LOW-COMPLEXITY USER AND ANTENNA SELECTION FOR MULTIUSER MIMO SYSTEMS WITH BLOCK DIAGONALIZATION

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ABSTRACT

Block diagonalization is a downlink precoding technique that precancels inter-user interference in multiuser MIMO systems. When there are a large number of users, the system throughput can be significantly increased by selecting a subset of users and a subset of receive antennas for each user. The optimal user and antenna subset can be obtained by exhaustively searching over all possible user and antenna combinations to find the one with the highest sum throughput. This brute-force solution, however, is prohibitively complicated. To reduce the complexity, in this paper we propose a low-complexity suboptimal user and antenna selection algorithm. For most system configurations, we show that our proposed algorithm achieves up to 98% of the optimal sum throughput of the exhaustive search, where the complexity is orders of magnitude lower than the exhaustive search method.

***Index Terms*—** MIMO systems, multiuser channels, scheduling.

1. INTRODUCTION

In a downlink multiuser multi-input multi-output (MU-MIMO) communication system, a base station (BS) transmits to multiple mobile stations (MS) simultaneously over the same frequency band, thereby greatly increasing the channel capacity. Dirty paper coding (DPC) has been proven to be the capacity optimal technique for MU-MIMO systems [1, 2], but it has a huge complexity associated with the successive encoding processing. Recently, block diagonalization (BD) has been extensively studied as practical MU-MIMO broadcast transmission technique, where each user's signal is premultiplied by a precoding matrix at the BS, such that inter-user interference can be perfectly eliminated at mobile station (MS) [3, 4, 7], given that all users' channel station information is perfectly known at the BS. The maximum number of simultaneously supported users with BD is upper bounded by the ratio of BS antennas and number of streams per MS. When there are a large number of users in the system, multiuser diversity can be exploited to significantly increase the system throughput by scheduling the optimum subset of users to serve [6, 8].

Previous scheduling algorithms for BD assume that each selected user applies all its receive antennas [6]. This assumption, however, leads to a suboptimal capacity result. The optimal solution is to select a subset of users and a subset of active receive antennas for each user, such that the capacity is maximized. In addition to multiuser diversity due to user scheduling [6, 8], antenna selection significantly boosts the system performance by using only the receive antennas with good channel conditions, and disabling the "bad" antennas in deep fade, such that the transmission resources are dynamically shared among the users. Additionally, because each selected user uses only a subset of its antennas, it is possible to increase the number of simultaneously supportable users in the system and reduce the scheduling delay per user. Antenna selection is particularly beneficial if channel correlation at MS is high. Antenna selection avoids using a group of highly correlated antennas at the same MS, but selects a distributed set of antennas at different MSs which are less correlated, thus effectively increases the spatial diversity and sum channel capacity.

Exhaustive user and antenna search can be used to find the optimum user and antenna set with the highest throughput [9]. This approach, however, is very complicated due to the huge number of possible user/antenna sets, especially as the number of users and the number of receive antennas per user increase. As a result, a low-complexity algorithm is critical to efficiently exploit the capacity gain promised by joint user and antenna selection [9], while keeping the search complexity low. To address this issue, in this paper we propose a low-complexity joint user/antenna selection algorithm. Following a greedy selection method, our proposed algorithm activates one receive antenna at a time, associated with the best user, until no active antenna can be added to the system. We analytically evaluate the computational complexity of our proposed algorithm, and show that it is orders of magnitude lower than that of the exhaustive search. Simulation results show that the proposed algorithm achieves 98% of the optimal sum throughput. Our proposed technique can be extended to incorporate other QoS constraint such as proportional fairness, which will be considered in future research. With our proposed solution, the multiuser and multi-antenna diversity for future MU-MIMO systems can be efficiently exploited to obtain superior capacity and error performance.

2. SYSTEM AND SIGNAL MODEL

In this section, the system model of the conventional BD is illustrated. Throughout this section, it is assumed that each MS uses all its receive antennas.

Consider a MU-MIMO system with K *active* users, where the BS has N_t antennas and user k has $N_{r,k}$ receive antennas. The transmit symbol of user k is denoted by a $L_k \times 1$ vector $\mathbf{x}_k$, where $\mathbf{Q}_k = E(\mathbf{x}_k \mathbf{x}_k^\dagger)$ is the transmit covariance matrix, subject to sum power constraint trace $(\mathbf{Q}_k) \leq P$. Data vector $\mathbf{x}_k$ is multiplied by a $N_t \times L_k$ precoding matrix $\mathbf{T}_k$ and sent to the BS antenna array.

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At receiver k, a $N_{r,k} \times L_k$ equalizer matrix $\mathbf{R}_k$ [1] is applied at the receive signal, and the post-processing signal is given as

$$\mathbf{y}_k = \mathbf{R}_k^\dagger \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{R}_k^\dagger \mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{T}_j \mathbf{x}_j + \mathbf{R}_k^\dagger \mathbf{n}_k, \qquad (1)$$

The additive white Gaussian noise (AWGN) is given by $\mathbf{n}_k$ with $E(\mathbf{n}_k \mathbf{n}_k^\dagger) = \sigma_n^2 \mathbf{I}$. $\mathbf{H}_k$ is the $N_{r,k} \times N_t$ channel matrix from the BS to the kth MS. We assume $\mathbf{H}_k$ has full rank, i.e. rank $(\mathbf{H}_k) = \min(N_{r,k}, N_t)$ and that $\{\mathbf{H}_k\}_{k=1}^K$ are independent, given that K MSs are sufficiently spaced apart. The principal idea of BD is to find the precoding matrix $\mathbf{T}_k \in \mathbb{U}(N_t, L_k)$ and equalizer matrix $\mathbf{R}_k \in \mathbb{U}(N_{r,k}, L_k)$, such that

$$\mathbf{R}_k^\dagger \mathbf{H}_k \mathbf{T}_j = \mathbf{0}, \quad \forall 1 \le k \neq j \le K, \qquad (2)$$

where $\mathbb{U}(n,k)$ denotes the set of $n \times k$ unitary matrices with orthonormal columns.

If (2) is satisfied, then inter-user interference is perfectly canceled at the BS. The received signal at MS k is

$$\begin{aligned}\mathbf{y}_k &= \mathbf{R}_k^\dagger \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{R}_k^\dagger \mathbf{H}_k \sum_{j=1, j\neq k}^{K} \mathbf{T}_j \mathbf{x}_j + \mathbf{R}_k^\dagger \mathbf{n}_k \\ &= \mathbf{R}_k^\dagger \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \mathbf{R}_k^\dagger \mathbf{n}_k. \end{aligned} \qquad (3)$$

Let $\tilde{\mathbf{H}}_k = \left[\mathbf{H}_1^\dagger \mathbf{R}_1, \cdots, \mathbf{H}_{k-1}^\dagger \mathbf{R}_{k-1}, \mathbf{H}_{k+1}^\dagger \mathbf{R}_{k+1} \cdots, \mathbf{H}_K^\dagger \mathbf{R}_K\right]^\dagger$. To satisfy the zero-interference constraint (2), precoder $\mathbf{T}_k$ should lie in the null space of $\tilde{\mathbf{H}}_k$, thus constraint (2) can be rewritten as

$$\tilde{\mathbf{H}}_k \mathbf{T}_k = \mathbf{0}, \quad 1 \le k \le K. \qquad (4)$$

As a result, the columns of $\mathbf{T}_k$ should lie in the null space of $\tilde{\mathbf{H}}_k$. To guarantee that the null space is not empty, a necessary condition of BD is specified as follows [5, 7]:

Lemma 1 *([3, 7]) To ensure that the null space is not empty, a necessary condition to perform BD is $N_t \ge \sum_{j=1}^K L_j$.* □

Lemma 1 shows that the number of transmit antennas must be larger than the total number of data streams to all users. If there are large number of $\hat{K}$ users in the system, a subset of K users must be property selected to maximize the channel capacity, where K is uniquely determined by N_t and $\{L_k\}_{k=1}^{\hat{K}}$.

3. LOW-COMPLEXITY USER AND ANTENNA SELECTION

In this section, we introduce the joint user and antenna selection scheme, and propose a low-complexity joint user/antenna selection technique that can substantially reduce the complexity of finding the optimum user/antenna set.

3.1. BD with User/Antenna Selection

In most conventional BD schemes [3, 6, 7], the number of data streams L_k for user k, defined as the mode, is fixed. If there are a large number of users in the system, a subset of users must be selected to maximize the sum throughput [6].

[1] $(\cdot)^\dagger$ denotes the complex conjugate transpose, $E_s(\cdot)$ denotes the expectation with respect to variable s, trace $(\cdot)$ denotes the summation of the diagonal element of a square matrix, $\|\cdot\|_F$ is the Frobenius norm, card $(\cdot)$ denotes the cardinality of a set.

Fixing the mode L_k of user k is obviously a suboptimal approach. The more general BD scheme is to adaptively optimize the mode L_k of user k, subject to $\sum_{k=1}^{\hat{K}} L_k \le N_t$, such that the sum capacity is maximized. Adaptively selecting the mode L_k can significantly increase the system performance by allowing a dynamic allocation of the transmission resources (i.e., N_t total data streams) over the users [10]. The optimal user/mode selection is to exhaustively search over $\sum_{k=1}^{N_t} C_{\hat{K}N_r}^k$ possible user and mode combinations, and find the one with the highest throughput. This method, however, is extremely complicated.

A joint user/antenna selection technique was proposed in [9]. In this scheme, each MS uses a subset of its receive antennas instead of all the receive antennas. Therefore the problem reduces to a joint user/antenna selection problem, where the objective is to find the optimum set of L_k receive antennas for user k such that

$$C_{\max} = \max_{L_k, \mathbf{T}_k, \mathbf{R}_k, \mathbf{Q}_k} \sum_{k=1}^{\hat{K}} \log_2 |\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{R}_k^\dagger \mathbf{H}_k \mathbf{T}_k \mathbf{Q}_k \mathbf{T}_k^\dagger \mathbf{H}_k^\dagger \mathbf{R}_k|, \qquad (5)$$

$$\mathbf{T}_k \in \mathbb{U}(N_t, L_k), \mathbf{R}_k \in \mathcal{R}^{(N_r, L_k)}, \qquad (6)$$

$$0 \le L_k \le N_r, \ \forall k; \ \sum_{k=1}^{\hat{K}} L_k \le N_t, \qquad (7)$$

where $\mathcal{R}^{(N_r, L_k)}$ is the set of $N_r \times L_k$ antenna selection matrices formed by taking L_k columns from $\mathbf{I}_{N_r}$. It is a special case of the joint user/mode selection, where $\mathbf{R}_k$ is purposely enforced to be an antenna selection matrix. The mode of user k is upper bounded by L_k, and is determined by the rank of the input covariance matrix $\mathbf{Q}_k$ after water-filling. Although it is naturally a suboptimal scheme, it reduces the computational complexity for *a fixed set of users and modes*. In addition, because $\mathbf{R}_k$ is restricted as a special set of antenna selection matrices, the amount of feedback (in number of bits) for BS to send $\mathbf{R}_k$ to user k is substantially reduced.

The straightforward user/antenna selection method is to exhaustively search over all possible user/antenna combinations [9]. Because there are $\hat{K}$ users and N_r antennas per user, $\sum_{i=1}^{N_t} C_{\hat{K}N_r}^i$ combinations need to be searched over, which is very complicated.

3.2. Low-Complexity User/Antenna Selection Algorithm

To avoid the computational complexity of the exhaustive search, a low-complexity joint user/antenna selection technique is proposed in this paper.

An intuitive interpretation of this algorithm is given as follows. If the SNR is at medium to high range, water-filling tends to pour water in every eigenmode. Hence for each user, the number of streams is equal to the number of selected receive antennas. Therefore this joint user/antenna selection problem reduces to allocating a maximum of N_t receive antennas to a group of users. The proposed algorithm follows an iterative allocation procedure. In each iteration, the BS transmits one more data stream, and assigns it to the best receive antenna of the best user that produces the maximum sum throughput with the already activated antennas. This newly activated antenna is allowed to cooperate with antennas associated with the same MS, but not allowed to cooperate with other users. The BS keeps adding more antennas to the system, until the number of total selected antennas reaches N_t, or if the sum throughput begins to decrease. Therefore this algorithm needs to undergo a maximum of N_t iterations, where in each iteration no more than $\hat{K}N_r$ antennas need to be considered. As a result, the size of search space is upper bounded by $\hat{K}N_rN_t$, which is much greatly simplified than the exhaustive search where $\sum_{i=1}^{N_t} C^{i}_{\hat{K}N_r}$ possible combinations have to be searched over.

Consider $\hat{K}$ users, and let $\mathcal{A}_k$ and $\mathcal{S}_k$ denote the index of the *unselected* and the *selected* antennas for MS k, where $\mathcal{A}_k$ and $\mathcal{S}_k$ are subsets of $\{1, 2, \ldots, N_r\}$. Let $L_k = \text{card}(\mathcal{S}_k)$ denote the number of substreams (e.g., selected antennas) for user k. For example, if $N_{r,k} = 4$ and $\mathcal{S}_k = \{1, 3\}$, it means that antenna 1 and antenna 3 of user k are chosen to receive two substreams. Let $\mathcal{K}$ denotes of the index of active users with $L_k \geq 1$, which is a subset of $\left\{1, 2, \ldots, \hat{K}\right\}$.

1. Stage $i = 0$: Set all antennas of all MSs inactive, by letting $L_1 = \ldots = L_{\hat{K}} = 0$, $\mathcal{K} = \phi$, $\mathcal{S}_1 = \ldots = \mathcal{S}_{\hat{K}} = \phi$, $\mathcal{A}_1 = \ldots = \mathcal{A}_{\hat{K}} = \{1, 2, \ldots, N_r\}$,
2. Stage $i = 1$.
   (a) Find the best antenna $\bar{j}$ of the best user $\bar{k}$,
   $$(\bar{k}, \bar{j}) = \arg \max_{k=1,\ldots,\hat{K};\ j=1,\ldots,N_r} \|\mathbf{h}_{k,j}\|_F^2, \quad (8)$$
   where $\mathbf{h}_{k,j}$ denotes the jth row of $\mathbf{H}_k$.
   (b) Let $\mathcal{S}_{\bar{k}} = \mathcal{S}_{\bar{k}} \cup \{\bar{j}\}$, $\mathcal{A}_{\bar{k}} = \mathcal{A}_{\bar{k}} - \{\bar{j}\}$, $L_k = \text{card}(\mathcal{S}_k)$, $\mathcal{K} = \mathcal{K} \cup \{\bar{k}\}$.
   (c) Calculate $C_{\text{temp}} = \log_2 \det(1 + \frac{P}{\sigma_n^2}\|\mathbf{h}_{\bar{k},\bar{j}}\|_F^2)$.
3. Stage $i = 2$,
   (a) For every user $k = 1 : \hat{K}$, for every inactive antenna $j \in \mathcal{A}_k$,
      i. Temporarily active antenna j of MS k, by setting $\bar{\mathcal{S}}_k = \mathcal{S}_k \cup \{j\}$, $\bar{\mathcal{A}}_k = \mathcal{A}_k - \{j\}$, $\bar{\mathcal{K}} = \mathcal{K} \cup (\{k\}$.
      ii. Find the precoding matrix $\mathbf{T}_k$ for each active MS $k \in \bar{\mathcal{K}}$, with the channel $\{\mathbf{H}_{\mathcal{S}_k}\}_{k \in \bar{\mathcal{K}}}$, where $\mathbf{H}_{\mathcal{S}_k}$ denotes the rows of $\mathbf{H}_k$ indexed by $\mathcal{S}_k$, i.e., the channel associated with the selected receive antennas.
      iii. Perform SVD on the effective channel $\mathbf{H}_{\mathcal{S}_k}\mathbf{T}_k$, and obtain the singular values $\{\lambda_{k,l}\}_{l=1}^{L_k}$, for all selected MSs $k \in \bar{\mathcal{K}}$. Perform water-filling over all selected MSs $\{\lambda_{k,l}\}_{l=1}^{L_k}$ for all $k \in \bar{\mathcal{K}}$. Find the sum throughput $C_{k,j}$.
   (b) Find the optimum antenna $\bar{j}$ and the optimum user $\bar{k}$ that provides the highest sum throughput
   $$(\bar{k}, \bar{j}) = \arg \max_{k=1,\ldots,\hat{K},\ j \in \mathcal{A}_k,\ \mathcal{A}_k \neq \phi} C_{k,j}. \quad (9)$$
   (c) If $C_{\bar{k},\bar{j}} \geq C_{\text{temp}}$, we will select antenna $\bar{j}$ of MS $\bar{k}$ in this iteration. Let $\mathcal{S}_{\bar{k}} = \mathcal{S}_{\bar{k}} \cup \{\bar{j}\}$, $\mathcal{A}_{\bar{k}} = \mathcal{A}_{\bar{k}} - \{\bar{j}\}$, $\mathcal{K} = \mathcal{K} \cup \{\bar{k}\}$. For all $k \in \mathcal{K}$, $L_k = \text{card}(\mathcal{S}_k)$. Let $i = i + 1$, $C_{\text{temp}} = C_{\bar{k},\bar{j}}$.
   (d) if $C_{\bar{k},\bar{j}} < C_{\text{temp}}$ or $\sum_{k=1}^{\hat{K}} L_k = N_t$, exit the algorithm.

4. COMPLEXITY ANALYSIS

The computational complexity of the proposed user/antenna selection algorithm is analytically compared to that of the exhaustive user/antenna search. Complexity is measured in terms of the number of flops φ, defined as a real floating point operation. A complex addition and multiplication have 2 and 6 flops, respectively.

4.1. Complexity of Exhaustive User/Antenna Selection

We provide a complexity *lower bound* of the exhaustive search. For a particular user/antenna combination, suppose there are totally $i \leq N_t$ active receive antennas, distributed over K active users. The flop count to calculate $\mathbf{T}_k$ from $\tilde{\mathbf{H}}_k$ is lower bounded by $24N_t + 48N_t^2 + 54N_t^3$. The flop count to perform SVD for $\mathbf{R}_k^\dagger \mathbf{H}_k \mathbf{T}_k$ is lower bounded by 1 (L_k=1). Water-filling over i streams requires $2i^2 + 6i$ flops. Therefore, flop count φ is lower bounded by

$$\begin{aligned}
\varphi &> \sum_{i=1}^{N_t} C^{i}_{\hat{K}N_r}(K(24N_t + 48N_t^2 + 54N_t^3 + 1) + 2i^2 + 6i) \\
&> \sum_{i=1}^{N_t} C^{i}_{\hat{K}N_r}(\lfloor \frac{i}{N_r} \rfloor (24N_t + 48N_t^2 + 54N_t^3) + 2i^2 + 6i) \\
&\approx \mathcal{O}(\frac{N_t^5}{N_r} C^{N_t}_{\hat{K}N_r}) \qquad (10)
\end{aligned}$$

4.2. Complexity of Proposed Low-Complexity Algorithm

We provide an *upper bound* of the complexity of our proposed algorithm.

1. $i = 1$: Compute the Frobenius norm of $\mathbf{h}_{k,j}$ takes $4N_t$ flops, $1 \leq k \leq \hat{K}$, $1 \leq j \leq N_r$. Therefore the total flop count is $4N_t N_r \hat{K}$.
2. $i = 2, \ldots, N_t$: Suppose there are $i \leq N_t$ active receive antennas, distributed over $K \leq N_t$ selected users. For each selected MS k, computing $\mathbf{T}_k$ from $\tilde{\mathbf{H}}_k$ requires fewer than $126N_t^3$ flops. Computing the eigenvalues of MS k requires fewer than $24N_r N_t^2 + 48N_r^2 N_t + 54N_t^3$ flops. Water-filling operated over the eigenmodes of the K selected MSs requires fewer than $2N_t^2 + 6N_t$ flops. Therefore, the number of total flops is upper bounded by

$$\begin{aligned}
\varphi &< \sum_{i=2}^{N_t} (\hat{K}N_r - i)[K(126N_t^3 + 24N_r N_t^2 \\
&\quad 48N_r^2 N_t + 54N_t^3) \\
&\quad + 2N_t^2 + 6N_t] + 4N_r N_t \hat{K} \\
&< \sum_{i=2}^{N_t} (\hat{K}N_r)[N_t(126N_t^3 + 24N_r N_t^2 + 48N_r^2 N_t \\
&\quad 54N_t^3) + 2N_t^2 + 6N_t] + 4N_r N_t \hat{K} \\
&\approx \mathcal{O}(\hat{K}N_r N_t^5) \qquad (11)
\end{aligned}$$

In summary, the complexity of the proposed user and antenna scheduling algorithm is linear with the total number of users $\hat{K}$, because no more than $\hat{K}N_r$ receive antennas have to be searched in each iteration. Exhaustive user and antenna scheduling, however, has complexity proportional to $C^{N_t}_{\hat{K}N_r}$. The complexity ratio of the two methods is upper bounded by

$$\eta \leq \frac{\hat{K}N_r N_t^5}{\frac{N_t^5}{N_r} C^{N_t}_{\hat{K}N_r}} = \frac{\hat{K}N_r^2}{C^{N_t}_{\hat{K}N_r}}. \qquad (12)$$

For example, η is less than 4.7189×10^{-8} for a MU-MIMO system with $\hat{K} = 20, N_t = 10, N_r = 2$, therefore the computational complexity is greatly reduced.

5. NUMERICAL RESULTS

We compare the sum throughput of the following schemes.

- Iterative water-filling for DPC (capacity upper bound) [1]
- Round-robin algorithm for $\hat{K}$ users (no user selection)
- BD with user selection but *without antenna selection* [6] (capacity based, near optimal)
- BD with proposed low-complexity user/antenna selection
- BD with optimal exhaustive user/antenna selection [9]

Fig. 2 depicts the sum throughput in bit/s/Hz versus the total number of users $\hat{K}$, for a $N_t = 12$, $N_r = 4$ MU-MIMO system and various SNR values, averaged over 2500 channel realizations. Compared to the BD with only user selection [6], performing additional antenna selection (i.e., optimizing the number of streams) with our proposed algorithm can increase the sum capacity by up to 10 bit/s/Hz, approximately a 16% throughput increase. This capacity gain is even higher compared to a round-robin BD without any user selection, where a capacity gain as large as 20 bit/s/Hz is achieved. The proposed low-complexity user/antenna selection algorithm performs very close to the exhaustive user/antenna search, achieving approximately 98% of the throughput gain of the brute-force method.

6. CONCLUSIONS

A joint user/antenna selection technique for downlink multiuser MIMO systems with BD is studied. The objective is to dynamically select a subset of users and receive antennas, such that the sum throughput is maximized. The optimal brute-force method is prohibitively complicated. In this paper we propose a near-optimal user/antenna selection algorithm, whose complexity grows on the linear scale of $\hat{K}$. Simulation results demonstrate that the proposed low-complexity algorithm achieves approximately 98% throughput of the exhaustive search method, with much less complexity.

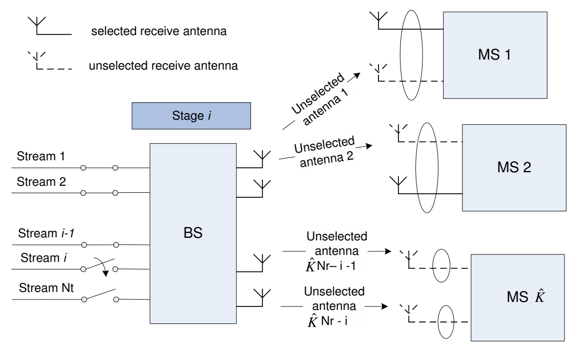

Fig. 1. Block diagram of the proposed low-complexity user/antenna selection algorithm: In stage i ($i \le N_t$), the remaining $\hat{K}N_r - i$ antennas are one by one evaluated, to find the optimal one that produces the maximum sum capacity with those already activated antennas.

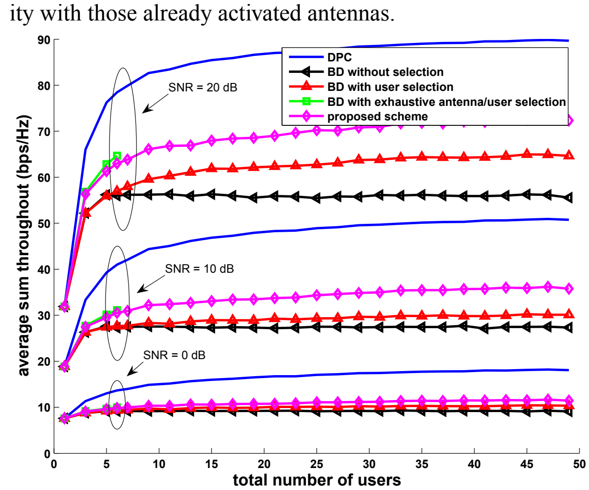

Fig. 2. Sum throughput for $N_t = 12$, $N_r = 4$, and different number of users $\hat{K}$.

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