CLOSED-FORM UPPER BOUNDS FOR THE CONSTELLATION-CONSTRAINED CAPACITY OF ULTRA-WIDEBAND COMMUNICATIONS

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ABSTRACT

This paper[1] focuses on the evaluation of performance limits for ultra-wideband communications in terms of constellation-constrained capacity. The major drawback of most of the existing capacity measures is that an exact and closed-form expression cannot be easily obtained. As a result, evaluation of these capacity measures must resort to numerical methods. In this paper, however, closed-form upper bounds for the constellation-constrained capacity of PPM modulation are proposed for both coherent and non-coherent receivers. The tightness of the proposed bounds is evaluated under the IEEE 802.15.3a/4 channel models where a good match is observed, in particular, for the low-SNR regime.

Index Terms— Ultra-wideband, PPM, capacity, bounds.

1. INTRODUCTION

Short-range wireless communication has become an essential part of everyday life thanks to the enormous growth in the deployment of wireless local/personal area networks. However, traditional wireless technology cannot meet the requirements of upcoming wireless services that demand high-data rates to operate. This issue has motivated an unprecedented resurgence of ultra-wideband (UWB) technology, a transmission technique that is based on the emission of extremely-short pulses with a very low power spectral density. Because of the particular characteristics of UWB signals, very high data rates can be provided with multipath immunity and high penetration capabilities.

Nevertheless, formidable challenges must be faced in order to fulfill the expectations of UWB technology. One of the most important challenges is to cope with the overwhelming distortion introduced by the intricate propagation physics of UWB signals [1]. In addition to this, UWB antennas behave like direction-sensitive filters such that the signal driving the transmitting antenna, the electric far field, and the signal across the receiver load may differ considerably in waveshape and spectral content [2]. As a result, matched filter correlation is difficult to be implemented at the receiver unless high computational complexity is dedicated for obtaining perfect waveshape estimation. Thus, UWB receivers may be implemented under a *coherent* or *non-coherent* approach depending on a tradeoff between complexity and performance.

On the one hand, coherent receivers are optimal in the sense that they have perfect knowledge of the end-to-end channel response. This channel state information is usually obtained by using channel estimation techniques prior to the symbol detection stage. With proper channel knowledge, coherent receivers consist of a traditional correlator-based architecture where a replica of the transmitted pulse is used to implement a matched filtering or RAKE receiver [3]. On the other hand, non-coherent receivers do not perform channel estimation and thus, they can be seen as a low-cost and low-power alternative to the more complex and computationally demanding coherent receivers [4]. The save in hardware complexity is especially important because channel estimation usually requires about 60 % of the total number of gates in an UWB coherent receiver [5].

From the above considerations, the main purpose of this paper is to analyze the asymptotic performance of both coherent and non-coherent UWB communication systems operating over multipath fading channels. To this end, the conditions for which an arbitrarily small error probability is achieved are analyzed by using the concept of *constellation-constrained* capacity. Since we focus on digital communication systems, constellation-constrained capacity is adopted because it provides a benchmark on the best rate it can be achieved with a given discrete input distribution [6]. This is in contrast with the traditional definition of capacity where continuous inputs are considered. One of the major problems when dealing with capacity measures is that rather difficult expressions are encountered and their evaluation must resort to numerical methods. In order to circumvent this limitation, this paper presents tight upper bounds for the constellation-constrained capacity of both coherent and non-coherent receivers. The major contribution is that simple and closed-form expressions are provided that tightly model the exact behavior of constellation-constrained capacity.

The paper is structured as follows. The signal model is presented in Section 2 and the notion of constellation-constrained capacity is introduced in Section 3. The proposed closed-form upper bounds for both coherent and non-coherent receivers are derived in Section 4. Finally, simulation results are enclosed in Section 5 and conclusions are drawn in Section 6.

2. SIGNAL MODEL

The mostly adopted modulation formats for UWB communication systems are pulse-amplitude modulation (PAM) and pulse-position modulation (PPM). On the one hand, PAM requires perfect channel state information in order to resolve the ambiguity introduced by the channel in the amplitude of the received signal. On the other hand, PPM can either be adopted in the presence or in the absence of channel state information. That is, PPM can be adopted by either coherent or non-coherent receivers. For this reason, PPM will be considered in the sequel.

When focusing on PPM, several variations are found including multipulse PPM (MPPM), overlapping PPM (OPPM) and differential PPM (DPPM), among other. Interestingly, they all can be regarded as a constrained version of on-off keying (OOK) modulation. In OOK a single bit is transmitted per channel use. That is, a pulse is transmitted within the symbol duration for representing ”1” and no pulse is transmitted for representing ”0”. An important point to be remarked is that, since PPM, MPPM, OPPM and DPPM are each a constrained version of OOK modulation, the capacity of uncon-

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strained OOK becomes the upper bound on the capacity of these pulse-position modulation formats. As it is shown in [7], the capacity of PPM is found to be near that of OOK for the low-SNR regime. Consequently, no other constrained version of OOK modulation (e.g. MPPM, OPPM or DPPM) can offer a significant improvement over traditional PPM for this region. Since most UWB communication systems must operate in the low-SNR regime because of the stringent spectral regulations, we will not consider MPPM, OPPM nor DPPM hereafter but only the standard and traditional PPM.

Let us consider the following real-valued discrete-time signal model for the received PPM signal,

$$\mathbf{y} = \mathbf{H}\mathbf{x}_i + \mathbf{w} \tag{1}$$

where $\mathbf{y} \in \Re^{N_{ss}\times 1}$ is the vector of received samples with N_{ss} the number of samples per symbol. The vector $\mathbf{w} \in \Re^{N_{ss}\times 1}$ incorporates the Gaussian contribution from both the thermal noise and possible multiple access interference with $\mathbf{C}_w \doteq \mathrm{E}\left[\mathbf{w}\mathbf{w}^T\right]$. The $(P \times 1)$ vector $\mathbf{x}_i$ corresponds to the i-th PPM symbol from the PPM codebook $\mathcal{X} : \{\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{P-1}\}$. Only the i-th entry in $\mathbf{x}_i$ is active. That is, $[\mathbf{x}_i]_i = 1$ and $[\mathbf{x}_i]_j = 0$ for all $j \neq i$.

The shaping matrix $\mathbf{H} \in \Re^{N_{ss}\times P}$ incorporates the end-to-end channel response between transmitter and receiver. The columns of the shaping matrix $\mathbf{H}$ are indicated by $\mathbf{h}_i$ with $\mathbf{H} = [\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_{P-1}]$, and they contain time-shifted replicas of the end-to-end channel response with the time-shift equal to N_Δ samples. The end-to-end channel response has a maximum length of N_g samples. Then, by taking into consideration the maximum delay spread of the channel and the maximum PPM time-shift, a guard interval is introduced in order to avoid intersymbol interference with the next received symbol. Since no time-hopping is assumed here for the sake of simplicity, the condition for avoiding intersymbol interference is $N_{ss} \geq N_g + (P-1)N_\Delta$. Later on, it will be useful to incorporate the N_g samples of the channel response into a vector indicated herein by $\mathbf{g}$, with $\mathbf{g} \in \Re^{N_g\times 1}$. Indeed, $\mathbf{g}$ is included in the columns of the shaping matrix $\mathbf{H}$ as follows,

$$\mathbf{h}_i = \Big[\underbrace{0,0,\ldots,0,0}_{iN_\Delta}, \mathbf{g}^T, \underbrace{0,0,\ldots,0,0}_{N_{ss}-iN_\Delta-N_g}\Big]^T. \tag{2}$$

Two different approaches are adopted in this paper depending on whether coherent or non-coherent receivers are considered.

- **Coherent approach:** The end-to-end channel response is assumed to be perfectly known at the receiver side so that the only nuisance parameter is the Gaussian contribution from the noise. As a result, the probability density function of the received signal $\mathbf{y}$ conditioned on the transmission of the PPM codeword $\mathbf{x}_i$ and a given channel response $\mathbf{g}$ becomes

$$f_{\mathrm{coh}}(\mathbf{y}|\mathbf{x}_i;\mathbf{g}) = \frac{\exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{h}_i)^T\mathbf{C}_w^{-1}(\mathbf{y}-\mathbf{h}_i)\right)}{(2\pi)^{N_{ss}/2}\det^{1/2}(\mathbf{C}_w)}. \tag{3}$$

- **Non-coherent approach:** The end-to-end channel response is assumed to be an unknown random Gaussian process. Since the unknown end-to-end channel response and the noise are statistically independent, the probability density function of the received signal $\mathbf{y}$ conditioned on the transmission of the PPM codeword $\mathbf{x}_i$ is

$$f_{\mathrm{non-coh}}(\mathbf{y}|\mathbf{x}_i) = \frac{\exp\left(-\frac{1}{2}\mathbf{y}^T(\mathbf{C}_w+\mathbf{C}_{\mathbf{h}_i})^{-1}\mathbf{y}\right)}{(2\pi)^{N_{ss}/2}\det^{1/2}(\mathbf{C}_w+\mathbf{C}_{\mathbf{h}_i})} \tag{4}$$

with $\mathbf{C}_{\mathbf{h}_i} \doteq \mathrm{E}\left[\mathbf{h}_i\mathbf{h}_i^T\right]$ the covariance matrix for the received waveform under the hypothesis $\mathcal{H}_i : \mathbf{x} = \mathbf{x}_i$.

Similarly to [2] and the references therein, the noise contribution will be assumed to be white. This assumption is well justified by the low duty cycle of UWB transmissions and the adoption of time hopping mechanisms for multiple access.

3. CONSTELLATION-CONSTRAINED CAPACITY

For the case of digital communication systems, the so-called *constellation-constrained capacity* establishes a benchmark on the achievable rates with a given discrete input distribution [6]. The constellation constrained capacity is indicated herein as $\mathsf{C_c}$ and it is based on the original definition of capacity where the maximization over the input distribution is omitted. That is,

$$\mathsf{C_c} \doteq \sum_{\mathbf{x}}\int_{\mathbf{y}} f(\mathbf{x},\mathbf{y})\log_2\frac{f(\mathbf{x},\mathbf{y})}{f(\mathbf{x})f(\mathbf{y})}d\mathbf{y} \quad \text{(bits/channel use).} \tag{5}$$

For the case of P-ary modulation with equiprobable transmitted symbols we have that $p(\mathbf{x}=\mathbf{x}_i) = \frac{1}{P}$. Then, after some straightforward manipulations, the contellation-constrained capacity is found to be given by a more insightful expression as follows,

$$\mathsf{C_c} = \log_2 P - \frac{1}{P}\sum_{i=0}^{P-1}\mathrm{E}_{\mathbf{y}|\mathbf{x}_i}\left[\log_2\sum_{j=0}^{P-1}\Lambda_{j,i}(\mathbf{y})\right] \tag{6}$$

where $\Lambda_{j,i}(\mathbf{y})$ is the likelihood ratio for deciding between the hypothesis $\mathcal{H}_i : \mathbf{x} = \mathbf{x}_i$ and the hypothesis $\mathcal{H}_j : \mathbf{x} = \mathbf{x}_j$. That is,

$$\Lambda_{j,i}(\mathbf{y}) \doteq \frac{f(\mathbf{y}|\mathbf{x}=\mathbf{x}_j)}{f(\mathbf{y}|\mathbf{x}=\mathbf{x}_i)}. \tag{7}$$

The result in (6) provides a valuable interpretation of the notion of channel capacity since the argument of the $\log_2(\cdot)$ operator is indeed a sum of likelihood ratios. In the sequel, the constellation-constrained capacity in (6) will be analyzed for coherent and non-coherent UWB receivers and closed-form upper bounds will be provided to avoid numerical evaluation.

4. CLOSED-FORM UPPER BOUNDING

4.1. Coherent Receivers

Since channel state information is available in coherent receivers, the likelihood ratio in (6) must be evaluated with the probability density function in (3). The result is found to be given by

$$\Lambda_{j,i}(\mathbf{y},\mathbf{g}) = \exp\left(\frac{1}{2\sigma_w^2}\left[2(\mathbf{h}_j-\mathbf{h}_i)^T\mathbf{y} + \mathbf{h}_i^T\mathbf{h}_i - \mathbf{h}_j^T\mathbf{h}_j\right]\right). \tag{8}$$

with $\mathrm{E}\left[\mathbf{w}\mathbf{w}^T\right] = \sigma_w^2\mathbf{I}$ and σ_w^2 the noise power. According to (8), the likelihood ratio for coherent detection depends on the particular realization of the end-to-end channel response $\mathbf{g}$ contained within $\mathbf{h}_i$. However, the channel response may vary between different transmissions and so does the capacity. Therefore, a meaningful measure for the channel capacity requires the expectation over the channel statistics. This leads to the so-called *ergodic* capacity defined as $\mathsf{C_{c\,|\,coh}} \doteq \mathrm{E}_{\mathbf{g}}[\mathsf{C_c}]$. In that case,

$$\mathsf{C_{c\,|\,coh}} = \log_2 P - \frac{1}{P}\sum_{i=0}^{P-1}\mathrm{E}_{\mathbf{g},\mathbf{w}}\left[\log_2\sum_{j=0}^{P-1}\exp\left(-\frac{\|\mathbf{h}_i-\mathbf{h}_j\|^2}{2\sigma_w^2}\right)\right.$$
$$\left.\times \quad \exp\left(\frac{1}{\sigma_w^2}(\mathbf{h}_i-\mathbf{h}_j)^T\mathbf{w}\right)\right] \tag{9}$$

where $\mathbf{y}$ has been substituted by the signal model $\mathbf{y} = \mathbf{h}_i + \mathbf{w}$ to be consistent with the conditioned expectation $\mathrm{E}_{\mathbf{y}|\mathbf{x}_i}[\cdot]$ in (6).

The main problem with the result in (9) is that an exact and closed-form expression is hard to find. The main difficulty is due to the discrete nature of the input alphabet which makes the argument of the $\log_2(\cdot)$ to consist on the sum of exponential terms. The problem of finding a simple expression for the logarithm of a sum of exponentials is a recurrent problem, for instance, in the field of turbo decoding. In particular, the so-called *max-log MAP* algorithm for turbo decoding is based on the approximation $\log\left(\sum_i \exp z_i\right) \approx \max_i z_i$. However, the $\max$ operator is still a nonlinear operator which does not help in providing a closed-form expression for (9).

Interestingly, a closed-form upper bound for the constellation-constrained capacity in (9) can be obtained when orthogonal PPM signaling is considered. Orthogonal signaling can directly be obtained by properly designing the transmitted signal such that non-overlapping time intervals are assigned to different PPM symbols. However, even when there exists some overlapping, the noise-like structure of the received waveforms makes the cross-correlation between different PPM hypothesis to be almost negligible.

With orthogonal signaling, the Euclidean distance in (9) turns out to be given by $\|\mathbf{h}_i - \mathbf{h}_j\|^2 = 2E_s$ for $i \neq j$, with $E_s \doteq \|\mathbf{g}\|^2$ the energy of the received waveform or, equivalently, the energy-per-symbol[2]. Then, the constellation-constrained capacity in (9) can be simplified as follows,

$$\begin{aligned}\mathsf{C}_{\mathsf{c}\,|\,\mathrm{coh}} &= \log_2 P - \frac{1}{P}\sum_{i=0}^{P-1} \mathrm{E}_{\mathbf{g},\mathbf{w}}\Bigg[\log_2\Bigg(1+\exp(-\rho) \\ &\times \exp\left(\frac{1}{\sigma_w^2}\mathbf{h}_i^T\mathbf{w}\right)\sum_{j\neq i}\exp\left(-\frac{1}{\sigma_w^2}\mathbf{h}_j^T\mathbf{w}\right)\Bigg)\Bigg] \quad (10)\end{aligned}$$

where the symbol-SNR ρ is defined as $\rho \doteq \frac{E_s}{\sigma_w^2} = 2\frac{E_s}{N_0}$ with $S_w(f) = \frac{N_0}{2}$ the double-sided noise spectral density. Next, by considering the law of large numbers in (10),

$$\sum_{j\neq i}\exp\left(-\frac{1}{\sigma_w^2}\mathbf{h}_j^T\mathbf{w}\right) \rightarrow (P-1)\mathrm{E}_{\mathbf{w}}\left[\exp\left(-\frac{1}{\sigma_w^2}\mathbf{h}_j^T\mathbf{w}\right)\right]. \quad (11)$$

The assumption above can be reasonably adopted provided that P is sufficiently large and taking into consideration that the product $\mathbf{h}_j^T\mathbf{w}$ does not vary significantly for different j. After some mathematical manipulations, the required expectation is found to be given by

$$\mathrm{E}_{\mathbf{w}}\left[\exp\left(-\frac{1}{\sigma_w^2}\mathbf{h}_j^T\mathbf{w}\right)\right] = \exp\left(\frac{1}{2\sigma_w^2}\|\mathbf{h}_j\|^2\right) = \exp\left(\frac{\rho}{2}\right). \quad (12)$$

Note that the result in (12) is indeed an exact result. No approximations were made at this point. Substituting the result in (12) into the constellation-constrained capacity in (10) results in

$$\begin{aligned}\mathsf{C}_{\mathsf{c}\,|\,\mathrm{coh}} &= \log_2 P - \frac{1}{P}\sum_{i=0}^{P-1}\mathrm{E}_{\mathbf{g},\mathbf{w}}\Bigg[\log_2\Bigg(1+(P-1)\exp\left(-\frac{\rho}{2}\right) \\ &\times \exp\left(\frac{1}{\sigma_w^2}\mathbf{h}_i^T\mathbf{w}\right)\Bigg)\Bigg]. \quad (13)\end{aligned}$$

The expression in (13) still requires numerical evaluation. However, a simple and closed-form expression can be obtained by introducing the Jensen's inequality. To this end, let us define the function

$$g(\mathbf{h}_i,\mathbf{w}) \doteq \log_2\left(1+(P-1)\exp\left(-\frac{\rho}{2}\right)\exp\left(\frac{1}{\sigma_w^2}\mathbf{h}_i^T\mathbf{w}\right)\right). \quad (14)$$

The function $g(\mathbf{h}_i,\mathbf{w})$ is a convex $\cup$ function. Consequently, the Jensen's inequality results in

$$\mathrm{E}_{\mathbf{w}}[g(\mathbf{h}_i,\mathbf{w})] \geq g(\mathbf{h}_i,\mathrm{E}_{\mathbf{w}}[\mathbf{w}]) = \log_2\left(1+(P-1)\exp\left(-\frac{\rho}{2}\right)\right)$$

because of the zero mean of the Gaussian noise, $\mathrm{E}_{\mathbf{w}}[\mathbf{w}] = \mathbf{0}$. Finally, substitution of the Jensen's inequality results in the following closed-form upper-bound for the constellation-constrained capacity of orthogonal PPM signaling.

$$\mathsf{C}_{\mathsf{c}\,|\,\mathrm{coh}} \leq \log_2 P - \log_2\left(1+(P-1)\exp\left(-\frac{\rho}{2}\right)\right). \quad (15)$$

4.2. Non-Coherent Receivers

For the case of non-coherent receivers, the end-to-end channel response is now assumed to be a random Gaussian process with covariance matrix $\mathbf{C}_{\mathbf{g}} = \mathrm{E}\left[\mathbf{g}\mathbf{g}^T\right]$. When the pulse-position modulation comes into action, the received waveform $\mathbf{g}$ creates a set of time-shifted replicas $\{\mathbf{h}_0,\mathbf{h}_1,\ldots,\mathbf{h}_{P-1}\}$ as indicated in the signal model in Section 2. These received waveforms $\mathbf{h}_k$ for $k=0,1,\ldots,P-1$ are characterized by the multivariate Gaussian probability density function in (4) so that the likelihood ratio $\Lambda_{j,i}(\mathbf{y})$ becomes,

$$\Lambda_{j,i}(\mathbf{y}) = \frac{\det^{1/2}(\mathbf{C}_w+\mathbf{C}_{\mathbf{h}_i})}{\det^{1/2}(\mathbf{C}_w+\mathbf{C}_{\mathbf{h}_j})}\frac{\exp\left(-\frac{1}{2}\mathbf{y}^T(\mathbf{C}_w+\mathbf{C}_{\mathbf{h}_j})^{-1}\mathbf{y}\right)}{\exp\left(-\frac{1}{2}\mathbf{y}(\mathbf{C}_w+\mathbf{C}_{\mathbf{h}_i})^{-1}\mathbf{y}\right)} \quad (16)$$

Since white Gaussian noise is being considered, the expression above can be simplified because the determinants on the form $\det(\mathbf{C}_w+\mathbf{C}_{\mathbf{h}_i})$ turn out to be independent of i. This statement can easily be proved by taking into consideration the properties of the determinant of block partitioned matrices. Then,

$$\Lambda_{j,i}(\mathbf{y}) = \exp\left(\frac{1}{2}\mathbf{y}^T\left[(\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_i})^{-1} - (\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_j})^{-1}\right]\mathbf{y}\right). \quad (17)$$

With the above considerations, the constellation-constrained capacity can be upper bounded as follows,

$$\begin{aligned}&\mathsf{C}_{\mathsf{c}\,|\,\mathrm{no-coh}} = \\ &\log_2 P - \frac{1}{P}\sum_{i=0}^{P-1}\mathrm{E}_{\mathbf{y}|\mathbf{x}_i}\Bigg[\log_2\sum_{j=0}^{P-1}\exp\Bigg(\frac{1}{2}\mathbf{y}^T\Big[(\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_i})^{-1} \\ &- (\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_j})^{-1}\Big]\mathbf{y}\Bigg)\Bigg] \quad (18) \\ &\leq \log_2 P - \frac{1}{P}\sum_{i=0}^{P-1}\log_2\sum_{j=0}^{P-1}\exp\Bigg(\frac{1}{2}Tr\Big(\Big[(\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_i})^{-1} \\ &- (\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_j})^{-1}\Big](\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_i})\Big)\Bigg). \quad (19)\end{aligned}$$

In (19), the Jensen's inequality was applied over the random matrix $\mathbf{y}\mathbf{y}^T$, that is,

$$\begin{aligned}\mathrm{E}_{\mathbf{y}|\mathbf{x}_i}\left[\mathsf{C}_{\mathsf{c}\,|\,\mathrm{no-coh}}\left(\mathbf{y}\mathbf{y}^T\right)\right] &\leq \mathsf{C}_{\mathsf{c}\,|\,\mathrm{no-coh}}\left(\mathrm{E}_{\mathbf{y}|\mathbf{x}_i}\left[\mathbf{y}\mathbf{y}^T\right]\right) \\ &= \mathsf{C}_{\mathsf{c}\,|\,\mathrm{no-coh}}\left(\sigma_w^2\mathbf{I}+\mathbf{C}_{\mathbf{h}_i}\right). \quad (20)\end{aligned}$$

To be more specific, it is interesting to particularize the result in (19) to the case of UWB received waveforms with uncorrelated scattering (US). These waveforms are obtained when adopting most of the channel models in the IEEE802.15.3a/4 standards. The advantage of US is that covariance matrices $\mathbf{C}_{\mathbf{h}_i}$ turn out to be diagonal. Thus, significant simplifications can be incorporated in (19). After some straightforward manipulations, the upper bound for the constellation-constrained capacity of non-coherent receivers with uncorrelated received samples is given by

$$\begin{aligned}C_{c\,|\,\mathrm{no-coh}}^{\mathrm{US}} &\leq \log_2 P \quad (21) \\ &- \frac{1}{P}\sum_{i=0}^{P-1}\log_2\sum_{j=0}^{P-1}\exp\left(-\frac{1}{2}\sum_{k=0}^{N_{ss}-1}\frac{\gamma_i(k)-\gamma_j(k)}{\sigma_w^2+\gamma_j(k)}\right)\end{aligned}$$

with $\gamma_i(k) = [\mathbf{C}_{\mathbf{h}_i}]_{k,k}$ the k-th entry of the power delay profile (PDP) of the received waveform under the hypothesis $\mathcal{H}_i : \mathbf{x} = \mathbf{x}_i$.

[2] For simplicity, frame repetition is not considered in this study.

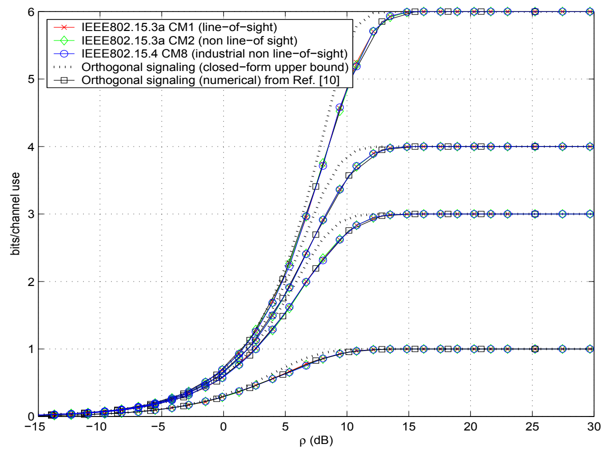

Fig. 1. Constellation-constrained capacity for UWB PPM coherent receivers.

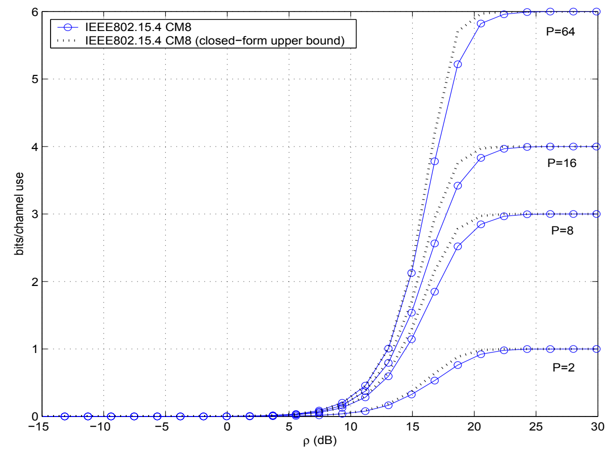

Fig. 2. Constellation-constrained capacity for UWB PPM non-coherent receivers.

5. NUMERICAL RESULTS

In this section, the proposed closed-form upper bounds are compared with the exact constellation-constrained capacity for both coherent and non-coherent receivers. The exact constellation-constrained capacity in (9) and (18) is numerically evaluated with UWB signals propagating through some of the IEEE 802.15.3a/4 channel models in [8] and [9], respectively. For non-coherent receivers, only the channel model CM8 from the IEEE 802.15.4 is considered. The reason is that it assumes an industrial environment with NLOS propagation where the small-scale fading statistics are found to be modeled by the traditional Rayleigh distribution. Thus, the received samples can still be modeled by random Gaussian variables as in (4).

The results are shown in Fig. 1 and Fig. 2 for the case of coherent and non-coherent receivers, respectively, with $P = \{2, 8, 16, 64\}$. For both simulation scenarios, the sampling time is set to $T_s = 0.5$ ns, the symbol duration is $T = 1.5$ μs and the PPM time-shift is $T_\Delta = 15$ ns. The delay spread of the simulated channel models is larger than the PPM time-shift so that some degree of overlapping is experienced. However, the randomness introduced by the propagation channel makes the cross-correlation between time-shifted waveforms to be almost negligible. To proof this fact, the exact capacity for orthogonal PPM in coherent receivers is also plotted in Fig. 1. This capacity was derived in [10] and requires numerical evaluation. When comparing the capacity for the simulated channel models with the exact capacity for orthogonal signaling in [10], both results do coincide. Therefore, we are virtually dealing with an orthogonal PPM signaling despite of the waveform overlapping.

From the observation of Fig. 1 and Fig. 2, it is found that the proposed closed-form upper bounds provide a really tight fit for the low-SNR regime. This is especially important, since this is the region where most UWB communication systems are forced to operate due to stringent spectral regulations. For the medium- to high-SNR, a good match is observed with just an slight deviation when entering the capacity saturation region.

6. CONCLUSIONS

Closed-form upper bounds have been presented for the constellation-constrained capacity of UWB communication systems operating with PPM modulation. Both coherent and non-coherent receivers are considered and simulation results are provided to show the tightness of the proposed bounds. Interestingly, an excellent match to the exact capacity is achieved for the low-SNR regime, the region where most UWB communication systems are forced to operate because of spectral regulations.

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