ZIV-ZAKAI TIME DELAY ESTIMATION BOUND FOR ULTRA-WIDEBAND SIGNALS

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ABSTRACT

The Ziv-Zakai bound (ZZB) provides a general mean-square error analytical baseline to evaluate time delay estimation (TDE) techniques for a wide range of time-bandwidth products and signal-to-noise ratios, but generally can only be numerically evaluated. The Weiss-Weinstein bound (WWB) further improves characterization of the attainable system performance, for narrowband and wideband signals with small to moderate fractional bandwidth. Similar to the WWB, here we find a simplified closed-form ZZB for TDE with ultra-wideband (UWB) signals. The resulting simplified bound is found over disjoint segments, separated by thresholds that characterize different regions of ambiguity. The closed-form simplified bound can be analytically studied, and approaches both the ZZB and the TDE performance of a maximum likelihood estimator.

INDEX TERMS

Ultra-wideband, time delay estimation, Ziv-Zakai bound, Weiss-Weinstein bound.

1. INTRODUCTION

Time delay estimation (TDE) is fundamental to many applications, such as array processing, localization, and tracking. Time delay estimator performance is often characterized via mean square error (MSE), that can then be compared to analytical MSE lower bounds. The Cramér-Rao bound (CRB) predicts maximum likelihood estimator (MLE) performance for sufficient time bandwidth (TB) product and signal to noise ratio (SNR) [1]. However, the CRB is a local bound that fails to characterize performance when the ambiguity-free condition is violated, for example, when a cross-correlation TDE incorrectly selects neighboring correlation peaks. Tighter bounds for TDE have been developed, including the Chapman-Robbins (Barankin) bound [2], and the Ziv-Zakai bound (ZZB) [3]. Though much tighter than the corresponding CRB, they are not easily manipulated into simple closed form expressions, and consequently often require numerical evaluation. Further improvements have been made by Bellini & Tartara [4], and Weiss & Weinstein [5], [6]. Extension to the vector case has also been developed [7]. The improved ZZBs in [5], [6] are applicable to either narrowband or wideband waveforms because of explicit assumptions made throughout the derivations. This facilitates closed form expressions in some cases, or close approximations that have simple forms. TDE for frequency hopping signals is considered in [8], and an overview of TDE bounds is given in [9].

Ultra-wideband (UWB) signals have long been employed in radar, and more recently for communications and geolocation, e.g., in the 3.1GHz to 10.6GHz band in the US [10, 11]. Timing acquisition, synchronization, and delay estimation are important and challenging aspects of these systems, with sensitivity to jitter and signal mismatch [12, 13]. We consider TDE in the UWB regime, assuming large fractional bandwidth, and simplify the ZZB for this case. Similar to the developments in [5, 6] for the narrowband and wideband cases, we develop the ZZB for the UWB case, and find a piece-wise approximation in simple closed form that is amenable to analysis in applications. For the UWB case, the correlation exhibits a significantly different pattern than for signals with smaller fractional bandwidth, such that there is no clearly defined phase or envelope ambiguity corresponding to different correlation peaks. Consequently, we develop a new interpretation of ambiguity, that depends on the lower and upper signal frequencies. Similarly, we define new threshold points. The new ambiguities arise in segments dominated by the envelope, phase and their mixtures, that in turn depend on TB product and SNR. Finally, a simulation study shows that our simplified ZZB closely predicts MLE performance, and approaches the true ZZB (obtained numerically) in the different regions.

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2. REVIEW OF ZZB AND WWB

Consider estimation of delay τ between two sensors. Each waveform is corrupted by additive noise so that

$$\begin{aligned} r_1(t) &= s(t) + n_1(t), \\ r_2(t) &= s(t-\tau) + n_2(t) \quad -T/2 \le t \le T/2, \end{aligned} \tag{1}$$

where T is the observation time, and $s(t)$, $n_1(t)$ and $n_2(t)$ are the sample functions of the signal and Gaussian noise with spectral densities $S(\omega)$, $N_1(\omega)$, and $N_2(\omega)$, respectively. The ZZB development begins with a binary decision problem based on an arbitrary estimate $\widehat{\tau}$ of τ,

$$\begin{aligned} &\text{Decide } H_0 : \tau = a && \text{if } |\widehat{\tau} - a| < |\widehat{\tau} - a - \theta|, \\ &\text{Decide } H_1 : \tau = a + \theta && \text{if } |\widehat{\tau} - a| > |\widehat{\tau} - a - \theta|. \end{aligned} \tag{2}$$

Without loss of generality, the two hypothesized delays are assumed to be equally likely to occur, so the probability of error is $P\{\widehat{\tau} - a > \theta/2 | \tau = a\}/2 + P\{\widehat{\tau} - a - \theta < -\theta/2 | \tau = a + \theta\}/2$. Denote the error in estimating τ by $\widehat{\tau}$ as $\epsilon = \widehat{\tau} - \tau$.

We are interested in the mean square error $\overline{\epsilon^2} = E(\epsilon\epsilon^T)$ with τ uniformly distributed within the interval $[-D/2, D/2]$, and θ within $[0, D]$. The basic ZZB is given by [3]

$$\overline{\epsilon^2} \ge \frac{1}{D} \int_0^D \theta \, d\theta \int_{-D/2}^{D/2-\theta} P_e(a, a+\theta) \, da, \tag{3}$$

where $P_e(a, a+\theta)$ is the minimum attainable probability of error associated with the likelihood ratio test between the two hypothesized delays. It immediately follows that

$$P_e(a, a+\theta) \leq \frac{1}{2}\{P(\epsilon > \theta/2|\tau = a) + P(\epsilon < -\theta/2|\tau = a+\theta)\}. \tag{4}$$

Bellini and Tartara further noticed that $\int_{-D/2}^{D/2-\theta} P_e(a, a+\theta)da$ is a nonincreasing function of θ [4]. Therefore, by using a nonincreasing function $\mathcal{V}[\cdot]$ that fills the valleys of the function inside the bracket, an improved ZZB is given by

$$\overline{\epsilon^2} \geq \frac{1}{D}\int_0^D \theta\, \mathcal{V}\left[\int_{-D/2}^{D/2-\theta} P_e(a, a+\theta)\, da\right] d\theta. \tag{5}$$

If $P_e(a, a+\theta) = P_e(\theta)$ is independent of a, the improved ZZB may be simplified to the WWB [5]

$$\overline{\epsilon^2} \geq \frac{1}{D}\int_0^D \theta\, \mathcal{V}[(D-\theta)P_e(\theta)]\, d\theta. \tag{6}$$

Typically, the observation time T is large compared to the correlation time $2\pi/W$ where W is the bandwidth. Then, $P_e(\theta)$ is closely approximated by

$$P_e(\theta) \approx e^{a(\theta)+b(\theta)}\Phi(\sqrt{2b(\theta)}), \tag{7}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\mu^2/2}\, d\mu$,

$$a(\theta) = -\frac{T}{2\pi}\int_0^\infty \ln[1 + \mathrm{SNR}(\omega)\sin^2(\omega\theta/2)]\, d\omega, \tag{8}$$

$$b(\theta) = \frac{T}{2\pi}\int_0^\infty \frac{\mathrm{SNR}(\omega)\sin^2(\omega\theta/2)}{1 + \mathrm{SNR}(\omega)\sin^2(\omega\theta/2)}\, d\omega, \tag{9}$$

$$\mathrm{SNR}(\omega) = \frac{[S(\omega)/N_1(\omega)][S(\omega)/N_2(\omega)]}{1 + S(\omega)/N_1(\omega) + S(\omega)/N_2(\omega)}. \tag{10}$$

In the derivation of (7), large WT is assumed. This condition is satisfied in most practical scenarios. Although it is difficult to simplify (8) and (9) in general, [5] and [6] considered narrowband and wideband waveforms, and derived closed form expressions for different disjoint segments separated by thresholds. The corresponding simplified ZZBs are very valuable for comprehensive analytical study of bound performance under different system operating conditions when the fractional bandwidth of the waveform is small or moderately large (the "narrowband" and "wideband" cases). In the following, we develop a simplified and analytically tractable ZZB for the UWB case with large fractional bandwidth.

3. SIMPLIFIED ZZB FOR UWB SIGNALS

We consider source signals and noises that are spectrally flat over the frequency band $[\omega_0 - W/2, \omega_0 + W/2]$ with spectral densities S and N respectively, where ω_0 denotes the center frequency. Cases for signals transmitted through frequency selective multipath channels will be studied in the future. Then, the SNR defined in (10) is constant with respect to W. We define $\omega_H = \omega_0 + W/2$, $\omega_L = \omega_0 - W/2$, $\eta = WT \cdot \mathrm{SNR}$, and $\Lambda = \mathrm{SNR}\sin^2(\omega\theta/2)$ (this last term appeared in (8) and (9)).

The most common estimator of τ is to find the peak of the cross-correlation of $r_1(t)$ and $r_2(t)$. In the absence of noise, this reduces to the signal auto-correlation

$$\begin{aligned} R_{ss}(\widehat{\tau}) &= \frac{\omega_H}{\pi}\mathrm{sinc}(\omega_H\widehat{\tau}) - \frac{\omega_L}{\pi}\mathrm{sinc}(\omega_L\widehat{\tau}) \\ &= \frac{W}{\pi}\cos(\omega_0\widehat{\tau})\mathrm{sinc}(W\widehat{\tau}/2). \end{aligned} \tag{11}$$

Although the peak occurs at the true delay τ, there are many other local maxima $\widehat{\tau} = \tau + \theta$ that appear quasi-periodically. In the narrowband and wideband cases, typically $\omega_0 \gg W$. Then, according to the second equality of (11), phase and envelope ambiguities occur at points approximately uniformly distributed with period $2\pi/\omega_0$ and $2\pi/W$, respectively [6]. However, as W grows closer to ω_0, $\cos(\omega_0\widehat{\tau})$ cannot be perceived as a carrier signal and $\mathrm{sinc}(W\widehat{\tau}/2)$ as the envelope. However, the first equality of (11) shows that $\mathrm{sinc}(\omega_H\widehat{\tau})$ changes more rapidly than $\mathrm{sinc}(\omega_L\widehat{\tau})$ when $\omega_H \gg \omega_L$. This fact permits us to similarly introduce the concept of phase and envelope ambiguities for the UWB case. The phase ambiguity points are caused by $\omega_H\mathrm{sinc}(\omega_H\widehat{\tau})/\pi$ with period $2\pi/\omega_H$, and the envelope ambiguity points are caused by $\omega_L\mathrm{sinc}(\omega_L\widehat{\tau})/\pi$ with period $2\pi/\omega_L$. Fig. 1 demonstrates this transition behavior of $R_{ss}(\widehat{\tau})$ with $\omega_0 = 20W$, $\omega_0 = 5W$ and $\omega_0 = W$ respectively.

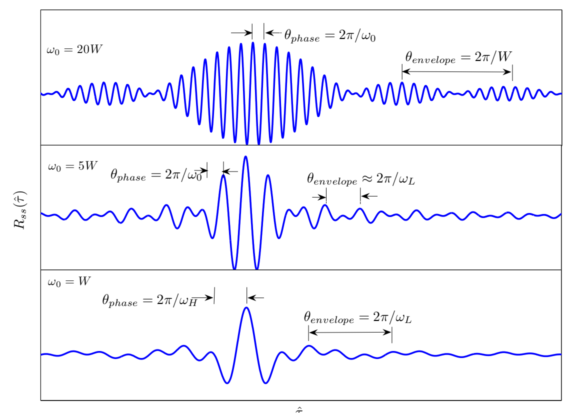

Fig. 1. $R_{ss}(\widehat{\tau})$ for narrowband, wideband, and UWB signals.

In order to obtain an analytically tractable expression for (6) suitable for UWB signals, we will partition the integration interval $[0, D]$ into sub-intervals separated by s_1, s_2 and s_3, defined below. These points will designate the effects of individual phase or envelope ambiguity, or their mixture. Then the valley-filling function $\mathcal{V}[(D-\theta)P_e(\theta)]$ will be closely bounded. Subsequently, an analytical expression for the integral of each sub-interval will be derived.

3.1. Simplification and Segmentation of $\mathcal{V}[(D-\theta)P_e(\theta)]$

As previously explained, the more critical ambiguity results from the phase of the autocorrelation function because of its oscillatory nature with period $2\pi/\omega_H$, and the secondary ambiguity is caused by the oscillatory nature of the envelope of the autocorrelation function with period $2\pi/\omega_L$. Therefore, for θ that is not very close to 0, a simplified bound on $\mathcal{V}[(D-\theta)P_e(\theta)]$ can be developed from the local maxima points (i.e., both the phase and the envelope ambiguity points) of $P_e(\theta)$, or equivalently the local maxima (ambiguity points) of the autocorrelation function. When $0 \leq \theta < s_1$, no ambiguity needs to be considered in the autocorrelation function, and this region is noise-dominated. When

$s_1 \le \theta < s_2$, the phase ambiguity dominates. When $s_2 \le \theta < s_3$, both the phase and envelope peaks of the autocorrelation function contribute to the ambiguity. Finally, in the $s_3 \le \theta < D$ region, the envelope ambiguity has the dominant effect. In the following we present the key results, and omit the derivations due to space limitations.

For the noise dominated region ($0 \le \theta < s_1$), the valley-filling function $\mathcal{V}[(D-\theta)P_e(\theta)]$ is bounded by

$$\mathcal{V}[(D-\theta)P_e(\theta)] \quad \ge \quad (D-\theta)e^{-d_1(\theta)}\Phi[\sqrt{c_1(\theta)}], \quad (12)$$

where

$$\begin{aligned} c_1(\theta) &= \theta^2 \cdot \frac{\eta}{48\pi}(12\omega_0^2 + W^2), \\ d_1(\theta) &= \theta^4 \cdot \frac{\eta \cdot \mathrm{SNR}}{5120\pi}(80\omega_0^4 + 40\omega_0^2 W^2 + W^4). \quad (13) \end{aligned}$$

In the phase ambiguity dominated region ($s_1 \le \theta < s_2$), the peaks of the autocorrelation function are nearly periodically distributed, and located approximately at $\widehat{\theta}_n = 2n\pi/\omega_H$($n = 1, 2, ...$). At each $\widehat{\theta}_{n-1} \le \theta \le \widehat{\theta}_n$, the valley-filling function can be closely bounded by

$$\begin{aligned} \mathcal{V}[(D-\theta)P_e(\theta)] \quad &\ge \quad (D - \frac{2\pi}{\omega_H} - \theta)e^{-d_2(\theta+2\pi/\omega_H)} \\ &\cdot \Phi[\sqrt{c_2(\theta+2\pi/\omega_H)}], \quad (14) \end{aligned}$$

where

$$\begin{aligned} c_2(\theta) &= \theta \cdot \frac{\omega_0 \eta}{2\pi\sqrt{1+2\mathrm{SNR}}}, \\ d_2(\theta) &= d_1(\theta) + \frac{\eta \mathrm{SNR}^2}{384\pi}\theta^6 \\ &\cdot(\omega_0^6 + \frac{5}{4}\omega_0^4 W^2 + \frac{3}{16}\omega_0^2 W^4 + \frac{W^6}{448}). \quad (15) \end{aligned}$$

In the region ($s_2 \le \theta < s_3$), where both phase and envelope ambiguities are dominant, the ambiguity points are the autocorrelation peaks not only caused by the highly oscillatory nature of the phase but also the nearby envelope. Suppose $\widetilde{\theta}_0$, $\widetilde{\theta}_1$, $\widetilde{\theta}_2$, ..., are the local maxima of $P_e(\theta)$, which are phase ambiguity points and are in the adjacent area of the envelope ambiguity points as well. Then, $2\pi/\omega_H \le \widetilde{\theta}_n - \widetilde{\theta}_{n-1} \le 2\pi/\omega_L$. The valley-filling function is strictly bounded as

$$\mathcal{V}[(D-\theta)P_e(\theta)] \ge (D - \frac{2\pi}{\omega_L} - \theta) \cdot e^{-d_3}\Phi(\sqrt{c_3}), \quad (16)$$

with

$$c_3 = \frac{2\omega_H T \cdot \mathrm{SNR}}{\pi^2\sqrt{1+2\mathrm{SNR}}}, d_3 = \frac{T}{2\pi^2}(\frac{3}{16}\omega_H \mathrm{SNR}^2 + \frac{5}{48}\omega_H \mathrm{SNR}^3). \quad (17)$$

Here, both c_3 and d_3 are independent of θ.

In the envelope ambiguity dominated region ($s_3 \le \theta < D$), $\theta_1, \theta_2, \ldots$, are the local maxima associated with the envelope ambiguity with $\theta_n = 2n\pi/\omega_L$. The valley-filling function can be closely bounded as

$$\mathcal{V}[(D-\theta)P_e(\theta)] \ge \begin{cases} (D - 2\pi/\omega_L - \theta)e^{-d_4}\Phi(\sqrt{c_4}) \\ \qquad\qquad (\theta \le D - 2\pi/\omega_L) \\ 0 \qquad\quad (\theta > D - 2\pi/\omega_L), \end{cases} \quad (18)$$

where $c_4 = 2b$ and $d_4 = -(a+b)$, and

$$a = -\frac{TW}{\pi}\ln\frac{1+\sqrt{1+\mathrm{SNR}}}{2}, b = \frac{TW}{2\pi}\frac{\sqrt{1+\mathrm{SNR}}-1}{\sqrt{1+\mathrm{SNR}}}. \quad (19)$$

Both c_4 and d_4 are independent of θ.

3.2. Simplified ZZB

Now we collect the above results to formulate a simplified ZZB, and consider some limiting cases. Partitioning the integration interval $[0, D]$ in (6) into segments, and substituting (12), (14), (16) and (18) into (6), the simplified ZZB for UWB signals is given by

$$\begin{aligned} \overline{\epsilon^2} \quad &\ge \quad \frac{1}{D}\int_0^{s_1} \theta(D-\theta)e^{-d_1(\theta)}\Phi(\sqrt{c_1(\theta)})\,d\theta \\ &+ \quad \frac{1}{D}\int_{s_1}^{s_2} \theta(D - \frac{2\pi}{\omega_H} - \theta)e^{-d_2(\theta+2\pi/\omega_H)} \\ &\quad \cdot \Phi(\sqrt{c_2(\theta+2\pi/\omega_H)})\,d\theta \\ &+ \quad \frac{1}{D}\int_{s_2}^{s_3} \theta(D - \frac{2\pi}{\omega_L} - \theta) \cdot e^{-d_3}\Phi(\sqrt{c_3})\,d\theta \\ &+ \quad \frac{1}{D}\int_{s_3}^{D-2\pi/\omega_L} \theta(D - \frac{2\pi}{\omega_L} - \theta)e^{-d_4}\Phi(\sqrt{c_4})\,d\theta. \end{aligned} \quad (20)$$

In this expression, s_1, s_2 and s_3 are found as follows.

Since in the range $[0, s_1)$, noise dominates without any ambiguity, we have

$$s_1 = 2\pi/\omega_H. \quad (21)$$

At $\theta = s_2$, the second term in (20) is approximately equal to the third term, therefore

$$c_2(s_2 + 2\pi/\omega_H) = c_3. \quad (22)$$

At $\theta = s_3$, the third term in (20) is approximately equal to the fourth term. Then s_3 can be found from

$$\frac{T\mathrm{SNR}}{\pi\sqrt{2(1+2\mathrm{SNR})}}[W + \frac{1}{2(s_3 - 2\pi/\omega_L)}] = c_4. \quad (23)$$

With these values for s_1, s_2 and s_3, the lower bound (20) can be analytically studied for different system parameters. Next we consider several limiting cases.

When $\eta \to 0$, the fourth term in (20) dominates, and the lower bound is close to

$$\begin{aligned} \overline{\epsilon^2} \quad &\ge \quad \frac{1}{D}\int_0^{D-2\pi/\omega_L} \theta(D - 2\pi/\omega_L - \theta)d\theta \cdot \Phi(\sqrt{c_4}) \\ &\approx \quad \frac{D^2}{6}\Phi(\sqrt{\eta/2\pi}) \approx \frac{D^2}{12}. \end{aligned} \quad (24)$$

When $1 \ll \eta \ll 8\sqrt{2}(W + 2\omega_0)/\omega_0$, i.e., when $s_2 < s_3$, the third term in (20) dominates, and the lower bound is very close to

$$\begin{aligned} \overline{\epsilon^2} \quad &\ge \quad \frac{s_3^2 - s_2^2}{2} \cdot e^{-d_3}\Phi(\sqrt{c_3}) \\ &\approx \quad \frac{1}{2}(\frac{2\pi}{\omega_L} + \frac{1}{2\sqrt{2}\eta W})^2\Phi(\sqrt{\frac{2\eta}{\pi^2}}) \\ &\approx \quad \frac{1}{4}(\frac{2\pi}{\omega_L} + \frac{1}{2\sqrt{2}W\eta})^2. \end{aligned} \quad (25)$$

If $\eta \gg 8\sqrt{2}(W + 2\omega_0)/\omega_0$, the first term in the summation becomes the dominant one, and the lower bound approaches

$$\begin{aligned} \overline{\epsilon^2} \quad &\ge \quad \frac{1}{c}\int_0^{2\pi\sqrt{c}/\omega_H} \theta e^{-\theta^4/(c^2/d)}\Phi(\theta)\,d\theta \\ &\approx \quad \frac{1}{4c} = \frac{\pi}{\eta(\omega_0^2 + W^2/12)}, \end{aligned} \quad (26)$$

where $c = c_1(\theta)/\theta^2 = \eta(12\omega_0^2 + W^2)/48\pi$.

The transition from (24) to (25) starts at the 3dB lower point $\eta = \alpha$, which satisfies $\Phi(\sqrt{\alpha/2\pi}) = \frac{1}{4}$ and completes at the point where the fourth term in (20) is equal to the third term, i.e., $\eta = \beta$ when

$$\frac{D^2}{6}\Phi(\sqrt{\beta/2\pi}) = \frac{1}{4}(\frac{2\pi}{\omega_L} + \frac{1}{2\sqrt{2}W\beta})^2. \quad (27)$$

Similarly, the transition from (25) to (26) starts at the 3dB lower point $\eta = \gamma$ where

$$\Phi(\sqrt{\frac{2\gamma}{\pi^2}}) = \frac{1}{2}\Phi(\sqrt{\frac{2\beta}{\pi^2}}), \quad (28)$$

and completes at $\eta = \delta$ where the third term in (20) is equal to the first term, i.e.,

$$\frac{1}{2}(\frac{2\pi}{\omega_L} + \frac{1}{2\sqrt{2}\delta W})^2\Phi(\sqrt{\frac{2\delta}{\pi^2}}) = \frac{\pi}{\delta(\omega_0^2 + W^2/12)}. \quad (29)$$

Accordingly, the lower bound for UWB signals has five regions, with two thresholds (the second and fourth lines) between three disjointed segments (the first, third and fifth lines) for different $\eta = WT \cdot$ SNR,

$$\overline{\epsilon^2} \geq \begin{cases} D^2/12 & \eta < \alpha \\ \text{threshold} & \alpha \leq \eta < \beta \\ \frac{1}{4}(\frac{2\pi}{\omega_L} + \frac{1}{2\sqrt{2}W\eta})^2 & \beta \leq \eta < \gamma \\ \text{threshold} & \gamma \leq \eta < \delta \\ \frac{\pi}{\eta(\omega_0^2 + W^2/12)} & \delta \leq \eta, \end{cases} \quad (30)$$

with threshold expressed in (20) as the sum of four terms.

4. SIMULATION

The UWB spectrum defined by the FCC is used in a simulation, with center frequency 6.85GHz, bandwidth 7.5GHz and EIRP emission -41.3dBm. Fig. 2 compares our simplified ZZB, the ZZB obtained numerically, and the MLE performance. (For this case, the WWB is not applicable.) The simulation shows that our simplified ZZB is close to both the numerically obtained ZZB, and the MLE. Our bound is divided into five segments, with two thresholds dividing the WT·SNR domain into three disjointed segments. When $WT \cdot$ SNR $< \alpha$, the envelope-related ambiguities are dominant and the MSE bound only depends on the expected maximum time delay D as $D^2/12$. Since $WT/2\pi \gg 1$, when $WT \cdot$ SNR < 1, that is, when SNR $\to 0$, the observed signals are completely dominated by noise and nearly useless for time delay estimation, i.e., the performance is equivalent to making a random decision. When $\beta \leq WT$SNR $< \gamma$, the lower bound is contributed by both the unresolved phase ambiguity and envelope ambiguity. β is a function of both ω_0, W and D. γ is defined as a 3dB lower point where the MSE is half of the one at β. Obviously, β and γ increase with D. The lower bound matches quite well with the CRB when $WT \cdot$ SNR is very large (linear region for $WT \cdot$ SNR > 20). This region is noise dominated. Both phase and envelope ambiguities can be resolved completely.

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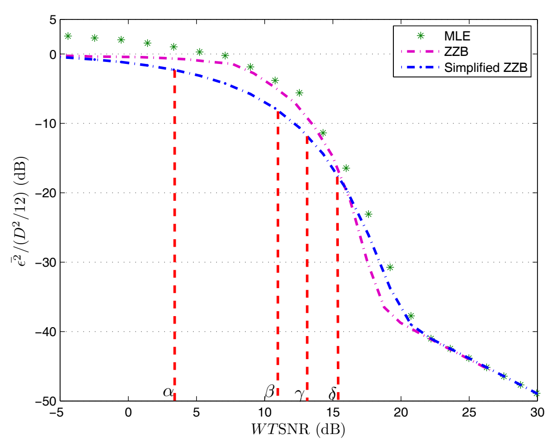

Fig. 2. Comparison of the simplified bound with the ZZB, and MLE.