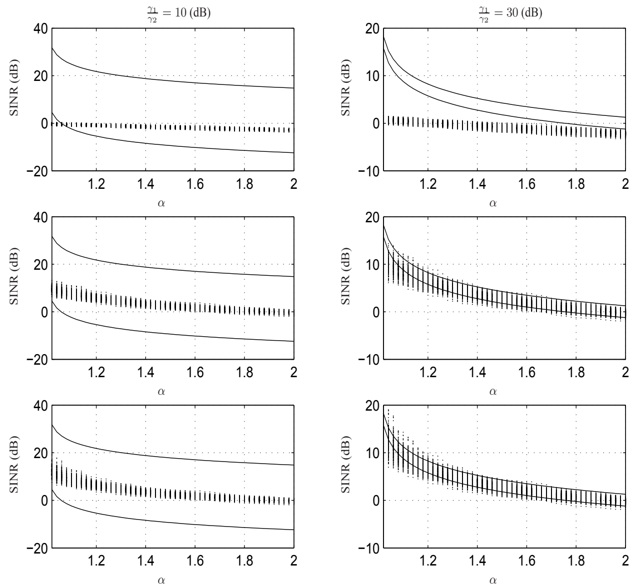

Fig. 1. $\mathrm{SINR}(\mathbf{f}_{\mathrm{m}}(\mathbf{e}_1))$ versus α for $\mathrm{tr}(\mathbf{\Gamma})/\sigma^2 = 20$ dB (upper subplots), 40 dB (middle subplots), and $+\infty$ (lower subplots).

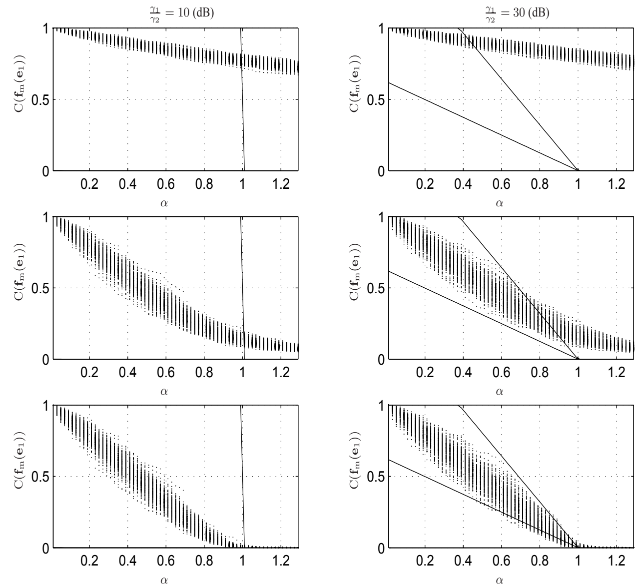

Fig. 2. $\mathrm{C}(\mathbf{f}_{\mathrm{m}}(\mathbf{e}_1))$ versus α for $\mathrm{tr}(\mathbf{\Gamma})/\sigma^2 = 20$ dB (upper subplots), 40 dB (middle subplots), and 60 dB (lower subplots).

Figure 2 shows downlink $\mathrm{C}(\mathbf{f}_{\mathrm{m}}(\mathbf{e}_1))$ versus α. The user signatures and the channel vectors are drawn using the same distributions as in Fig. 1. Again, in the left and the right subplots γ_1/γ_2 is equal to 10 and 30 dB, respectively. Moreover, the upper, the middle, and the lower subplots correspond to the scenarios in which $\mathrm{tr}(\mathbf{\Gamma})/\sigma^2$ is equal to 20, 40, and 60 dB, respectively. The lower and upper bounds of $\eta(\mathbf{f}_{\mathrm{m}}(\mathbf{e}_1)) = \lim_{\sigma^2 \to 0} \mathrm{C}(\mathbf{f}_{\mathrm{m}}(\mathbf{e}_1))$ given in (28) are also shown. It can be observed from Fig. 2 that as $\mathrm{tr}(\mathbf{\Gamma})/\sigma^2$ increases, these bounds become more relevant. Moreover, the derived lower and upper bounds of (28) are specially useful for the case of strong line-of-sight which, in this example, is realized by choosing $\gamma_1/\gamma_2 = 30$ dB. It can be observed from the lower subplots that, as SNR increases, $\mathrm{C}(\mathbf{f}_{\mathrm{m}}(\mathbf{e}_1))$ converges to zero for $\alpha > 1$. The latter observation verifies (29).

5. CONCLUSIONS

In this paper, we have analyzed the performances of the blind MOE receiver and the Capon channel estimation technique in the case when the noise power tends to zero and both the spreading factor and the number of users go to infinity with the same rate. We showed that if the system load, i.e., the ratio of the number of users to the spreading factor, is less than one, then, for any arbitrary interference powers the SINR of the blind MOE receiver goes to infinity as the noise power converges to zero. For the case when the system load is larger than one, we derived lower and upper bounds on the SINR of the MOE receiver. We also obtained the bounds on efficiency and asymptotic efficiency of the MOE receiver and analyzed particular cases when the so-obtained upper and lower bounds become equal.

It was also shown that if the system load is less than one, then the Capon channel estimate converges to a scaled version of the channel vector of the user of interest as the noise power approaches to zero. The latter result shows that the channel identifiability condition derived for the conventional (bounded number of users) scenario reduces to a much simpler condition as the number of users goes to infinity.

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