THE GENERALIZED LINEAR DECOMPOSITION OF MULTILEVEL CPM SIGNALS

W. Van Thillo[1,2], *J. Nsenga*[1,2], *F. Horlin*[1], *A. Bourdoux*[1], *R. Lauwereins*[1,2]

[1]Interuniversity Micro Electronics Center IMEC
Wireless Research
Kapeldreef 75
B-3001 Leuven, Belgium

[2]Dept. Electrotechnical Eng. ESAT
Katholieke Universiteit Leuven
Kasteelpark Arenberg 10
B-3001 Leuven, Belgium

ABSTRACT

Multilevel Continuous Phase Modulated (CPM) signals feature a perfectly constant envelope, attractive spectral properties and excellent power efficiency. However, their nonlinear nature makes them less tractable and their processing more complex. Fortunately, a linear decomposition exists, allowing to apply linear signal processing techniques.

This decomposition was originally developed for *binary* CPM schemes only, and is not suited for schemes with an *integer* modulation index. It was extended to multilevel CPM schemes, by decomposing the multilevel input sequence in a product of binary subsequences, and applying the binary decomposition to these subsequences. When one or more of these subsequences has an integer modulation index though, this technique fails.

We present a general solution, and prove how many pulses are needed to represent the CPM signal in this particular case. The decomposition of the quaternary $3RC$ system with $h = \frac{1}{2}$ is given as an example. A receiver based on this solution is presented.

Index Terms— Continuous Phase Modulation (CPM), Pulse Amplitude Modulation (PAM) decomposition, integer modulation index, multilevel CPM, linear representation

1. INTRODUCTION

CPM signals feature a perfectly constant envelope and very attractive spectral properties due to their continuous phase [1]. Moreover, excellent power efficiency can be obtained by moving from bilevel to multilevel signaling as this increases the minimum distance of the CPM scheme. The GSM and Bluetooth standards use Gaussian Minimum Shift Keying (GMSK), which is a member of the CPM family. CPM is also an attractive candidate for 60 GHz short range, high data rate communication because it lowers the constraints on the analog front end, which is critical [2].

The nonlinear nature of CPM signals though, makes them less tractable and their processing more complex. Fortunately, Laurent [3] devised a linear representation for *binary* CPM schemes with *noninteger* modulation index, which we will call the binary Laurent Decomposition (2-LD). It is commonly used to simplify receivers [4]. Mengali and Morelli [5] extended his work to *multilevel* schemes, which we will denote M-LD. This extension has been used for reduced-complexity detection and phase synchronization of multilevel CPM signals [6]. Huang and Li later presented a solution for schemes with an *integer* modulation index [7].

The M-LD is based on the expression of the M-ary input sequence as a product of binary subsequences. Each subsequence is then modulated by a binary CPM scheme, which produces binary subsignals. Applying the 2-LD to each subsignal and performing the product then leads to the desired result. As the 2-LD is not applicable to subsequences with an integer modulation index though, this strategy fails in particular cases. We therefore present an alternative method, based on the further decomposition of the subsequence(s) with integer modulation index in a product of (sub)subsequences. This approach can be applied recursively until all the final subsequences can be decomposed with the 2-LD.

Section 2 reviews the CPM signal structure and the M-LD. In Section 3, this M-LD is extended to the particular case where one or more subsequences have an integer modulation index. A proof is given of the number of pulses needed to exactly represent these schemes. An application example and some simulation results are then presented in Section 4, and conclusions are drawn in Section 5.

2. M-ARY CPM PAM DECOMPOSITION

2.1. CPM signal model

The complex envelope of a CPM signal has the form

$$s(t, \boldsymbol{\alpha}) = e^{j\,\psi(t,\boldsymbol{\alpha})} \tag{1}$$

where $\boldsymbol{\alpha} = \{\alpha_0, \alpha_1, \ldots, \alpha_{N_s-1}\}$ is a vector containing the sequence of M-ary data symbols $\alpha_n = \pm1, \pm3, \ldots \pm(M-1)$. The transmitted information is contained in the phase:

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$$\psi(t,\boldsymbol{\alpha}) = 2\pi h \sum_n \alpha_n \cdot q(t-nT) \tag{2}$$

where h is the modulation index, T is the signaling interval and $q(t)$ is the *phase response*, related to the *frequency response* $f(t)$ by the relationship

$$q(t) = \int_{-\infty}^{t} f(\tau)\,\mathrm{d}\tau. \tag{3}$$

The pulse $f(t)$ is a smooth pulse shape over a finite time interval $0 \le t \le LT$ and zero outside. Thus L is the length of the pulse per unit T. The function $f(t)$ is normalized such that $\int_{-\infty}^{\infty} f(t)\,\mathrm{d}t = \frac{1}{2}$.

2.2. Multilevel Laurent decomposition (M-LD)

As shown in [5], the M-ary data sequence $\boldsymbol{\alpha}$ can be expressed as a product of P *binary* subsequences $\boldsymbol{\gamma}_l$, l=0...$P-1$, where P is an integer satisfying

$$2^{P-1} < M \le 2^P. \tag{4}$$

Each subsequence $\boldsymbol{\gamma}_l$ is modulated by a binary CPM scheme with modulation index $h^{(l)} = 2^l h$. The 2-LD is applied to all of the resulting subsignals:

$$s^{(l)}(t,\boldsymbol{\gamma}_l) = \sum_{k=0}^{Q-1} \sum_n b_{k,n}^{(l)} c_k^{(l)}(t-nT),\ l = 0...P-1 \tag{5}$$

where $b_{k,n}^{(l)}$ and $c_k^{(l)}(t)$ are the k-th *pseudo-symbols* and binary *Laurent function* (2-LF) respectively, of the l-th subsignal $s^{(l)}(t,\boldsymbol{\gamma}_l)$. The multilevel CPM signal is then obtained by performing the product

$$s(t,\boldsymbol{\alpha}) = \prod_{l=0}^{P-1} s^{(l)}(t,\boldsymbol{\gamma}_l) \tag{6}$$

which leads to the M-LD

$$s(t,\boldsymbol{\alpha}) = \sum_{k=0}^{N-1} \sum_n a_{k,n} g_k(t-nT) \tag{7}$$

where

$$g_k(t) = \prod_{l=0}^{P-1} c_{d_{j,l}}^{(l)}(t + e_{j,l}^{(m)} T) \tag{8}$$

are the N multilevel Laurent Functions (M-LF's) and

$$a_{k,n} = \prod_{l=0}^{P-1} b_{d_{j,l},n-e_{j,l}^{(m)}}^{(l)} \tag{9}$$

are the multilevel *pseudo-symbols*. The time shifts $e_{j,l}^{(m)}$ and the indices $d_{j,l}$ are calculated as follows [5]. Consider the radix-Q representation of an integer j belonging to the interval $0 \le j \le Q^P - 1$

$$j = \sum_{l=0}^{P-1} Q^l d_{j,l} \tag{10}$$

and collect the coefficients $d_{j,l}$ into a vector $\boldsymbol{d}_j$ as follows:

$$\boldsymbol{d}_j = \{d_{j,P-1}, d_{j,P-2}, \ldots, d_{j,0}\}. \tag{11}$$

Call $D_{j,l}$ the duration of $c_{d_{j,l}}^{(l)}(t)$ and form the vector

$$\boldsymbol{D}_j = \{D_{j,P-1}, D_{j,P-2}, \ldots, D_{j,0}\} \tag{12}$$

for $0 \le l \le P-1$. For all $\boldsymbol{D}_j$ seek the P-tuples

$$\boldsymbol{e}_j^{(m)} = \{e_{j,P-1}^{(m)}, e_{j,P-2}^{(m)}, \ldots, e_{j,0}^{(m)}\} \tag{13}$$

with integer components and satisfying $0 \le e_{j,l}^{(m)} \le D_{j,l} - 1$ and $\prod_{l=0}^{P-1} e_{j,l}^{(m)} = 0$.

3. EXTENSION TO SUBSEQUENCES WITH INTEGER MODULATION INDEX

3.1. Decomposition of subsequences with integer h

When $h^{(l)}$ takes an integer value for any subsequence l, the 2-LD of that subsignal (5) fails [3]. In [7] a solution is proposed, and it is shown that an extra data-independent periodic component shows up in the 2-LD. This extra term makes this solution unsuitable for the method proposed in [5].

As suggested in [5] though, this subsequence l can be thought of as the product of two subsequences, each with $h^{(l)'} = \frac{h^{(l)}}{2} = h^{(l-1)}$. If $l \neq 0$, there are now three bilevel subsequences with $h = h^{(l-1)}$. Two of them contain the same pseudo-symbols, and all three share the same set of 2-LF's as their h is the same. This approach can be applied recursively for all values of $h^{(l)}$ and $h^{(l)'}$ which are still integer. In the end this will yield a product of K subsequences which all share the same set of 2-LF's. This is illustrated in Figure 1.

3.2. Needed number of M-LF's

As we now have a product of $K > P$ subsequences, the M-LD will consist of more PAM components. We present a general method to calculate the number of components needed to exactly represent the CPM signal.

As shown in [3], any binary CPM signal can be decomposed as a sum of 2^{L-1} 2-LF's modulated by pseudo-symbols. It can easily be shown that the sum of the durations of these 2-LF's equals 2^L symbol intervals. Thus, in any symbol interval, the CPM signal is composed of 2^L contributions. Among these contributions, 2^{L-1} are originating from the 2-LF's modulated by pseudo-symbols of the *current* interval, the 2^{L-1} others come from intersymbol interference (ISI).

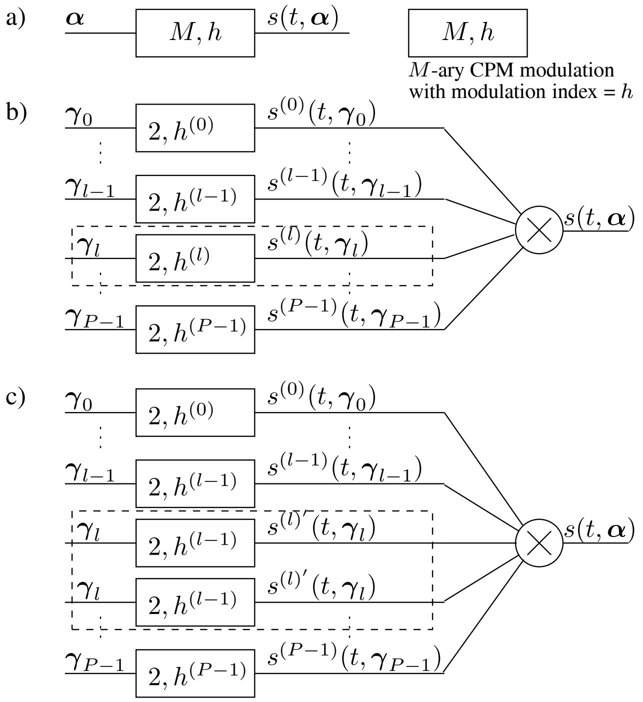

Fig. 1. Decomposition of a) the M-ary input sequence in b) a product of P binary subsequences with one integer $h^{(l)}$ and c) the decomposed substream l and the final K subsequences.

An M-LF in the *multilevel* CPM signal arises from the product of K of these contributions. In our particular case, the sets of 2-LF's on all K subsequences are the same, so only different *combinations* (as opposed to permutations) with repetition of these 2^L contributions will create new M-LF's. As the number of combinations with repetition of r elements out of a set of n elements is given by

$$\binom{n+r-1}{n-1} = \frac{(n+r-1)!}{r!(n-1)!} \tag{14}$$

where $\binom{\cdot}{\cdot}$ denotes the binomial coefficient, and we have 2^L components on each of the K streams, this yields

$$G = \binom{2^L+K-1}{2^L-1} \tag{15}$$

different combinations. Only combinations with at least one contribution of a 2-LF starting in the current interval belong to this interval though. We should thus exclude the number of combinations which consist only of the 2^{L-1} 'ISI contributions'. The final result, the number of M-LF to exactly represent the multilevel CPM signal composed of K binary subsequences with identical sets of 2-LF's, is then

$$I = \binom{2^L+K-1}{2^L-1} - \binom{2^{L-1}+K-1}{2^{L-1}-1}. \tag{16}$$

It is interesting to note that the number of M-LF's in the general case of [5] can be obtained by considering permutations instead of combinations. As the number of permutations with repetition of r elements out of a set of n elements is given by n^r, this indeed yields

$$N = (2^L)^P - (2^{L-1})^P = 2^{(L-1)P}(2^P-1). \tag{17}$$

3.3. Calculation of the M-LF's

The general case of [5] concerns the multiplication of P substreams with a *different* set of 2-LF's on each stream. In our case though, the K subsequences to be multiplied are all composed of the *same* set of 2-LF's. Thus, if we define the vector of pairs $(d_{j,l}, e_{j,l}^{(m)})$

$$\boldsymbol{s}_k = \{(d_{j,K-1}, e_{j,K-1}^{(m)}), (d_{j,K-2}, e_{j,K-2}^{(m)}), \ldots, (d_{j,0}, e_{j,0}^{(m)})\} \tag{18}$$

with the mapping $(j,m) \to k$ as described in [5], it can be seen that all vectors $\boldsymbol{s}_k$ of which the elements are mere permutations of each other, will bring about the same M-LF in (8). Grouping these permutations in sets $\mathcal{S}_i$, $i = 1 \ldots I$ we can now calculate the reduced number of M-LF's as

$$g_i(t) = \prod_{l=0}^{K-1} c_{d_{j,l}}^{(l)}(t + e_{j,l}^{(m)}T) \tag{19}$$

where $\boldsymbol{s}_k$ is any member of $\mathcal{S}_i$. The corresponding pseudo-coefficients are calculated as

$$a_{i,n} = \sum_{\mathcal{S}_i} \prod_{l=0}^{K-1} b^{(l)}_{d_{j,l}, n-e_{j,l}^{(m)}}. \tag{20}$$

4. EXAMPLE: LD OF THE QUATERNARY $3RC$ SYSTEM WITH $h = 0.5$

According to (4), an $M = 4$, $L = 3$, $h = 0.5$ system can be decomposed as the product of $P = \log_2 M = 2$ binary subsequences, with $h^{(0)} = 0.5$ and $h^{(1)} = 1$ respectively. If it were not for the integer value of $h^{(1)}$, according to (17) this would yield a decomposition with $N = 48$ M-LF's.

Subsequence $l = 1$ has to be decomposed though in two binary subsequences, with each $h = 0.5$. This yields three subsequences with the same set of 2-LF's. According to (16), we now need $I = 100$ pulses to exactly represent this CPM scheme. These pulses are shown in Figure 2.

In [4] and [6], receivers for CPM based on the LD are presented. They consist of a matched filterbank and a Viterbi detector. The filters are matched to the different M-LP's. The optimal receiver thus needs $I = 100$ matched filters, and has a trellis with 64 states. As can be seen from Figure 2 though,

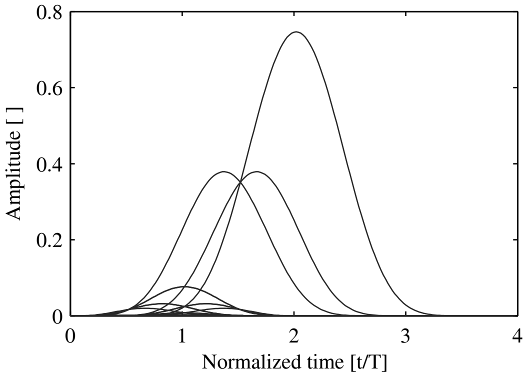

Fig. 2. Laurent Functions for quaternary $3RC$ with $h = 0.5$.

the 3 main pulses contain the largest part of the signal energy. Using the technique described in [6], we construct a receiver with a filterbank of 3 filters, matched to these main pulses. The number of states in the trellis is then reduced to 4. The performance of the optimal receiver is compared to this reduced-complexity receiver in Figure 3. The large performance loss is due to the emergence of parallel paths in the reduced-complexity trellis.

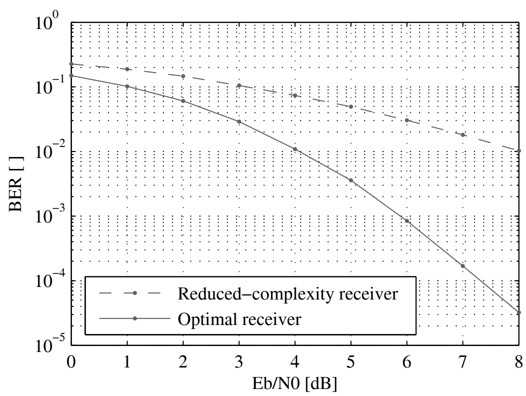

Fig. 3. Error performance of Laurent-based receivers for quaternary $3RC$ with $h = 0.5$.

5. CONCLUSIONS

We have generalized the Laurent decomposition to include a particular class of M-ary CPM signals. The technique of Mengali [5] decomposes the M-ary input sequence in a product of binary subsequences. Then the 2-LD is applied to each subsignal and the M-LD is found by performing the product of these subsignals. This technique fails when the 2-LD can not be applied to one or more of the subsignals because it has an integer modulation index h.

We have proposed a solution to calculate the M-LD in this particular case. It is based on a further decomposition of the subsequences with integer h until none of the final subsequences has an integer h anymore. Now the 2-LD can be applied to all subsignals and the M-LD can be found as their product. We have shown how many M-LF's are needed to exactly represent the M-ary CPM signal in this particular case. Our theory also confirms the number of M-LF's needed in the general case of [5].

The approach was illustrated with the decomposition of the $M = 4$, $h = 0.5$ $3RC$ system, and it was shown an optimal receiver can be based on it. Starting from this exact decomposition, reduced-complexity receivers can be built using general techniques as the one described in [5].

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