COMPRESSED SENSING FOR ULTRAWIDEBAND IMPULSE RADIO

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ABSTRACT

In this paper, ultrawideband (UWB) channel estimation based on the novel theory of Compressive Sensing (CS) is developed. The proposed approach relies on the fact that transmitting an ultra-short UWB pulse through a multipath channel leads to a received UWB signal that can be approximated by a linear combination of a few atoms from a pre-defined dictionary, yielding thus a sparse representation of the received signal. The CS reconstruction capabilities are exploited to recover the composite pulse-multipath channel from a reduced set of random projections using the Matching Pursuit algorithm. This reconstructed signal is subsequently used as a referent template in a correlator based detector. Extensive simulations show that for different propagation scenarios and UWB communication environments, the CS detector outperforms traditional correlators using just 1/3 of the sampling rate leading thus to a reduced use of analog-to-digital resources in the channel estimation stage.

Index Terms— Ultrawideband communications, Signal detection, Signal reconstruction, Compressive Sensing, Channel Estimation.

1. INTRODUCTION

Ultrawideband (UWB) communications has emerged as a promising technology for wireless communications systems that require high bandwidth, low-power consumption and shared spectrum resources [1]. In Impulse Radio UWB communications, an ultra-short duration pulse, typically on the order of nanoseconds, is used as the elementary pulse-shaping to carry information, achieving thus simplicity in the transmitter, signal power spreading, and rich multipath diversity. UWB receivers, however, face several challenges including interference cancellation, antenna design, timing synchronization, and channel estimation, among others [1, 2]. Digital UWB receiver architectures have been proposed in [3] as an alternative to implement UWB receivers since digital detectors offer considerable flexibility and technology scaling benefits. However, the extremely high bandwidth of the received UWB signal (up to 7.5 GHz) requires high-speed analog-to-digital converters (ADCs). Furthermore, oversampling of the received UWB signal may be required to improve timing synchronization and channel estimation. For instance, in [4] the required sampling rate is in excess of 25 GHz for accurate UWB channel estimation. Such formidable sampling rates are not feasible with state-of-the-art ADC technology. New approaches for UWB receivers are needed to attain the required sampling rates and bit resolution.

This paper focuses on this goal by casting the problem of UWB channel estimation and detection into the emerging framework of compressive sensing [5, 6]. CS is a new concept based on the theoretical results of signal reconstruction with random basis coefficients. The remarkable result of CS reveals that with high probability, a signal, f, with a large number of data points that is M-sparse[1] in some dictionary Ψ of basis functions or tight-frames, can be exactly reconstructed using only a few number of random projections of the signal onto a random basis Φ that is incoherent with Ψ. The number of projections, in general, is much smaller than the number of samples in the original signal leading to a reduced sampling rate and, hence, to a reduced use of ADCs resources [6].

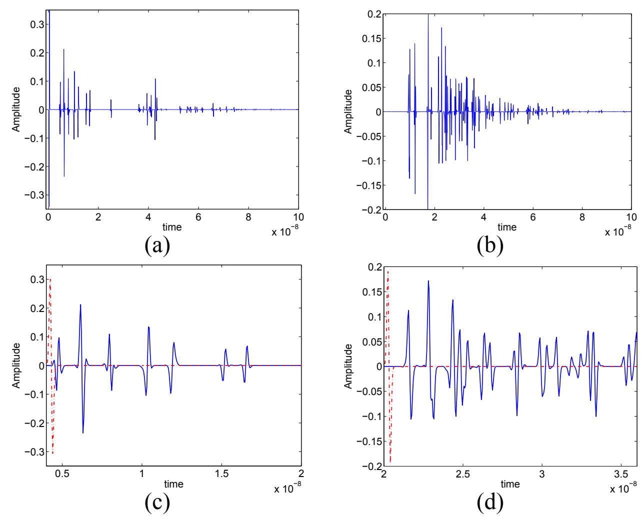

Fig. 1. Effect of UWB channel (indoor residential environment) on the transmitted pulse for two different propagation scenarios (a) LOS; (b) NLOS; (c) Zoom-in of (a); (d) Zoom-in of (b); Transmitted pulse $(- \cdot -)$ is also shown in (c) and (d).

We begin with the basic assumption that when the short duration (high frequency) pulses used in UWB communications propagate through multipath channels, the received signals remain sparse in some domain and thus compressed sensing is indeed applicable. To illustrate this concept, consider a Gaussian monocycle as the information carrier UWB pulse having a duration of 0.65 ns. Furthermore, consider the pulse propagating through two different noiseless propagation scenarios. Figure 1(a) shows the received signal per frame for an UWB channel that models an indoor residential environment with line-of-sight (LOS), while Fig. 1(b) shows the received signal per frame for the same communication environment but with non-line-of-sight (NLOS) [7]. The time observation window is 100 ns, that is a typical frame time for UWB systems [1].

Upon closer examination of these figures, it can be seen rela-

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[1]By M-sparse, a signal f can be written as a sum of M known basis functions, i.e. $f = \sum_{i=1}^{M} \alpha_i \psi_i$

tively long time intervals between clusters and rays where the signal takes on zero or negligible values. It is precisely this signal sparsity of the received UWB signal that is exploited in this work aimed at UWB channel estimation and detection.

In this paper, we show that the CS framework can indeed be used for the processing of UWB signals. We not only show that the received UWB signals can be reconstructed from a set of random projections, leading to a reduced sampling rate but also that the CS framework can be extended to other statistical inference tasks suitable in UWB communications such as symbol detection. Thus, the reconstruction capability of CS using the Matching Pursuit (MP) algorithm [8] is exploited to estimate the composite pulse-multipath channel $(p(t) * h(t))$ by random projecting a set of received pilot waveforms for training. This reconstructed signal is subsequently used as a template for correlator based detector that demodulates the transmitted information symbols.

2. ULTRAWIDEBAND COMPRESSIVE SENSING

2.1. Compressive Sensing Overview

Suppose f is the N-point discrete-time representation of an analog signal of interest. Also, suppose a set of K measurements, y, are acquired that are a linear combinations of the points in f. More precisely, $y = \Phi f$, where Φ is a $K \times N$ matrix, hereafter called measurement matrix. For $K << N$, it can be shown that if f is sparse then f can be recovered from y, with high probability as long as the measurement matrix Φ is incoherent with the dictionary Ψ.

The vector $\Theta = [\theta_1, \theta_2, \ldots, \theta_Z]^\dagger$ is a vector that contains M nonzeros coefficients. The index of the non-zero coefficient defines which element in the dictionary composes the signal and the coefficient value the contribution of that element in defining the signal f. The signal f can be recovered from the solution of a convex, nonquadratic optimization problem. Formally, with very high probability, Θ is the unique solution to

$$\min \|\Theta\|_1 \quad \text{subject to} \quad y = \Phi\Psi f. \tag{1}$$

Note that the only *a priory* knowledge required is that f is sparse in some dictionary of parameterized waveforms, called atoms, and may contain Fourier basis, Wavelet basis, cosine packets, chirplets basis, Gabor functions, or even a combination of basis and tight-frame [9]. In general, the structure of the signal of interest leads to the definition of the dictionary.

Solving the optimization problem in (1) is computationally expensive and is not suitable for real-time applications. Faster and more efficient reconstruction algorithms exist that use iterative greedy-based algorithms, at the expense of slightly more measurements. In particular, MP is a computationally simple iterative greedy algorithm that tries to recover the signal by finding, in the measurement signal, the strongest component, removing it from the signal, and searching again the dictionary for the strongest atom that is presented in the residual signal.

2.2. Processing UWB Signals Using CS

Consider the simple communications model of transmitting a pulse $p(t)$ throughout a noiseless UWB communication channel $h(t)$. The received UWB signal can be modeled as

$$g(t) = p(t) * h(t) = \sum_{\ell=0}^{L-1} \alpha_\ell p(t - \tau_\ell) \tag{2}$$

where $p(t)$ is the ultra-short pulse used to convey information with a time duration in the order of nanoseconds. Typically, a Gaussian pulse or its derivatives are used as the UWB pulses.

In Eq. (2), $h(t)$ is the impulse response of the UWB channel where τ_ℓ and α_ℓ are, respectively, the delay and gain associated with the ℓ-th path of the UWB channel and L is the number of propagation paths. In our analysis, the set of delays and gains are generated according to the models proposed by the IEEE 802.15.4a working group in [7] for different communication environments and different propagation scenarios. Without loss of generality, we restrict our analysis to real-valued UWB channel models where there is not pulse distortion. This restriction, however, does not represent a limitation on the proposed approach since it can be readily extended to include pulse distortion.

2.2.1. UWB signal reconstruction using time sparsity model

A first approach to reconstructing the composite pulse-multipath channel from a set of random projections assumes that the signal is sparse in the time domain. This signal model is adequate for the UWB channel provided that there are only a few propagation paths as is the case of UWB channels in industrial environments with LOS propagation [7].

Let $\mathbf{g}$ be a discrete-time representation of the composite pulse-multipath channel. Define the measurement matrix, Φ, as $K \times N$ random matrix with entries i.i.d. taken from a normal distribution with zero-mean and unit variance. Since we are assuming sparsity in the time domain, the dictionary Ψ reduces to the identity matrix. Running the MP algorithm yields the results show in Fig. 2.

Figure 2(a) shows the 2048-point composite pulse-multipath channel for a realization of an indoor residential channel with LOS propagation obtained from [7]. This is the signal targeted for reconstruction from a reduced set of random projections. Figure 2(b) depicts the reconstructed signal obtained using 500 random measurements with this time sparsity model. Note that although CS is able to recover the most significant values of the underlying signal, it fails to recover many of the signal details yielding, in general, a poor performance. Note also that several spurious components are introduced in the reconstructed signal leading to a cumulative square error of 0.9276.

To improve the reconstruction performance, one may be tempted to increase the number of random projections since a large number of random projections increases the probability of exact reconstruction [10, 5, 6], this, however, leads to a higher sampling rate, and, therefore more demanding ADC resources. A more appealing approach, described next, is to design a dictionary of parameterized waveforms where the received UWB signal can be compactly represented, increasing thus the sparsity of the underlying signal.

2.2.2. UWB signal reconstruction using multipath diversity

UWB channels in general are rich in multipath diversity motivating the construction of basis functions offering higher energy compaction, sparseness, and higher probability for CS reconstruction. Since CS theory relies on the fact that the underlying signal is sparse in some dictionary of basis or tight-frames, it is important to define a suitable dictionary to represent the underlying UWB signal. We can explore a great variety of dictionaries that have been defined in the context of atomic decomposition to find the best basis that match our problem [9]. Alternatively, we can generate a new dictionary just inspecting the characteristic of the received UWB waveform.

Since the received UWB signal is formed by scaled and delayed versions of the transmitted pulse and since the dictionary should contain elements (atoms) that can fully represent the signal of interest, it is natural to think that the elementary function to generate the atoms of the dictionary should be the pulse waveform used to covey information. This leads to a set of parameterized waveforms given by

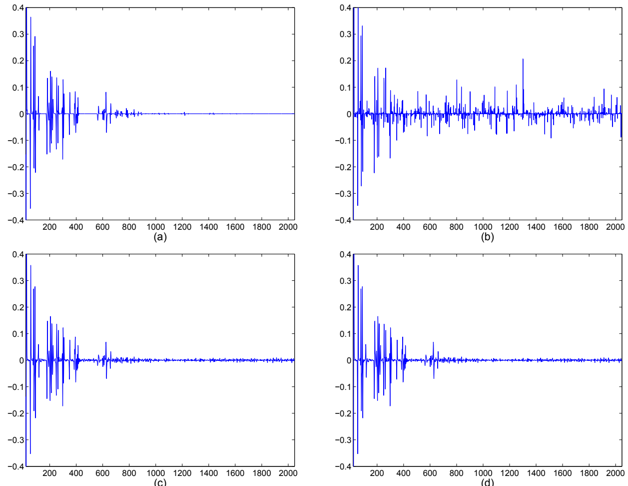

Fig. 2. (a) Received UWB signal; (b) CS reconstruction using time-sparsity model, $K = 500$. CS reconstruction using multipath diversity with: (c) $K = 500$ and (d) $K = 250$.

$$d_j(t) = p(t-j\Delta) = p_n(t-j\Delta)e^{-\frac{(t-j\Delta)^2}{2\sigma^2}} \qquad j = 0, 1, 2, \ldots \quad (3)$$

that define the dictionary $\mathcal{D} = \{d_0(t), d_1(t), d_2(t), \ldots, \}$ where Δ is a shifting parameter. The atoms in the dictionary are thus delayed versions of the UWB transmitted pulse. Note that a more general dictionary could be constructed by time-shifting, scaling and modulating the basic pulse shape taking into account the pulse distortion that may be produced by the UWB channel.

Having defined a suitable dictionary that matches the UWB signal, we next return to the reconstruction problem. Consider the composite pulse-multipath channel given by Eq. (2) that has been sampled to define the discrete-time vector $\mathbf{g}$. Furthermore, let $\mathbf{y} = \Phi\mathbf{g}$ be the random projected signal where Φ is the $K \times N$ measurement matrix with $\phi_{i,j} \sim N(0, 1)$. The MP algorithm is then applied on the random projected signal, $\mathbf{y}$, and the dictionary Ψ, where Ψ is the discrete-time dictionary defined by uniformly sampling the atoms of the dictionary $\mathcal{D}$.

Figures 2(c) and 2(d) show the reconstructed signal using 500 and 250 random measurements, respectively. CS successfully recovers the desired signal from random projections yielding cumulative square errors of 0.0262 and 0.1110 using just 500 and 250 measurements, respectively. Note also that UWB signal reconstruction using multipath diversity outperforms UWB signal reconstruction using time sparsity model yielding a reconstruction error that is 35-fold smaller. Therefore, by building a dictionary that is closely matched to the underlying waveform, a notable performance gain is achieved in the reconstruction.

Note that by sampling the random projected signal at a notably reduced sampling rate (1/8 of the original samples), it is possible to reconstruct the unprojected signal with a very small cumulative reconstruction error. Note also that the signal prior to the projection stage does not have to be a discrete-time signal, since the random projections can be performed in the analogous domain by a bank of synchronized high speed analog mixers with PAM waveform random generators followed by low-rate sampling.

Figure 3 depicts the probability of successful reconstruction as a function of the number of measurements (averaged over 1000 trails) for two different propagation scenarios (LOS and NLOS) in an indoor residential environment[2]. As expected, the NLOS propagation

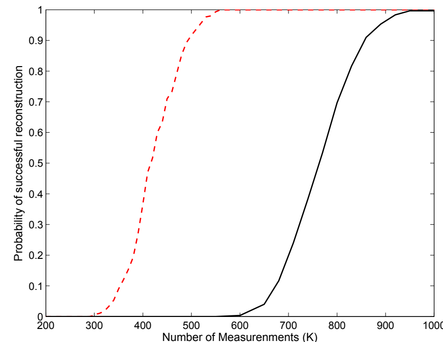

Fig. 3. Probability of reconstruction for LOS (- - -) and NLOS (—).

scenario requires more random measurements than that required by the LOS propagation scenario since NLOS channels are more dispersive and thus, have more multipath components than LOS channels, consequently the NLOS received signal is less sparse than LOS received signal demanding more measurements.

3. COMPRESSIVE SENSING UWB DETECTION

The CS theory can be further extended to address the detection problem under the framework of data-aided channel estimation followed by symbol demodulation [1, 11, 4]. In this framework, a set of training symbols, also known as pilot signals, are used to estimate a referent template for subsequent correlation detection [1, 11]. In this light of work, we use the CS framework for template reconstruction leading naturally to a new method for signal detection.

Consider a peer-to-peer UWB communication system as in [11] where the k-th binary information symbol is transmitted by sending N_f ultra-short pulses in the symbol interval T_s, that is $s(t) = \sum_k b(k) \sum_{j=0}^{N_f-1} p(t - jT_f - kT_s)$ where $T_f = T_s/N_f$ is the frame time, and $b(k) \in \{-1, 1\}$. T_p is the pulse duration with $T_p << T_f$. Hence, N_f non-overlapped pulses are transmitted for each information symbol.

Following [4, 11], consider that the channel is static during a burst of N_s consecutive symbols. Furthermore, let $T_f \geq \tau_{L-1} + T_p$, where τ_{L-1} is the maximum delay spread of the multipath channel. The received waveform is as follows

$$r(t) = \begin{cases} \sum_{k=0}^{N_w-1} b_p(\lfloor k/N_f \rfloor) \sum_{l=1}^{L} \alpha_l p(t - kT_f - \tau_l) + \eta(t) \\ \text{for} \quad 0 < t \leq T_w \\ \sum_{k=N_w}^{(N-N_p)N_f-1} b_i(\lfloor k/N_f \rfloor) \sum_{l=1}^{L} \alpha_l p(t - kT_f - T_w - \tau_l) + \eta(t) \\ \text{for} \quad T_w < t \leq N_s N_f T_f \end{cases} \quad (4)$$

where $b_p(\cdot)$ and $b_i(\cdot)$ are the pilot and the information symbols, respectively, N_w is the total number of pilot waveforms ($N_w = N_p N_f$) and T_w is the time duration of the pilot waveforms. $\eta(t)$ is a zero-mean additive white Gaussian (AWG) noise that models thermal noise and other interferences [4].

The optimal template for demodulation is the composite pulse-multipath channel, $p(t) * h(t) = \sum_{\ell=1}^{L} \alpha_\ell p(t - \tau_\ell)$ [1, 11]. Thus, the receiver performs frame-rate sampling on the correlator output to generate sufficient statistics for the detection of the transmitted information symbol [1, 11].

Unlike the traditional approach where $p(t)*h(t)$ is estimated using analog-delay units that delay and average frame-long segments [11], we use N_p known pilots symbols in each packet to estimate the

[2] A signal is considered to be successfully reconstructed if the reconstruction error is less than 1% of the signal's energy, i.e. $||g - \hat{g}||_2^2 < 0.01||g||_2^2$.

composite pulse-multipath channel. Therefore, by observing the received UWB signal in a frame-long interval and random projecting the observed signal, a noisy template can be recovered using MP algorithm. To mitigate the effect of the additive noise, the random projected signals corresponding to the received pilot waveforms are averaged and input to the MP algorithm for template reconstruction.

Thus, CS template reconstruction is achieved by random projecting the frame-long received signals, ensemble averaging the random projected signals, and using MP algorithm to recover an estimate of the composite pulse-multipath channel.

Note that in the reconstruction of the optimal template using MP algorithm, a denoising operation is implicitly performed. To be more precise, by building a dictionary that is closely matched to the transmitted signal, we expect that the dictionary will initially be most correlated with the underlying transmitted pulse than to the noise. Furthermore, since the reconstructed template is a linear combination of the atoms in the dictionary, it does not contain any noise components. However, other type of errors may appear in the reconstructed template coming from spurious atoms that may have been wrongly identified in the received pilot signal.

To overcome this limitation, the temporal correlation between consecutive received pilot waveforms can be exploited by increasing the observation window to project more than one received pilot waveforms at the same time. MP algorithm is then suitably adapted such that the observed signal is compared to each elements in the dictionary that is periodically repeated several times to match the signal length. Thus, an atom is found in the observed signal if its signal contribution appears periodically each T_f seconds adding thus robustness to the signal reconstruction.

Once the template has been estimated, it can be used as correlator template to enable integrate-and-dump demodulation at frame-rate sampling. Since each symbol is present in N_f frames, the decision statistics for the k-th symbol is formed by adding up the N_f correlator output samples related to the transmitted symbol.

4. SIMULATION RESULTS

In this section, numerical results are presented showing the potential of CS for UWB signal detection. The performance of the proposed CS based detectors are compared to that of the traditional correlator based detectors used in [11]. We use the average over 200 different realizations for each UWB channel model to compute the bit error rate (BER) at the receiver as a function of signal-to-noise ratio (SNR) as a performance criterion. We select the first derivative of the Gaussian pulse as the transmitted pulse waveform, $p(t)$, that has been normalized to have unit energy and a pulse duration of 0.650 ns. For the sake of simplicity in our simulation, the negligible taps at the tail of the multipath impulse response are cut off to make the maximum delay spread of the multipath channel equal to 99.35 ns.

Figure 4 shows the BER performance of CS-correlator for different number of random projections. The UWB communication channel is modelled as an indoor residential environment [7]. As expected, the CS-correlator's performance improves as the number of projections increases. Furthermore, the reconstructed template using CS framework, is more reliable for symbol detection than the one obtained by averaging the received pilot signal. This performance is expected since a denoising operation is inherently applied on the recovered signal yielding a template that is a linear combination of the transmitted pulses. More interesting, by sampling the random projected signal at 30% of the signal's sampling rate, CS-Correlator achieves the same performance than that yielded by the traditional correlator. Thus, with a reduced ADC resources, CS framework is able to reconstruct a template as good as the one obtained sampling

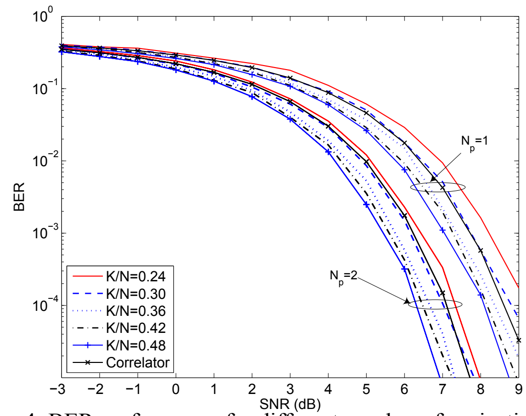

Fig. 4. BER performance for different number of projections.

the received UWB signal at much higher sampling rate.

5. CONCLUSIONS

In this paper, we have shown that a reduced number of random projections of the received UWB signal contains most of the relevant information useful not only for UWB signal reconstruction but also for UWB symbol detection. We also show that by CS reconstructing the composite pulse-multipath channel from the set of random projections a denoising operation is implicitly applied yielding a performance improvement on a correlator based detector that uses the reconstructed signal as a referent template. CS can be used in other UWB communications scenarios such as in transmitted reference (TR) systems. Such work will be reported in future publications.

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