DECENTRALIZED ESTIMATION FOR BANDWIDTH CONSTRAINED SENSOR NETWORKS IN CLUSTERED ENVIRONMENTS

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ABSTRACT

This paper extends the homogeneous wireless sensor network (WSN) model to possess (possibly) varying levels of noise that represent environment clusters in a WSN. The estimation of the source parameter is approached from Maximum Likelihood (ML) perspective. The Cramer Rao Lower Bound (CRLB) for any unbiased estimator operating in a clustered WSN environment is derived. Noting that the ML estimate cannot be found in closed–from, we resort to a numerical search. Although numerically determined, the ML estimate is guaranteed to converge to the optimal solution since it is shown here that the log–likelihood function is concave. Also considered is the estimation of a random parameter with a *priori* information, which is approached from Maximum A Posteriori (MAP) perspective. Finally, the proposed MAP and ML optimal are validated and compared to theoretical bounds with illustrative numerical examples.

Index Terms— decentralized estimation, clustering.

1. INTRODUCTION

Many wireless sensor networks (WSNs) is constrained by the fact the that bandwidth is limited, imposing the use and transmission of quantized binary versions of the original noisy observations. Many recent efforts address the estimation of a deterministic source signal from quantized noisy observations [1–4]. When the probability density function (pdf) of the sensor noise is known, transmitting a single bit per sensor leads to minimal loss in estimator variance compared with a clairvoyant estimator (estimator based on unquantized measurements) [1, 4]. Alternatively, when the sensor noise pdf is unknown, pdf–unaware estimators based on quantized sensor data have also been introduced recently [3].

Prior works consider a homogeneous sensor network environment, simplifying the environment characterization to a single (spread) parameter. Although this assumption yields tractable and closed–form solutions, unfortunately, this is not the case in practical WSN scenarios where the sensors are distributed in large–scaled environments that (possibly) possess different characteristics. In this paper, we extend the WSN model to admit clusters of sensors that have varying levels of reliability, i.e., varying levels of noise. Utilizing this clustered WSN model, we derive the maximum likelihood (ML) estimate of the source signal. The Cramer Rao Lower Bound (CRLB) for any unbiased estimator operating on a clustered WSN environment is also derived. Noting that the source parameter cannot be found in closed–from, we resort to a numerical search approach, namely Newton's algorithm. Albeit numerically determined, the estimate is guaranteed to converge to the optimal solution since we show here that the log–likelihood function is concave. Also addressed is the design of efficient estimators for the case in which the parameter of interest is stochastic with density function known a *priori*. In particular, we develop Maximum a Posteriori (MAP) estimators for decentralized parameter estimation schemes characterized by clustered statistics. Finally, numerical examples validating and comparing the developed optimal techniques are presented.

2. PROBLEM FORMULATION

Consider a set of K distributed sensors, each making an observation on a source signal θ. The sensors are, however, not in a homogeneous environment, i.e., the sensors are grouped in clusters according to their locations, exhibiting local statistics. Let $\{\mathcal{C}_m : m = 1, 2, \ldots, M\}$ denote the sensor clusters with corresponding variances $\{\sigma_m^2 : m = 1, 2, \ldots, M\}$, where M is the total number of clusters. The sensor observations are corrupted with additive noise, the variance of which is cluster dependent, and are given as

$$x_m(k) = \theta + n_m(k), \quad m = 1, 2, \ldots, M \tag{1}$$

and $k = 1, 2, \ldots, \#(\mathcal{C}_m)$, where $\#(\mathcal{C}_m)$ denotes the number of sensors in cluster $\mathcal{C}_m$. Note that $K = \sum_{m=1}^{M} \#(\mathcal{C}_m)$. Due to bandwidth limitations in WSNs, the observations $\{x_m(k) : m = 1, 2, \ldots, M \text{ and } k = 1, 2, \ldots, \#(\mathcal{C}_m)\}$ have to be quantized. To this end, we consider the quantization as the construction of a set of indicator variables consisting of binary observations [1–4]

$$b_m(k) = \mathbf{1}\{x_m(k) \in (\tau, +\infty)\}, \quad k = 1, 2, \ldots, K \tag{2}$$

where $\tau \in \mathbf{Z}$ is a threshold defining $b_m(k)$, $\mathbf{Z}$ denotes the set of real numbers, and $\mathbf{1}\{\cdot\}$ is the indicator function. Note that we consider the case of a single threshold for all sensors. The effect of the varying sensor thresholds are also recently studied for a homogeneous WSNs [1].

The bandwidth constraint manifests itself in dictating that θ must be estimated based on the binary observations $\{b_m(k) : m = 1, 2, \ldots, M$ and $k = 1, 2, \ldots, \#(\mathcal{C}_m)\}$. Instrumental to the ensuing scheme is the fact that each $b_m(k)$ is a Bernoulli random variable with parameter

$$q_m(\theta) \triangleq \Pr\{b_m(k) = 1\} = 1 - F_m(\tau - \theta) \tag{3}$$

where $F_m(\cdot)$ denotes the cumulative distribution function corresponding to cluster $\mathcal{C}_m$.

We hence consider a WSN model where the sensors are grouped into clusters according to environment statistics and each sensor observes a corrupted version of the deterministic source signal θ. The corruption level experienced by an individual sensor depends on the cluster $\mathcal{C}_m$ to which it belongs. All sensors send their quantized observations to a fusion center, $\Psi(\cdot)$. The fusion center estimates the source signal θ utilizing the clusters' statistics and binary observations. Throughout the rest of the paper we assume that the observation noise is characterized by the Gaussian density function $f_m(u) = 1/(\sigma_m\sqrt{2\pi}) \exp\left(-u^2/(2\sigma_m^2)\right)$ where σ_m denotes the spread parameter associated with cluster $\mathcal{C}_m$.

3. MLE BASED ON BINARY OBSERVATIONS IN CLUSTERED ENVIRONMENTS

The clustered WSN model, which incorporates varying levels of noise, represents a more realistic WSN scenario compared to the single level, uniform noise case. This section details the estimation of the source parameter approached from a ML perspective in such environments. The CRLB of any unbiased estimator operating on clustered WSN environments is also derived.

Note that the pdf of $b_m(k)$ is given by $f_{b_m}(b) = \delta(b - 1)q_m(\theta) + \delta(b)(1 - q_m(\theta))$. This formulation, however, makes the ML estimation intractable. To avoid this problem, we rewrite the density function as $f_{b_m}(b) = [q_m(\theta)]^b[1 - q_m(\theta)]^{1-b}$ noting that $b \in \{0, 1\}$. Furthermore, we suppose that the cluster $\mathcal{C}_m$, from which the observation $b_m(k)$ emanates, is known at the fusion center. Now define $\mathbf{b} \triangleq \{b_m(k) : m = 1, 2 \ldots, M$ and $k = 1, 2, \ldots, \#(\mathcal{C}_m)\}$. The log–likelihood function, $\Lambda_{\mathrm{L}}(\mathbf{b}, \theta)$, is, due to noise independence, given by

$$\begin{aligned}\Lambda_L(\mathbf{b}, \theta) = \sum_{m=1}^{M} \sum_{k=1}^{\#(\mathcal{C}_m)} & b_m(k) \log(q_m(\theta)) \\ & + (1 - b_m(k)) \log(1 - q_m(\theta))\end{aligned} \tag{4}$$

from which we can define the the ML estimate of θ, given $\mathbf{b}$, as

$$\hat{\theta} = \arg\max_{\theta}\{\Lambda_L(\mathbf{b}, \theta)\}. \tag{5}$$

As $\hat{\theta}$ in (4) and (5) cannot be found in closed form, we resort to the numerical search Newton's algorithm, which is based on the iteration

$$\hat{\theta}(j+1) = \hat{\theta}(j) - \frac{\Lambda_L^{(1)}(\mathbf{b}, \hat{\theta}(j))}{\Lambda_L^{(2)}(\mathbf{b}, \hat{\theta}(j))} \tag{6}$$

where

$$\begin{aligned}\Lambda_L^{(1)}(\mathbf{b}, \theta) = \sum_{m=1}^{M} & \#(\mathcal{C}_m)\hat{q}_m(\theta) \frac{f_m(\tau - \theta)}{1 - F_m(\tau - \theta)} \\ & - \#(\mathcal{C}_m)(1 - \hat{q}_m(\theta)) \frac{f_m(\tau - \theta)}{F_m(\tau - \theta)}\end{aligned} \tag{7}$$

and

$$\begin{aligned}\Lambda_L^{(2)}(\mathbf{b}, \theta) = & \sum_{m=1}^{M} \#(\mathcal{C}_m)\hat{q}_m(\theta) \\ & \times \frac{[1 - F_m(\tau - \theta)] f_m^{(1)}(\tau - \theta) - f_m^2(\tau - \theta)}{[1 - F_m(\tau - \theta)]^2} \\ & - \#(\mathcal{C}_m)(1 - \hat{q}_m(\theta)) \\ & \times \frac{F_m(\tau - \theta) f_m^{(1)}(\tau - \theta) + f_m^2(\tau - \theta)}{[F_m(\tau - \theta)]^2}.\end{aligned} \tag{8}$$

Although numerically determined, the ML estimate is guaranteed to converge to the optimal solution as the following proposition proves that $\Lambda_L(\mathbf{b}, \theta)$ is concave.

Proposition 1 *The log–likelihood function $\Lambda_L(\mathbf{b}, \theta)$ is concave on θ, i.e., for all $\{\theta_1, \theta_2\} \in \Re$, where $\Re$ denotes the set of real numbers, the following holds*

$$\Lambda_L(\mathbf{b}, (1 - \lambda)\theta_1 + \lambda\theta_2) \geq (1 - \lambda)\Lambda_L(\mathbf{b}, \theta_1) + \lambda\Lambda_L(\mathbf{b}, \theta_2) \tag{9}$$

for $\lambda \in [0, 1]$.

Proof Sketch 1 *The concavity is shown utilizing the facts that the $q_m(\theta)$ is log–concave since it is integral of a log–concave function, and that $\log(\cdot)$ is a concave function.*

The concavity of $\Lambda_L(\mathbf{b}, \theta)$ guarantees the convergence of Newton's iteration to the global maximum, regardless of the initialization.

Consider next the CRLB of any unbiased estimator operating in a clustered WSN. The CRLB in this case is given by

$$\mathcal{B}(\mathbf{b}, \theta) \triangleq -\left(E\left[\frac{\partial^2 \Lambda_L(\mathbf{b}, \theta)}{\partial \theta^2}\right]\right)^{-1}. \tag{10}$$

More specifically, the CRLB for the source estimation problem in inhomogeneous environments is established in the following proposition.

Proposition 2 *The CRLB for any unbiased estimator operating on clustered fusion center observations is*

$$\mathcal{B}(\mathbf{b}, \theta) = \frac{1}{K}\left[\sum_{m=1}^{M} \psi_m \frac{f_m^2(\tau-\theta)}{[1-F_m(\tau-\theta)]F_m(\tau-\theta)}\right]^{-1} \tag{11}$$

where $\psi_m = \#(\mathcal{C}_m)/K$.

Proof Sketch 2 *To obtain the CRLB, take the expected value of the second derivative of the log–likelihood function with respect to* b_m. *Note that* $\hat{q}_m(\theta)$ *is unbiased indicating that* $E(\hat{q}_m(\theta)) = q_m(\theta)$. *Also the terms involving* $f_m^{(1)}(\tau-\theta) \triangleq \partial f_m(\tau-\theta)/\partial\theta$ *disappear and the CRLB simply follows.*

As expected, the CRLB based on the clustered $\mathbf{b}$ reduces to the homogeneous case CRLB when the noise is uniform across sensors. In addition, as in the simplified WSN model case, it is straightforward to show that the minimum of the CRLB given in Proposition 3 is achieved when $\tau = \theta$. That is, $\mathcal{B}_{\min}(\mathbf{b}, \theta) = [\mathcal{B}(\mathbf{b}, \theta) : \tau = \theta]$ which is given by

$$\mathcal{B}_{\min}(\mathbf{b}, \theta) = \frac{\pi}{2K}\left[\sum_{m=1}^{M} \frac{\psi_m}{\sigma_m^2}\right]^{-1} \tag{12}$$

where we utilized the facts that $F_m(0) = 1/2$ and $f_m^2(0) = (\sigma_m^2 2\pi)^{-1}$. Iterative algorithm such as the ones detailed [1,5] can be utilized to set the threshold τ close to θ.

4. EXTENSIONS TO RANDOM PROCESSES

This section addresses the design of efficient estimators when the parameter of interest is a random variable with density function known a *priori*. In particular, we develop Maximum a Posteriori (MAP) estimators for decentralized parameter estimation schemes characterized by clustered statistics.

Let $f_\theta(\cdot)$ denote the density function of the source. Accordingly, the MAP estimate of θ is given by

$$\hat{\theta}_{MAP} = \arg\max_{\theta}\{\Lambda(\mathbf{b}, \theta) f_\theta(\theta)\}. \tag{13}$$

Taking the natural log of the above gives

$$\hat{\theta}_{MAP} = \arg\max_{\theta}\{\Lambda_L(\mathbf{b}, \theta) + \log(f_\theta(\theta))\}. \tag{14}$$

Note that $\Lambda_L(\mathbf{b}, \theta) + \log(f_\theta(\theta))$ is concave if $f_\theta(\theta)$ is a log–concave density since $\Lambda_L(\mathbf{b}, \theta)$ is concave and summation preserves concavity. Numerical methods, such as the one discussed in Section 3, can thus be utilized to obtain the optimal solution.

For the estimation of random parameters, bounds on mean square error (MSE) can be obtained by computing the pertinent Fisher Information $\mathcal{J}$. Specifically, the MSE of the estimate is bounded by

$$\text{MSE}(\hat{\theta}) \geq (\mathcal{J})^{-1}. \tag{15}$$

The following proposition establishes a performance bound for the random parameter estimation case in a decentralized estimation scheme characterized by clustered statistics. Specifically, the commonly encountered Gaussian prior density case is addressed.

Proposition 3 *Let* $\mathcal{J}$ *denote the Fisher Information regarding the clustered decentralized random parameter estimation problem. When prior* $f_\theta(\theta)$ *is taken to be Gaussian with mean* μ_θ *and variance* σ_θ^2, $\mathcal{J}$ *is bounded by*

$$\mathcal{J} \geq \frac{2K}{\pi}\sum_{m=1}^{M} \frac{\psi_m}{\sigma_m\sqrt{\sigma_m^2+\sigma_\theta^2}} \exp\left(-\frac{\Delta^2}{2(\sigma_m^2+\sigma_\theta^2)}\right) + \frac{1}{\sigma_\theta^2} \tag{16}$$

where $\Delta = \tau - \mu_\theta$.

Proof Sketch 3 *The proof utilizes the (tight) chernoff bound and integration by parts and is omitted here for brevity.*

Note that the minimum of the derived bound is achieved when $\Delta = 0$, i.e. $\tau = \mu_\theta$, which is in agreement with the results presented in Section 3 and previous literature considering the simplified single parameter case [1,3,4]. In addition, the derived bound is corroborated with numerical examples in Section 5 that show the bound to be very tight. Consequently, the bound provides a good theoretical means for characterizing the performance of decentralized MAP estimators operating in a bandwidth–constrained WSNs.

5. NUMERICAL EXAMPLES

This section reports the variance of the ML estimator determined through simulations, and contrast it with the analytical results derived in previous sections, i.e. the variances of ML is compared against the CRLB. The derived analytical MAP estimator results are also compared with simulation results in the following.

Consider a WSN that is composed of three clusters, i.e., $M = 3$, with channel parameters $\sigma_1 = 1$, $\sigma_2 = 2$ and $\sigma_3 = 1.5$. The sensor thresholds are $\tau = 0$ and the source parameter to be estimated is set to unity, i.e., $\theta = 1$. The cluster sizes are the same for each cluster, i.e., $\#(\mathcal{C}) = \#(\mathcal{C}_1) = \#(\mathcal{C}_2) = \#(\mathcal{C}_3)$ and the cluster size is varied in the range of $\#(\mathcal{C}) = [100, 105, \ldots, 1000]$ during simulations. The output variance of the optimal ML (circles) is plotted in Fig. 1 (a) along with the CRLB (solid). Note that each result is an ensemble average of 5000 trials. The ML estimator provide performance close the CRLB. Similar experiments are also run for the optimal $\tau = \theta$ case, the results of which is plotted in Fig. 1 (b).

Next, consider the MAP estimator operating on simulated sensor network defined by $M = 3, \sigma_1 = 1, \sigma_2 = 2, \sigma_3 = 1.5$, $\mu_\theta = 0$. The performance of the MAP estimator is plotted in plotted in Fig. 2 for the cases of (a) varying number of sensors

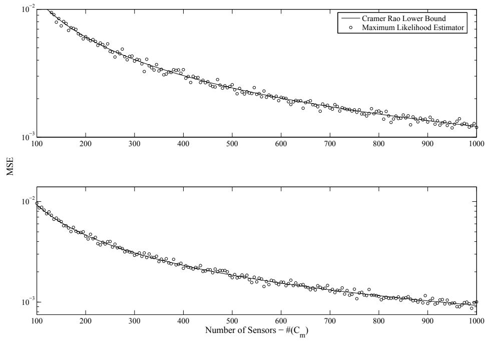
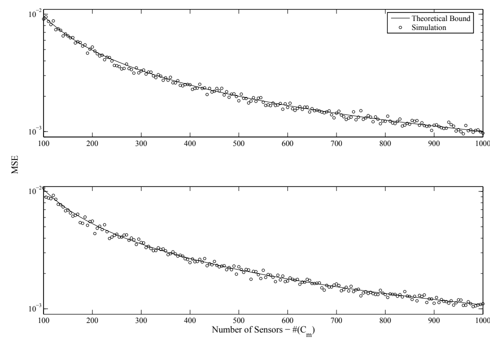


Fig. 1. Illustration of ML (circles) estimator output variance along with the CRLB (solid) where $M = 3$, $\sigma_1 = 1$, $\sigma_2 = 2$, $\sigma_3 = 1.5$, $\theta = 1$ (top:) $\tau = 0$ and (bottom:) $\tau = 1$.

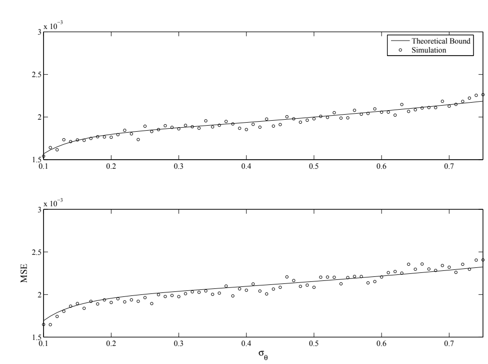
and $\sigma_\theta = 0.5$, (top:) $\tau = 0$ and (bottom:) $\tau = 0.5$, and (b) varying σ_θ and $\#(\mathcal{C}) = 500$, (top:) $\tau = 0$ and (bottom:) $\tau = 0.5$. Simulated MSE values are tightly scattered around the analytical values, corroborating the theoretical bound. As expected, the MSE of the MAP estimator based on clustered binary observations increases with σ_θ. Also, the simulated MSEs closely follow the analytical results.

6. CONCLUSIONS

The homogeneous wireless sensor network (WSN) model is extended to admit clusters of environments reflecting the varying levels of noises in large–scaled WSNs. The estimation of a source parameter is approached from a maximum likelihood (ML) perspective. The Cramer Rao Lower Bound (CRLB) for any unbiased estimator operating in a clustered WSN environment is derived. Noting that the source parameter cannot be found in a closed–from, we resort to a numerical search, namely Newton's algorithm. Although numerically determined, the estimate is guaranteed to converge to the optimal solution since it is shown that the log–likelihood function is concave. Also considered here is the estimation of parameter, with a *priori* information, approached from a Maximum A Posteriori (MAP) perspective, and a bound on the corresponding Fisher Information. Numerical experiments are presented to evaluate and compare the performances of the proposed estimators.



(a)



(b)

Fig. 2. Illustration of MAP estimator output variances along with the theoretical bound (solid). The parameter are: $M = 3$, $\sigma_1 = 1$, $\sigma_2 = 2$, $\sigma_3 = 1.5$, $\mu_\theta = 0$ (a) for varying number of sensors and $\sigma_\theta = 0.5$, (top:) $\tau = 0$ and (bottom:) $\tau = 0.5$ and (b) for varying σ_θ and $\#(\mathcal{C}) = 500$, (top:) $\tau = 0$ and (bottom:) $\tau = 0.5$.

7. REFERENCES

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