JOINT ITERATIVE DEMODULATION AND DECODING OF DIFFERENTIAL FREQUENCY HOPPING SIGNALS

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ABSTRACT

In this paper, joint iterative demodulation and decoding of differential frequency hopping (DFH) signals based on the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm is considered. The DFH is a nonlinear modulation with memory in the frequency sequence of the successive transmitted symbols, which can be represented by a trellis diagram. The proposed receiving scheme is primarily composed of an *a posteriori* probability (APP) decoder/filter and an APP demodulator, by which the extrinsic information on the coded bits extracted from the output of the APP filter can be utilized again by the demodulator as updated *a priori* information. Performance over AWGN and time-varying Rayleigh flat fading channels is investigated by Monte Carlo simulation. By comparison between the Symbol-by-Symbol maximum *a posteriori* probability (SBS-MAP) detection with soft decision Viterbi decoding and the proposed method, it can be shown that considerable improvement can be achieved for both static and time-varying fading channels.

Index Terms— Iterative processing, differential frequency hopping, MAP estimation, Rayleigh fading channels

1. INTRODUCTION

Differential frequency hopping (DFH) technique, proposed in [1], is identified as a spread spectrum modulation with memory in the frequency sequence of the transmitted signals. It has proved effective in the applications in which lower transmitted power is more desirable than economical use of bandwidth, such as military communications [2] and powerline communications [3]. Recent study, [3, 4, 5], has been focused on the design of DFH demodulator based on a trellis diagram. Both maximum likelihood sequence estimation (MLSE) by the Viterbi algorithm (VA) and symbol-by-symbol maximum *a posteriori* probability (SBS-MAP) detection using the BCJR algorithm are considered. By SBS-MAP detection, the demodulator in [3] and [5] can produce soft decision output for coded bits and hence outperforms the system employing MLSE detection. The demodulator, however, can not utilize the information exploited by the decoder for further improvement of the DFH system.

Iterative processing technique has been successfully applied to the decoding of serial concatenated codes (SCC) [6] and the joint demodulation and decoding of signaling techniques with recursive nature, such as DPSK and CPM signals, and trellis codes [7], [8], and [9]. In this paper, the iterative processing technique is applied to a new type of nonlinear modulation with memory, i.e. DFH signals, in which the concatenation of an *a posteriori* probability (APP) decoder/filter, defined in [8], and an APP demodulator by random interleaving is employed. The fundamental principle is that by the extraction of extrinsic information on the coded bits from the output of the APP filter, which is obtained by both of the input and the constraint of the code structure, the result can be used as refined *a priori* information of the coded bits and fed back to the demodulator to improve the performance of the demodulation. The performance of the proposed scheme is investigate by Monte Carlo simulation, in which both AWGN and time-varying Rayleigh flat fading channels are considered. A rate-$1/2$ convolutional code with constraint length 3 and a random interleaver are used in the concatenation of the channel encoder and the DFH modulator. Comparison between DFH signals with SBS-MAP detection and iterative processing is performed based on the simulation result.

2. SYSTEM MODEL

The DFH modulator can be modeled as a first-order Markov source, with the frequencies allowed to be used as the possible states. The number of branches starting from and ending in each state of the trellis is determined by a parameter of DFH, i.e. fanout, denoted by $\mathscr{F}$. The relation between $\mathscr{F}$ and the number of bits mapped onto each transmitted symbol is $\mathscr{F}_o = 2^{\nu}$, where ν is the number of bits transmitted per hop. Hence, the characterization of DFH signaling is similar to that of a binary nonsystematic convolutional code, with the exception that the associated transmitted symbol with each branch can be an M-ary symbol and is determined by the ending state of that branch. Throughout the rest of this paper, it is assumed that the size of the set of available frequencies is M.

The DFH signals can be modeled as M-ary FSK signals with noncoherent demodulation [1]. An equivalent baseband receiver is illustrated in Fig. 1.

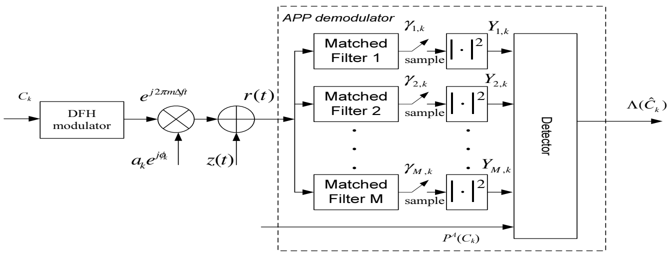

Fig. 1. Equivalent baseband DFH receiver with an APP demodulator.

Given a frame of N successive transmitted symbols, assuming that the mth symbol was transmitted, the complex-valued sampled output of the matched filter is

$$r_{i,k} = \sqrt{E_s} a_k e^{j\phi_k} \delta(i,m) + z_{i,k}, \tag{1}$$

where $i, m \in \{1, 2, \ldots, M\}$. a_k is a Rayleigh random variable. ϕ_k is a uniformly-distributed random variable ranging within $[0, 2\pi]$. $z_{i,k}$ is complex-valued Gaussian random variables. When transmitted through AWGN channels, a_k is a constant equal to one. $\delta(i,m)$ is the Kronecker delta function. According to [3], $Y_{1,k}, Y_{2,k}, \ldots, Y_{M,k}$ are statistically independent random variables. Furthermore, as for the cases of $i = m$ and $i \neq m$, $Y_{i,k}$'s are respectively non-central chi-square-dis-tributed random variable and central chi-square-distributed random variable with two degrees of freedom, whose conditional probability density functions (PDF's) are

$$\begin{aligned} & p(Y_{i,k} = y_{i,k} | X_{m,k} = x_{m,k}) \\ & = \begin{cases} \frac{1}{2\sigma^2} \exp\left(-\frac{a_k^2 E_s + y_{m,k}}{2\sigma^2}\right) I_0\left(\frac{a_k\sqrt{E_s y_{m,k}}}{\sigma^2}\right), i = m; \\ \frac{1}{2\sigma^2} \exp\left(-\frac{y_{i,k}}{2\sigma^2}\right) \qquad\qquad\qquad\quad , i \neq m, \end{cases} \end{aligned} \tag{2}$$

where $X_{m,k}$ is the transmitted symbol at time k, and $I_0(\bullet)$ is the zeroth-order modified Bessel function of the first kind.

3. APP DEMODULATION OF DFH SIGNALS WITH MODIFIED BCJR ALGORITHM

In this section, we assume that the signal has been coded before entering the modulator. When applied to the APP demodulation of DFH signals, the branch metric at time k, denoted by $\gamma_k(s', s)$, can be expressed as

$$\begin{aligned} \gamma_k(s', s) = \sum_{X_k} & P(S_k = s | S_{k-1} = s') \cdot \\ & P(X_k | S_k = s, S_{k-1} = s') \prod_{i=1}^{M} P(Y_{i,k} | X_k), \end{aligned} \tag{3}$$

where S_k and X_k are the state and transmitted symbol at time k, respectively. $\overrightarrow{Y}_k = \{Y_{1,k}, Y_{2,k}, \ldots, Y_{M,k}\}^T$ is the input vector of the detector at time k as shown in Fig. 1. The first term of (3) is the joint *a priori* probability of coded bits input to the modulator at one time. By sufficient interleaving, it can be written as the product of marginal *a priori* probabilities,

$$P(S_k = s | S_{k-1} = s') = \prod_{p=1}^{\nu} P^A(c_k^p), \tag{4}$$

where ν is the number of bits modulated in each symbol interval. This term will be updated by the feedback from the APP filter in iterative processing. As for the second term of (3), there is a zero-one relationship, such as (5), between a transmitted symbol and a branch, which in turn represents a state transition on the trellis.

$$P(X_k | S_k = s, S_{k-1} = s') = \begin{cases} 1, & \text{if } X_k \text{ transmitted;} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

Finally, the third term is the conditional probability of receiving Y_k, which is a symbol corrupted by additive Gaussian noise and fading, assuming X_k was transmitted, as in (2). By substituting (2), (4), and (5) into (3), the branch metric can be calculated by

$$\begin{aligned} \gamma_k(s', s) = & \frac{\prod_{p=1}^{\nu} P^A(c_k^p)}{2^M \sigma^{2M}} \cdot \\ & \exp\left(-\frac{a_k^2 E_s + \sum_{i=1}^{M} y_{i,k}}{2\sigma^2}\right) I_0\left(\frac{a_k\sqrt{E_s y_{m,k}}}{\sigma^2}\right). \end{aligned} \tag{6}$$

The log-likelihood ratio (LLR) of the output of the APP demodulator can be expressed as

$$\Lambda_{c_k^j} = \Lambda_{c_k^j}^a + \ln \frac{\sum\limits_{\{s',s\} \in C_k^{j,1}} \alpha_{k-1}(s') A_k(s', s) \beta_k(s)}{\sum\limits_{\{s',s\} \in C_k^{j,0}} \alpha_{k-1}(s') A_k(s', s) \beta_k(s)}, \tag{7}$$

where $\Lambda_{c_k^j}$ and $\Lambda_{c_k^j}^a$ are respectively the *a posteriori* and *a priori* LLR's of the jth bit of the group of coded bits input to the modulator at time k. $\alpha_k(s)$ and $\beta_k(s)$ are forward and backward state metrics, and

$$A_k(s', s) = \prod_{\substack{p=1 \\ p \neq j}}^{\nu} P^A(c_k^p) \prod_{i=1}^{M} P(Y_{i,k} | X_k). \tag{8}$$

By decomposing the output of the APP demodulator as (7), the APP demodulator is suitable for iterative processing, as shown in the next section.

4. JOINT ITERATIVE DEMODULATION AND DECODING OF DFH SIGNALS

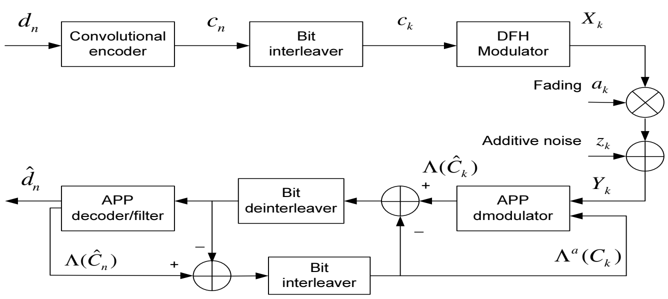

Fig. 2. Block diagram of the DFH system with joint iterative demodulation and decoding.

We shall use the following notation for expositional convenience. $\Lambda^{(q)}_{\textbf{dec}}(\hat{d}_n)$ is the *a posteriori* LLR of an information bit, i.e. the output on an information bit by the APP decoder. $\Lambda^{a(q)}_{\textbf{dec}}(d_n)$ denotes the *a priori* LLR of an information bit, i.e. the input on an information bit of the APP decoder. $\Lambda^{(q)}_{\textbf{fil}}(\hat{c}_n)$ represents the *a posteriori* LLR on a coded bit by the APP filter. $\Lambda^{i(q)}_{\textbf{fil}}(c_n)$ is the input LLR on a coded bit of APP decoder/filter. $\Lambda^{(q)}_{\textbf{dem}}(\hat{c}_k)$ is *a posteriori* LLR on a coded bit by APP demodulator. Finally, $\Lambda^{a(q)}_{\textbf{dem}}(c_k)$ denotes *a priori* LLR of a coded bit, i.e. the input LLR on a coded bit of the APP demodulator. The superscript q represents the qth iteration.

As illustrated in Fig. 2, in a DFH system employing iterative processing, the channel encoder and modulator is serially concatenated at the transmitting end, which is due to the following facts. A basic principle of the iterative processing is that the the information exploited by a certain stage in the previous iteration must not be feedback to itself as the input of the current iteration. To satisfy this condition, it must be guaranteed that the input to the current stage is separable from the output of that stage as in (7). Therefore, a bit interleaver is employed to connect the channel encoder and the DFH modulator and to make the input to both the APP demodulator and APP decoder as uncorrelated as possible. At the receiving end, the APP demodulator calculates the $\Lambda^{(q)}_{\textbf{dem}}(\hat{c}_k)$ for each coded bit and by subtracting the *a priori* information $\Lambda^{a(q)}_{\textbf{dem}}(c_k)$ from the *a posteriori* information as in (9), the result will be used by the APP decoder/filter after deinterleaving.

$$\Lambda^{i(q)}_{\textbf{fil}}(c_k) = \Lambda^{(q)}_{\textbf{dem}}(\hat{c}_k) - \Lambda^{a(q)}_{\textbf{dem}}(c_k) = \Lambda^{(q)}_{c_k} - \Lambda^{a(q)}_{c_k}, \quad (9)$$

where $\Lambda^{i(q)}_{\textbf{fil}}(c_k)$ is to be deinterleaved. Since the second iteration, the *a priori* information $\Lambda^{a(q)}_{c_k}$ of the coded bits will be updated by the extrinsic information extracted from output of the APP filter in the last iteration, i.e. $\Lambda^{(q-1)}_{\textbf{fil}}(\hat{c}_n)$, after interleaving. The APP /decoder filter operates in a similar way as the APP demodulator with the difference lying in that the *a priori* LLR of the information bits, $\Lambda^{a(q)}_{\textbf{dec}}(d_n)$, are always set to zero. By subtracting $\Lambda^{i(q)}_{\textbf{fil}}(c_n)$ from the *a posteriori* LLR calculated by the APP decoder/filter, the extrinsic information $\Lambda^{e(q)}_{c_n}$ on each coded bit can be computed by (10) and used as the *a priori* information of the next iteration after interleaving.

$$\Lambda^{a(q+1)}_{c_n} = \Lambda^{e(q)}_{c_n} = \Lambda^{(q)}_{\textbf{fil}}(\hat{c}_n) - \Lambda^{i(q)}_{\textbf{fil}}(c_n). \quad (10)$$

In the last iteration, the APP decoder will calculate the *a posteriori* LLR $\Lambda^{(q)}_{\textbf{dec}}(\hat{d}_n)$ and make hard decision on the information bits accordingly.

5. SIMULATION RESULT

The bit error rate (BER) performance of DFH system with iterative processing under both AWGN and rayleigh flat fading channels are investigated using Monte Carlo simulation. A rate-$1/2$ convolutional code with generator $[7, 3]$ is used as the channel coding scheme. A random interleaver is employed. The case of $\nu = 1$ is considered. The APP demodulation and decoding/filtering are performed in a frame-by-frame manner. The state of the channel encoder is forced to return to zero, whereas the DFH modulator is left open. Noncoherent demodulation is used in the APP demodulator because of the difficulty in the acquisition of the phase of the received fast frequency hopping signals such as DFH.

Results of AWGN channel are shown in Fig. 3. It has been concluded in [5] that the performance of SBS-MAP detection with soft-decision Viterbi decoding of convolutional coded DFH signals is better than that of MLSE detection of DFH signals with the same channel coding. Therefore, we only draw the comparison between the iterative processing and the SBS-MAP detection with soft-decision Viterbi decoding of DFH signals. Because the input *a priori* information on the information bits of the APP decoder is always zero, based on which the MAP criterion and ML criterion are equivalent to each other, the performance of the first iteration is similar to that of the system without iterative processing. Since the second iteration, the performance of the APP demodulator is improved by using the extrinsic information, which in turn augments the signal-to-noise ratio (SNR) for the decoder. Therefore, better BER performance can be achieved as shown in Fig. 3. It can be observed that about 2.5dB of gain is achieved by the joint demodulation and decoding at the BER of 10^{-3} after 5 iterations relative to the SBS-MAP detection with soft-decision decoding. In addition, the BER performance begins to converge after the 5th iteration.

Results over Rayleigh flat fading channel are shown in Fig. 4. A fading rate of $0.05R$, i.e. $f_{Dmax}T_s = 0.05$, has been selected. The comparison between SBS-MAP detection with soft-decision Viterbi decoding and the iterative processing is made again. The similarity between the performance of SBS-MAP detection with soft-decision Viterbi decoding

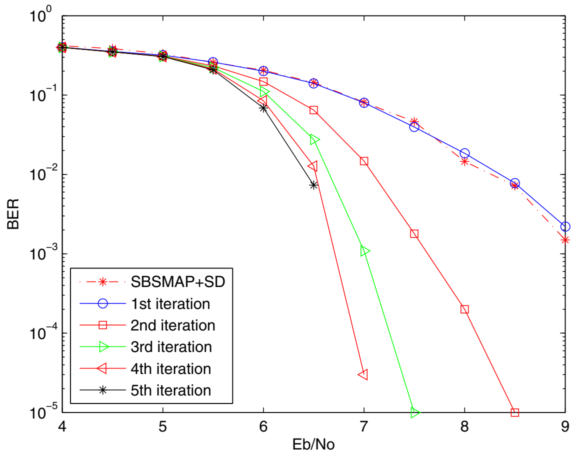

Fig. 3. BER performance of joint iterative demodulation and decoding of DFH signals over AWGN channel, compared with the SBS-MAP detection with soft-decision Viterbi decoding of DFH signals.

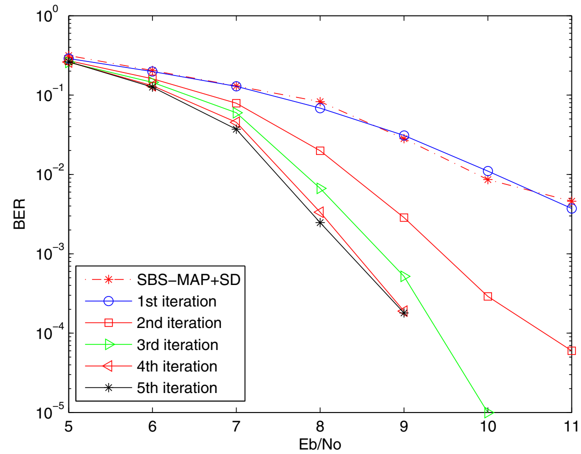

Fig. 4. BER performance of joint iterative demodulation and decoding of DFH signals over Rayleigh flat fading channel with fading rate $0.05R$, compared with the SBS-MAP detection with soft-decision Viterbi decoding of DFH signals.

and the first iteration of the iterative processing is due to the same reason as explained in the case of AWGN channels. At the BER of 10^{-3}, we can achieve 3dB of gain after 5 iterations relative to the other method simulated. The performance tends to converge after the 5th iteration.

6. CONCLUSION

A DFH system with joint iterative demodulation and decoding is proposed. The performance is investigated under AWGN and Rayleigh flat fading channels. It has been shown that the DFH modulator can be represented by a trellis diagram, making the iterative processing an effective solution to exploit the system performance as much as possible. Simulation results show that by joint iterative demodulation and decoding, the BER performance can be improved considerably over the systems without employing iterative processing. Future work would be further investigation on the system performance over frequency selective fading channels, a typical channel model as a platform to test the communication algorithm for powerline communications.

7. REFERENCES

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