DISTRIBUTED DETECTION OVER MULTIPLE ACCESS CHANNELS

Ying Lin, Biao Chen[*] | *Lang Tong*

Syracuse University | Cornell University

ABSTRACT

We address the design of binary local sensor quantizers for decentralized detection over multiple access channels. Our goal is to minimize the error probability at the fusion center using a single snapshot of local observations. Considering both the synchronized and asynchronous transmissions among sensors, we establish the optimality of a likelihood ratio test for both cases. For the case of asynchronous transmissions, to compensate for the unknown fading channel parameters and transmission delays, we propose a structure consisting of a RAKE receiver and a square-law detector. Simulations results are presented to demonstrate effectiveness of the design procedure.

***Index Terms*—** Wireless sensor networks, distributed detection, multiple access channel, likelihood ratio quantizers

1. INTRODUCTION

Distributed detection schemes that integrate the transmission and processing to achieve better performance with practical constraints have been studied recently [1, 2]. A prevailing model used for such applications is the parallel channel model where sensors communicate to the fusion center via orthogonal channels.

One of the disadvantages of transmitting local decisions over orthogonal channels is the large bandwidth consumption as the number of local sensors K increases. An appealing alternative is to allow multiple sensors share a common channel, i.e., communicating via a multiple access channel (MAC). Indeed, MAC has been adopted for decentralized detection ([3–5]). In [4] and [5], for type-based distributed detection, optimal fusion rules have been developed in the asymptotic case as $K \to \infty$ under the assumption of perfect synchronization among sensor transmissions. However, the design of optimal local decision rules has not been addressed. Furthermore, the assumption of synchronized sensor transmissions over MAC may not be realistic. In practical large-scale sensor networks, maintaining perfect synchronization among a large number of sensors may be prohibitive due to stringent resource constraints and geographical dispersiveness. This motivates our current work.

In this paper, we try to answer the following questions: What is the optimal local detector structure for distributed detection over MAC under asynchronous sensor transmissions, and, given the optimal structure, a practically feasible procedure to optimize the parameters (i.e., thresholds) for both synchronous and asynchronous cases. To address these questions, we consider the binary quantizer design for a binary hypothesis testing problem where sensors transmit their finite alphabet local messages through MAC. We restrict ourselves to the case of using a single snapshot of conditionally independent local observations. The case of multiple bits sensor outputs can be straightforwardly generalized based on the results of binary sensor outputs. In this work, we adopt the Bayesian criterion, i.e., one wants to minimize the error probability at the fusion center. For both synchronized and asynchronous cases, we show that the optimal local decision rules are in the form of likelihood ratio test (LRT). In the asynchronous case, to ease the implementation of the local decision rules and compensate for the unknown delays among sensor transmissions, we further propose a structure consisting of a RAKE receiver with a square-law detector (RAKE-SL) to produce the fusion statistic.

The organization of this paper is as follows. In the next section, we introduce the problem formulation for the general case of decentralized detection over MAC. We consider the synchronized case in Section 3 and derive the optimal local decision rules. In Section 4, we investigate the local decision rules for the asynchronous case. Design examples are provided in Section 5 and we conclude in Section 6.

2. PROBLEM FORMULATION

Consider a binary hypotheses testing problem with K distributed sensors, as illustrated in Fig. 1. Upon collecting X_k generated by one of the two hypotheses H_0/H_1 that under test, the kth sensor makes a local decision U_k that takes values from a finite alphabet. Assume the observations are independent and identically distributed (i.i.d.) given each hypothesis and the prior probability is given by $\pi_0 = P(H_0)$ and $\pi_1 = P(H_1) = 1 - \pi_0$. For simplicity, we assume binary sensor signaling, i.e., $U_k = \gamma_k(X_k) \in \{1, 0\}$. Sensors are divided into two groups by their U_k values. Only sensors with

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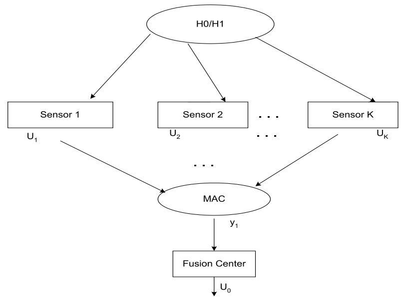

Fig. 1. A block diagram for a wireless sensor network tasked with binary hypothesis testing over multiple access channels.

$U_k = 1$ transmit their local decisions through a multiple access channel using a common waveform $s(t)$. Alternatively, one can have the two groups of sensors transmitting in two separate channels, as in the type based schemes, and the results obtained in this paper can be trivially extended to such a case. The bandwidth consumption is dramatically reduced compared to the case of requiring K orthogonal channels. The autocorrelation function of $s(t)$ is defined as $c_s(\tau) = \int_{t=0}^{T} s(t)s^*(t-\tau)dt$, where T denotes the symbol period, and assume $c_s(0) = \int_{t=0}^{T} |s(t)|^2 dt = E$; i.e., E is the energy required to transmit one symbol to take into account the energy constraints of the system.

At the fusion center, the received signal is [1]:

$$y(t) \;=\; \sum_{k=1}^{K_1} h_k s(t-d_k) + w(t), \qquad 0 \le t \le T \quad (1)$$

where

- K_1 is the total number of sensors that decide $U_k = 1$, i.e, $K_1 = \sum_{k=1}^{K} U_k$. It is unknown at the fusion center.
- h_k, $k = 1, 2, \cdots, K_1$, denote the channel fading coefficients. In the current work, we assume each link experiences the flat Rayleigh fading, thus, h_k's are i.i.d. complex Gaussian with zero mean and variance σ_h^2.
- d_k, $k = 1, 2, \cdots, K_1$, denote the unknown delay for different sensors. For the synchronous case, all d_k's are identical and is assumed to be zero without loss of generality.
- $w(t)$ is the complex Gaussian channel noise with zero mean and variance σ_w^2.

The fusion center implements the optimal fusion rule based on the channel output, i.e., $U_0 = \gamma_0(y(t))$. An error happens if the global decision U_0 differs from the true hypothesis.

[1] For the general case where U_k takes M possible values, $M-1$ groups of sensors transmit through $M-1$ MACs. The output will consist a vector with $M-1$ components

Our objective is to develop optimal local decision rules that minimize the error probability at the fusion center. Given the model in Eq.(1), we consider the local decision rule design for both synchronous and asynchronous cases.

3. SYNCHRONOUS CASE

In the synchronized case, we have: $y(t) = \sum_{k=1}^{K_1} h_k s(t) + w(t), \quad 0 \le t \le T$. For this case, the matched filter based receiver is optimal for the detection problem as its output provides a sufficient statistic. The output signal of the matched filter, denoted by r, can be expressed as $r = \int_0^T y(t)s^*(t)dt = \sum_{k=1}^{K_1} h_k E + Q_1$ where $Q_1 \triangleq \int_0^T w(t)s^*(t)dt$, is a complex Gaussian random variable with zero mean and variance $\sigma_w^2 E$. We can show that given K_1, r is also complex Gaussian distributed. In particular, $r|K_1 \backsim \mathcal{CN}(0, \sigma_1^2)$, where $\sigma_1^2 = K_1\sigma_h^2 E^2 + \sigma_w^2 E$.

Equivalently, the optimal fusion rule can be implemented based on the matched filter output, i.e, $U_0 = \gamma_0(r)$. Specifically, the fusion center employs the maximum a posteriori probability rule with threshold $\frac{\pi_0}{\pi_1}$.

Under the Bayesian framework, we aim to design the optimal local quantizers to minimize the error probability at the fusion center. The results are summarized below:

Theorem 1 *Assume that X_k's are conditionally independent, sensor transmissions are perfectly synchronized, and the fusion rule and the kth local decision satisfy*

$$\begin{aligned} P(U_0 = 1|\mathbf{u}^{k1}) - P(U_0 = 1|\mathbf{u}^{k0}) &\ge 0, \quad (2)\\ P(U_0 = 0|\mathbf{u}^{k0}) - P(U_0 = 0|\mathbf{u}^{k1}) &\ge 0. \quad (3)\end{aligned}$$

Then the optimal local decision rule for the kth sensor amounts to the following LRT

$$P(U_k = 1|X_k) = \begin{cases} 1 & if\ \frac{p(X_k|H_1)}{p(X_k|H_0)} > \tau_k \\ 0 & if\ \frac{p(X_k|H_1)}{p(X_k|H_0)} \le \tau_k, \end{cases}$$

where

$$\begin{aligned} \mathbf{u}^k &= [U_1, \cdots, U_{k-1}, U_{k+1}, \cdots, U_K], \mathbf{u} = [\mathbf{u}^k, U_k] \\ \mathbf{u}^{k1} &= [U_1, \cdots, U_{k-1}, U_k = 1, U_{k+1}, \cdots, U_K] \\ \mathbf{u}^{k0} &= [U_1, \cdots, U_{k-1}, U_k = 0, U_{k+1}, \cdots, U_K] \\ \tau_k &= \frac{\pi_0 \sum_{\mathbf{u}^k} P(\mathbf{u}^k|H_0)[P(U_0 = 1|\mathbf{u}^{k1}) - P(U_0 = 1|\mathbf{u}^{k0})]}{\pi_1 \sum_{\mathbf{u}^k} P(\mathbf{u}^k|H_1)[P(U_0 = 0|\mathbf{u}^{k0}) - P(U_0 = 0|\mathbf{u}^{k1})]} \end{aligned} \quad (4)$$

Eq. (2) and (3) amount to using a monotone fusion rule. A sketch of the proof is given in Appendix. Clearly, the threshold τ_k at the kth sensor is coupled with those at other sensors. To obtain the optimal thresholds τ_k, we devise an iterative algorithm described in Section 5.

4. ASYNCHRONOUS CASE

In the asynchronous case, $y(t)$ is given as in Eq. (1). In the current setting, we assume each sensor experiences a different delay and all delays are within $[0, T_{max}]$, where T_{max} denotes the maximum delay and is assumed known.

Similarly we can show that the optimal local decision rules amount to the LRT, as summarized in the theorem below.

Theorem 2 *Assume that X_k's are conditionally independent, sensor transmissions are asynchronous, and further that the fusion rule and the kth local decision satisfy Eq. (2) and (3), then the optimal local decision rule for the kth sensor amounts to the LRT and threshold τ_k is defined as in Eq. (4).*

The unknown delays in the received signal model make it impossible to implement the LRT decision rules described in Theorem 2. For example, it is untractable to evaluate the likelihood functions, e.g., $P(y(t)|H_j)$, for $j = 0, 1$. The simple matched filter structure in the synchronized case is not applicable here. In this work, we propose a RAKE-SL receiver structure at the fusion center, and use the corresponding output as the fusion statistic. The motivation is from the common practice in digital communication systems where RAKE-SL is used to compensate for the unknown delays and channel coefficients [6]. This RAKE-SL structure allows us to carry out the LRT based fusion rule as described below. Such structure also makes it tractable to optimize the local decision rules.

The received signal $y(t)$, is processed by a RAKE-SL receiver with $s(t)$ as the reference signal. By appropriately choosing L, the number of taps of the RAKE-SL, and signal bandwidth W, we can project the delays d_k to the taps of the RAKE-SL. The output, R, is

$$R = \sum_{l=1}^{L} \left| \int_0^T y(t) s^*(t - \frac{l}{W}) dt \right|^2 .$$

In general, the probability density function (pdf) of R, is hard to characterize, which amounts to finding the pdf of the sum of correlated random variables. However, we show that under some condition, R is the sum of two independent gamma distributed variables and admits an exact pdf. The lemma below summarizes the results. We skip the proof.

Lemma 1 *When the following is true: $\int_{t=0}^{T} s(t - \frac{n}{W}) s^*(t - \frac{m}{W}) dt \approx 0$, for $n \neq m$, then $R|_{K_1}$ is the sum of two independent gamma random variables. In particular, $R = U_1 + V_1$ where $U_1|_{K_1} \sim gamma(K_1, \sigma_U^2)$, $V_1|_{K_1} \sim gamma(L - K_1, \sigma_V^2)$, further, $\sigma_U^2 = E^2\sigma_h^2 + E\sigma_w^2$ and $\sigma_V^2 = E\sigma_w^2$.*

Referred to as the orthogonal property of a signal, the condition specified in Lemma 1 is valid in certain practical scenarios. For instance, this condition can be satisfied by choosing appropriate PN code with spread-spectrum techniques [6].

The pdf of the sum of two independent gamma distributions was established in [7]. We can express the conditional density function $P(R|K_1)$ in a similar manner. This will enable the implementation of the fusion rule and computation of τ_k. Next, we validate the proposed design procedures by examples for both synchronized and asynchronous cases.

5. EXAMPLES

In this section, we use examples to demonstrate how to obtain the optimal local thresholds. Consider the detection of a known signal in additive Gaussian noises that are i.i.d. among sensors, i.e.,

$$\begin{aligned} H_0 &: X_k = N_k \\ H_1 &: X_k = S + N_k \end{aligned}$$

for $k = 1, 2, ..., K$ with N_k being i.i.d. $\mathcal{N}(0, \sigma^2)$. Let $K = 10$, $S = 1$, and $\sigma_h^2 = 1$. For the Gaussian problem, the likelihood ratio threshold can be converted to the observation threshold which we use throughout this section.

5.1. Synchronized transmissions

Under synchronized transmissions, the optimal local thresholds can be determined using Eq. (4). The iterative algorithm for decision rule optimization is described below.

1. Initialize τ_k, for $k = 1, 2, ..., K$.
2. Obtain the optimal fusion rule for fixed τ_k.
3. For fixed fusion rule and τ_j, $j = 2, ..., K$, calculate τ_1 using (4).
4. Repeat the previous step for all sensors.
5. Check convergence, i.e., if the obtained τ_k, $k = 1, ..., K$ are identical (up to a prescribed precision) to that from the previous iteration then stop. Otherwise, go to 2.

Table 1 lists results for $\pi_0 = 0.5$, channel $SNR = 0dB$, and for different observation SNRs. We compare the thresholds obtained by the iterative algorithm and by the exhaustive search. Clearly the results match very well. Furthermore, all τ_k's converge to the same value, i.e., identical thresholds at local sensors. This observation is consistent with the results obtained in [8] where it was shown that all local thresholds converge to a common value asymptotically. Table 1 shows that at high observation SNR, the optimal threshold approaches to the local optimal threshold, $\tau = 0.5$ (the LR threshold $\tau_{LR} = \pi_0/\pi_1 = 1$), the threshold that achieves minimum error probability at local sensors.

Table 1. Thresholds under synchronized case

$SNR(dB)$	0	5	10	20
iterative τ	1.3102	0.9327	0.7024	0.5235
exhaustive τ	1.3107	0.9325	0.7024	0.5233

5.2. Asynchronous transmissions

Now we consider the asynchronous case. The output of the RAKE-SL, R, is used as the fusion statistic. We let $K = L = 10$. τ_k, $k = 1, 2, ...K$, can be determined using Eq.

Table 2. Thresholds under asynchronous case

$SNR(dB)$	0	5	10	20
iterative τ	0.9111	0.7597	0.6578	0.5214
exhaustive τ	0.9106	0.7599	0.6580	0.521

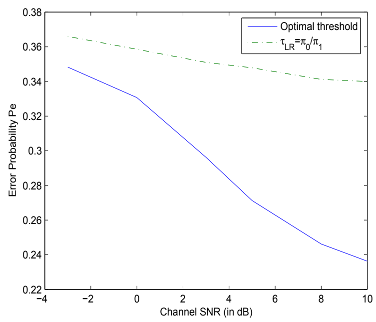

Fig. 2. Error prob. versus channel SNR, asynchronous case.

(4) and the iterative algorithm describe above. Table 2 lists the analytically calculated results at $\pi_0 = 0.5$ and channel $SNR = 5dB$ for both the iterative algorithm and the exhaustive search method. Again, the optimal threshold converges to the local optimal threshold 0.5 at high observation SNR.

In Fig. 2, we plot the simulated error probability curves as a function of channel SNR at $\pi_0 = 0.5$ for the asynchronous case, where we use $K = L = 10$, observation $SNR = 5dB$, 10000 Monte Carlo runs. d_k are generated uniformly from $[0, L-1]$. We adopt a m-sequence with length of 2^8 to generate $s(t)$. Fig. 2 shows that the optimal threshold outperforms the local optimal threshold as channel SNR increases.

6. CONCLUSIONS

For distributed detection in a wireless network, integrating transmission schemes and sensor decision rule design may prove useful in resource constrained applications. In this work, aiming to reduce the bandwidth consumption, we consider decentralized detection over MAC. We investigate two cases: synchronized and asynchronous transmissions. In both scenarios, the optimality of the LRT for local sensor decisions are established under the Bayesian criterion. Numerical examples demonstrate that carefully designed local sensor decision rules significantly outperform the naive approach of minimizing the local sensor error probability.

7. APPENDIX-PROOF OF THEOREM 1

In the synchronized case, $U_0 = \gamma_0(y(t))$, $y(t)$ is a function of K_1, h_k, $s(t)$, and $w(t)$, thus a function of $\mathbf{u}^k$ and U_k. Let $A \triangleq (h_k, s(t), w(t))$. Similarly as in [9], we define

$$J \triangleq E\{C(\gamma_0(\mathbf{u}^k, U_k, A), \mathbf{u}^k, U_k, A, H)\}.$$

and $F(.) = C(\gamma_0(\mathbf{u}^k, U_k, A), \mathbf{u}^k, U_k, A, H)$. Since $\mathbf{u}^k$ and U_k are conditional independent, results established in [9] are directly applicable to our setup. To achieve minimum P_e, we have $J = Pr(\gamma_0(y(t)) \neq H) = E[\mathbb{I}(U_0 \neq H)]$ where $\mathbb{I}(U_0 \neq H)$ is an indicator function. Thus, as derived in [9, 10], the optimal local decision rules are given by the LRT and the threshold is $\tau_k = \frac{\pi_0[\alpha_k(H_0,d=1)-\alpha_k(H_0,d=0)]}{\pi_1[\alpha_k(H_1,d=0)-\alpha_k(H_1,d=0)]}$ which reduces to Eq. (4). If r, the matched filter output is used, similarly, we can show that same results hold.

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