MINIMIZING TRANSMIT-POWER FOR COHERENT COMMUNICATIONS IN WIRELESS SENSOR NETWORKS USING QUANTIZED CHANNEL STATE INFORMATION*

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ABSTRACT

We consider minimizing average transmit-power with finite-rate feedback for coherent communications in a wireless sensor network (WSN), where sensors communicate with a fusion center (FC) using adaptive modulation and coding over a wireless fading channel. By viewing the coherent WSN setup as a distributed space-time multi-input single-output (MISO) system, we develop beamforming and resource allocation strategies and design optimal quantizers when the sensors only have available quantized (Q-) channel state information at the transmitters (CSIT) through a finite-rate feedback channel. Numerical results reveal that our novel design based on Q-CSIT yields significant power savings even for a small number of feedback bits.

Index Terms— Minimum energy control, Quantization, MISO systems, Optimization methods, Multi-sensor systems.

1. INTRODUCTION

A wireless sensor network (WSN) comprises a large number of spatially distributed signal processing devices (nodes), each with a non-rechargeable battery and thus limited computing and communication capabilities [1]. A main objective of current WSN research is to design power-efficient devices and algorithms to support different aspects of network operations (see e.g., [6], [9]). The WSN in these works includes a fusion center (FC) with which sensors are linked.

When these links are fading, communication performance across the WSN coverage area is severely degraded. A well-known approach to mitigate the adverse effects of fading relies on transmissions adapting to channel state information (CSI). In practice, CSI at transmitters (CSIT) is typically acquired through a limited-rate feedback channel from the receiver, and thus, only *quantized* (Q-) CSIT is available [3]. This finite-rate feedback model is pragmatically affordable and is robust to channel estimation errors, feedback delay and jamming. Adaptive transmissions and/or beamforming based on Q-CSIT have been optimized for multi-input multi-output (MIMO) systems to maximize rate or receive-signal-to-noise-ratio (SNR) [5], or in multi-user systems to optimize power-efficiency [6].

This paper deals with a *distributed* multi-input single-output (MISO) communication system in the power-limited regime of a WSN where sensor transmissions arrive coherently at the FC [7]. Based on Q-CSIT (i.e. when the sensors only have quantized knowledge of their links with the FC) the sensors' average transmit-power is minimized subject to average rate and BER constraints. Specifically, we derive the corresponding optimal adaptive modulation,

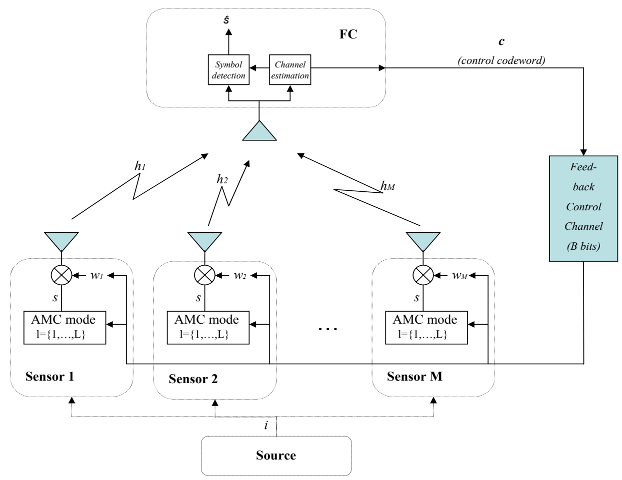

Fig. 1. System model.

power loading and beamforming strategies as well as optimal channel quantizers needed to form the required Q-CSIT.[1]

2. MODELING PRELIMINARIES

We consider a WSN setup where M sensors wish to communicate an information message (say the value of a random variable they track or information they relay) to the FC; see Fig. 1. We assume that: **(as1):** *the information is common to all sensors and arrives coherently at the FC*. With $\{h_m\}_{m=1}^M$ denoting block fading channel coefficients between sensors and the FC, we further assume that: **(as2):** $\{h_m\}_{m=1}^M$ *are independent*[2] *and identically distributed (i.i.d.) according to a complex Gaussian distribution with zero mean and unit variance, i.e.,* $h_m \sim \mathcal{CN}(0,1)$*; and each block fading channel is ergodic;* and, **(as3):** *the FC feeds back to the sensors the CSI indicated by* B *bits per channel realization, without error and with*

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[1]Notation: We use boldface lower-case letters to denote column vectors, ^{T} to denote transposition, † conjugate, $^\mathcal{H}$ conjugate transposition, and $\|\cdot\|$ the Euclidean norm. For a random variable x, $f_x(x)$ will denote its probability density function (PDF), and $F_x(x)$ its cumulative distribution function (CDF). Furthermore, $\mathcal{CN}(\mu,\sigma^2)$ will denote the complex-Gaussian distribution with mean μ and variance σ^2, $\lceil x \rceil$ the minimum integer $\geq x$, and $E_x[\cdot]$ the expectation operator over x.

[2]The extension to correlated channels is possible but is left for future research.

negligible delay[3].

Given a pool of adaptive modulation and coding (AMC) pairs, we suppose that each sensor supports a finite number L of AMC modes indexed by $l \in \{1, \ldots, L\}$, with each mode having constellation size M_l and transmission rate $r_l := \log_2(M_l)$. To guarantee quality of service, the rates $\{r_l\}_{l=1}^{L}$ must be delivered with a prescribed BER ϵ_0. To mitigate the effects of fading, the sensors coordinate their transmission strategy. Using a given AMC mode, the mth sensor transmits the common symbol s multiplied by a complex steering weight w_m. Let $\mathbf{w} := [w_1, \ldots, w_M]^T$ denote the distributed beamforming vector and $\mathbf{h} := [h_1, \ldots, h_M]^T$ the fading MISO channel. The received symbol y at the FC can be expressed as

$$y = \mathbf{w}^T \mathbf{h} s + v := \sqrt{P}\mathbf{u}^T \mathbf{h} s + v \quad (1)$$

where $\sqrt{P} := \|\mathbf{w}\|$, $\mathbf{u} := \mathbf{w}/\|\mathbf{w}\|$, and v denotes the additive complex Gaussian noise with zero mean and variance N_0. Notice that both the phase and the modulus of $\mathbf{w}$ can be tuned to effect not only distributed beamforming but also power allocation per fading state $\mathbf{h}$. Let $\mathbf{c}$ denote the B-bit Q-CSI codeword that the FC feeds back to the sensors per (as3). Based on $\mathbf{c} = \mathbf{c}(\mathbf{h})$, the sensors adapt their transmit-parameters to one of $N = 2^B$ prescribed modes specifying the transmission rate $r = r(\mathbf{c})$, transmit-power $P = P(\mathbf{c})$ and beamforming vector $\mathbf{u} = \mathbf{u}(\mathbf{c})$.

3. SOLUTION BASED ON Q-CSIT

Our goal in this section is to optimally design the channel quantizer which yields $\mathbf{c}(\mathbf{h})$ as well as adapt $r = r(\mathbf{c})$, $P = P(\mathbf{c})$, and $\mathbf{u} = \mathbf{u}(\mathbf{c})$, so that the total average power transmitted by all sensors is minimized subject to average rate and BER requirements.

In the context of collocated space-time MISO communications, the power and beamformer adaptation to maximize the capacity can be solved separately without loss of optimality (w.l.o.o.) [8]. Extending this result to our distributed power minimization set-up allows splitting the original problem into two separate subproblems.

This way, per fading realization, the FC quantizes $\mathbf{h}$ to find separately the optimal $\mathbf{u}$ according to a quantizer $Q_u(\cdot)$, and the optimal AMC mode and P according to a different quantizer $Q_t(\cdot)$. Concatenating the beamforming vector index $\mathbf{c}_u = Q_u(\mathbf{h})$ with the transmission mode index $\mathbf{c}_t = Q_t(\mathbf{h})$, the FC feeds back the B-bit Q-CSI codeword $\mathbf{c} = [\mathbf{c}_u; \mathbf{c}_t]$, where $B = B_u + B_t$, $B_u :=$ *length*$(\mathbf{c}_u)$, and $B_t :=$ *length*$(\mathbf{c}_t)$. Using $\mathbf{c}$, the sensors will adapt $r = r(\mathbf{c}_t)$, $P = P(\mathbf{c}_t)$, and $\mathbf{u} = \mathbf{u}(\mathbf{c}_u)$ to minimize their total average transmit-power.

3.1. Optimal Distributed Beamformer

With only B_u bits available, the beamforming vector $\mathbf{u}$ is chosen from a finite set $\mathcal{U} := \{\mathbf{u}_i\}_{i=1}^{N_u}$, where $N_u = 2^{B_u}$. Since $\|\mathbf{u}\| = 1$, to minimize power we can select $\mathbf{u}$ to maximize the receive-SNR in (1) per channel realization $\mathbf{h}$ for any fixed $r(\mathbf{c}_t)$ and $P(\mathbf{c}_t)$. The optimal[4] beamforming vector minimizing the transmit-power is clearly

$$\mathbf{u}^*(\mathbf{h}) = \arg\max_{\mathbf{u}_i \in \mathcal{U}} \gamma(\mathbf{h}) = \arg\max_{\mathbf{u}_i \in \mathcal{U}} \left|\mathbf{u}_i^T \mathbf{h}\right|^2 \quad (2)$$

where $\gamma(\mathbf{h})$ represents the receive-SNR in (1). Optimal codebooks $\mathcal{U}$ have been designed for collocated MISO systems [5, 10]. Under various criteria, optimal codebooks minimize chordal distance between unitary codewords, $d_{ch}(\mathbf{u}_i, \mathbf{u}_j) := \left(1 - |\mathbf{u}_i^T \mathbf{u}_j|^2\right)^{\frac{1}{2}}$ (see [5]). Therefore, the optimization in (2) can be expressed as

$$\mathbf{u}^*(\mathbf{h}) = \arg\min_{\mathbf{u}_i \in \mathcal{U}} d_{ch}\left(\mathbf{u}_i, \tfrac{\mathbf{h}}{\|\mathbf{h}\|}\right) \quad , \quad (3)$$

and the optimal codebook $\mathcal{U}^*$ as

$$\mathcal{U}^* = \max_{\{\mathbf{u}_i\}_{i=1}^{N_u}} \min_{\forall i \neq j} d_{ch}(\mathbf{u}_i, \mathbf{u}_j). \quad (4)$$

For arbitrary beamformer sizes M and codebook sizes $N_u > M$, numerically solutions of (4) are available; see, [4]. With the optimal codebook $\mathcal{U}^*$ available to both the FC and the sensors, the Q-CSI and optimal beamforming vector are then determined as

$$\mathbf{c}_u = Q_u(\mathbf{h}) := \arg_i \min_{\mathbf{u}_i \in \mathcal{U}^*} \left\{ d_{ch}\left(\mathbf{u}_i, \frac{\mathbf{h}}{\|\mathbf{h}\|}\right)\right\} \quad (5)$$

$$\mathbf{u}^*(\mathbf{h}) = \mathbf{u}_{\mathbf{c}_u}. \quad (6)$$

3.1.1. Statistical Description of the Equivalent Scalar Channel

Let E_s denote the energy per symbol and N_0 the noise density. Defining $g := \|\mathbf{h}\|^2 E_s/N_0$ and $z := \min_{\mathbf{u}_i \in \mathcal{U}} d_{ch}^2(\mathbf{u}_i, \mathbf{h}/\|\mathbf{h}\|)$ we can write the receive-SNR in (1) for $\mathbf{u} = \mathbf{u}^*(\mathbf{h})$ as

$$\gamma(\mathbf{h}) = Pg[1 - d_{ch}(\mathbf{u}^*(\mathbf{h}), \mathbf{h}/\|\mathbf{h}\|)] = Pg[1 - z] := P\tilde{g}, \quad (7)$$

where g can be interpreted as the scalar channel gain when the optimum beamformer corresponds to $\mathbf{u}^*(\mathbf{h}) = \mathbf{h}^\dagger/\|\mathbf{h}\|$ (i.e., $N_u = \infty$); z as the receive-SNR loss due to quantization; and $\tilde{g} := g(1 - z)$ as the instantaneous scalar channel gain when quantization via (5) and (6) is implemented.

Assuming, without loss of generality that $E_s/N_0 = \bar{g} = 1$, per (as2), g adheres to a chi-squared distribution with PDF $f_g(g) = \left(g^{M-1}\exp(-g)\right) / \left(\bar{g}^M \Gamma(M)\right)$ where $\Gamma(b, x) := \int_x^\infty t^{b-1} e^{-t} dt$ is the incomplete Gamma function and $\Gamma(b) := \Gamma(b, 0)$. On the other hand, based on the union bound, the CDF of z, can be upper-bounded tightly as: $F_z(z)\tilde{F}_z(z)$ where $\tilde{F}_z(z) = N_u z^{M-1}$ if $0 \leq z \leq z_{\max}$, $\tilde{F}_z(z) = 1$ if $z \geq z_{\max}$, and $z_{\max} := N_u^{-1/(M-1)}$ (see [10]).

Because g and z are independent [c.f. (as2)], using the approximation $F_z(z) \simeq \tilde{F}_z(z)$, we can obtain the CDF of $\tilde{g}$ as $F_{\tilde{g}}(x) = \Pr\{g(1 - z) < x\}$

$$\begin{aligned} F_{\tilde{g}}(x) &= \int_{g=0}^{\frac{x}{1-z_{\max}}} \int_{z=\max(0,1-x/g)}^{z_{\max}} f_z(z) dz f_g(g) dg \\ &= \int_{g=0}^{x} \int_{z=0}^{z_{\max}} f_z(z) dz f_g(g) dg + \\ &\int_{g=x}^{\frac{x}{1-z_{\max}}} \int_{z=1-x/g}^{z_{\max}} f_z(z) dz f_g(g) dg = 1 - \tfrac{\Gamma(M, x/(1-z_{\max}))}{\Gamma(M)} \\ &-N\exp(-x)\left(1 - \tfrac{\Gamma(M, x z_{\max}/(1-z_{\max}))}{\Gamma(M)}\right) \end{aligned} \quad (8)$$

The PDF of $\tilde{g}$ can be in turn obtained as $f_{\tilde{g}}(\tilde{g}) = \partial F_{\tilde{g}}(\tilde{g})/\partial \tilde{g}$.

3.2. Optimal Rate and Power and Allocation

It follows that once $\mathbf{u}^*(\mathbf{h})$ is determined via (5) and (6), the MISO channel (1) is fully characterized by an equivalent scalar channel with power gain $\tilde{g}$. This implies that solving for the optimal $r^*(\mathbf{h})$ and $P^*(\mathbf{h})$ is equivalent to finding the optimal $r^*(\tilde{g})$ and $P^*(\tilde{g})$. Notice that since $\mathbf{h}$ (and thus $\tilde{g}$) varies from one realization to the next, rate and power will be adapted across time in order to minimize the *average* transmit-power under an *average* rate constraint.

[3] This can be easily guaranteed with sufficiently strong error control codes, since the feedback channel has typically low rate.

[4] Henceforth, x^* will denote the optimal value of x.

We order the AMC modes such that $r_l < r_{l+1}\ \forall l > 1$ and let the first mode represent the inactive mode with zero rate and power ($r_1 = 0$). When only finite-rate feedback is available, the FC needs to quantize $\tilde{g}$ using a finite number of regions. In this case, identifying each quantization region with an AMC mode selection emerges as a natural framework. We will consider L different quantization regions $\{\mathcal{R}_l := [\tilde{\tau}_l, \tilde{\tau}_{l+1})\}_{l=1}^L$, with $\tilde{\tau}_1 = 0$ and $\tilde{\tau}_{L+1} = \infty$, and we associate with them the vector of thresholds $\tilde{\boldsymbol{\tau}} := [\tilde{\tau}_1, \ldots, \tilde{\tau}_{L+1}]^T$. The lth transmission mode is characterized by the rate-power pair $(r_l, \tilde{P}_l)$ in the quantization region $\mathcal{R}_l$. While r_l is fixed for the given AMC mode, we will select the fixed $\tilde{P}_l$ to satisfy the BER requirement. Clearly, the average BER $\tilde{\epsilon}_l^Q$ for the region $\mathcal{R}_l$ can be obtained as the expected number of erroneous bits divided by the expected number of transmitted bits; i.e.,

$$\tilde{\epsilon}_l^Q(\tilde{\tau}_l, \tilde{\tau}_{l+1}, \tilde{P}_l, r_l) := \mathbb{E}_{\tilde{g}\in[\tilde{\tau}_l,\tilde{\tau}_{l+1})}\left[r_l \epsilon(\tilde{g}, \tilde{P}_l, r_l)\right] / \mathbb{E}_{\tilde{g}\in[\tilde{\tau}_l,\tilde{\tau}_{l+1})}[r_l]. \quad (9)$$

where $\epsilon(\cdot)$ denotes the instantaneous BER function as a function of the channel gain $\tilde{g}$, the loaded power $\tilde{P}_l$ and the transmit-rate r_l. To satisfy the BER requirement ϵ_0, we need to set $\tilde{\epsilon}_l^Q(\tilde{\tau}_l, \tilde{\tau}_{l+1}, \tilde{P}_l, r_l) = \epsilon_0$, substituting the latter into (9) yields

$$\begin{aligned} f_\epsilon(\tilde{\tau}_l, \tilde{\tau}_{l+1}, \tilde{P}_l, r_l, \epsilon_0) := \quad & \int_{\tilde{\tau}_l}^{\tilde{\tau}_{l+1}} \epsilon(\tilde{g}, \tilde{P}_l, r_l) f_{\tilde{g}}(\tilde{g}) d\tilde{g} \\ & -\epsilon_0 \int_{\tilde{\tau}_l}^{\tilde{\tau}_{l+1}} f_{\tilde{g}}(\tilde{g}) d\tilde{g} = 0 \quad . \end{aligned} \quad (10)$$

Note that f_ϵ can be analytically found for many modulations schemes. Based on the latter, $\tilde{P}_l$ can be found by a one-dimensional search over f_ϵ. If we define this solution as $\tilde{P}_l(\tilde{\tau}_l, \tilde{\tau}_{l+1}, r_l, \epsilon_0)$, and considering that lth AMC mode will be chosen if $g \in [\tau_l, \tau_{l+1})$, the rate and power allocation $\forall \tilde{g}$ can be expressed as

$$\tilde{r}(\tilde{g}) = r_l; \quad \text{if } \tilde{g} \in [\tilde{\tau}_l, \tilde{\tau}_{l+1}) \quad (11)$$

$$\tilde{P}(\tilde{g}, \epsilon_0) = \begin{cases} 0, & \tilde{g} \in [\tilde{\tau}_1, \tilde{\tau}_2) \\ \tilde{P}_l(\tilde{\tau}_l, \tilde{\tau}_{l+1}, r_l, \epsilon_0), & \tilde{g} \in [\tilde{\tau}_l, \tilde{\tau}_{l+1}),\ l > 1. \end{cases} \quad (12)$$

3.2.1. Constrained Power Minimization

Equations (11) and (12) imply that to find the optimal rate and power allocations, we only need to search for the optimal $\tilde{\boldsymbol{\tau}}^*$ which solves the following constrained minimization problem:

$$\begin{cases} \min\limits_{\tilde{\boldsymbol{\tau}}} \quad \bar{P} := \sum\limits_{l=1}^{L} \tilde{P}_l(\tilde{\tau}_l, \tilde{\tau}_{l+1}, r_l, \epsilon_0) \int_{\tilde{\tau}_l}^{\tilde{\tau}_{l+1}} f_{\tilde{g}}(\tilde{g}) d\tilde{g} \\ \text{subject to}:\ C1. \sum\limits_{l=1}^{L} r_l \int_{\tilde{\tau}_l}^{\tilde{\tau}_{l+1}} f_{\tilde{g}}(\tilde{g}) d\tilde{g} \geq r_0 \end{cases} \quad (13)$$

where both transmit-power in the objective as well as transmit-rate in $C1$ are averaged over all channel regions (quantization states). Notice that the loaded power does not vary with the channel gain, but only with the region index. In fact, $\int_{\tilde{\tau}_l}^{\tilde{\tau}_{l+1}} f_{\tilde{g}}(\tilde{g}) d\tilde{g} = F_{\tilde{g}}(\tilde{\tau}_{l+1}^*) - F_{\tilde{g}}(\tilde{\tau}_l^*)$ can be interpreted either as the probability of falling into the lth quantization region (or selecting the lth AMC mode).

Next we use the Karush-Kuhn-Tucker (KKT) conditions to find $\tilde{\tau}_l^*$. If $\tilde{\lambda}$ denote the Lagrange multiplier associated with **to** the rate constraint $C1$ the KKT condition at the optimal $\tilde{\tau}_l^*$ dictates

$$\begin{aligned} \frac{\partial \mathcal{L}(\tilde{\lambda}^*, \tilde{\boldsymbol{\tau}}^*)}{\partial \tau_l} &= f_{\tilde{g}}(\tilde{\tau}_l^*) \Big[\tilde{P}_{l-1}(\tilde{\tau}_{l-1}^*, \tilde{\tau}_l^*, r_{l-1}, \epsilon_0) - \tilde{\lambda}^* r_{l-1} \\ &-\tilde{P}_l(\tilde{\tau}_l^*, \tilde{\tau}_{l+1}^*, r_l, \epsilon_0) + \tilde{\lambda}^* r_l\Big] + \frac{\partial \tilde{P}_{l-1}}{\partial \tilde{\tau}_l}(\tilde{\tau}_{l-1}^*, \tilde{\tau}_l^*, r_{l-1}, \epsilon_0) \\ &\times \int_{\tilde{\tau}_{l-1}^*}^{\tilde{\tau}_l^*} f_{\tilde{g}}(\tilde{g}) d\tilde{g} + \frac{\partial \tilde{P}_l}{\partial \tilde{\tau}_l}(\tilde{\tau}_l^*, \tilde{\tau}_{l+1}^*, r_l, \epsilon_0) \int_{\tilde{\tau}_l^*}^{\tilde{\tau}_{l+1}^*} f_{\tilde{g}}(\tilde{g}) d\tilde{g} = 0. \end{aligned} \quad (14)$$

Since $\tilde{P}_l(\tilde{\tau}_{l-1}^*, \tilde{\tau}_l^*, r_{l-1}, \epsilon_0)$ is an implicit function, to calculate $\frac{\partial \tilde{P}_l}{\partial \tau_l}$ we rely on the implicit differentiation theorem: $df_\epsilon = \frac{\partial f_\epsilon}{\partial x} dx + \frac{\partial f_\epsilon}{\partial y} \frac{\partial y}{\partial x} dx = 0$, which yields $\frac{\partial y}{\partial x} = \frac{-\partial f_\epsilon / \partial x}{\partial f_\epsilon / \partial y}$. Therefore, $\forall l \in \{2, \ldots, L\}$ and $\forall i \in \{1, \ldots, L\}$ we have $\frac{\partial \tilde{P}_i}{\partial \tilde{\tau}_l}(\tilde{\tau}_i, \tilde{\tau}_{i+1}, r_i, \epsilon_0)$

$$= \begin{cases} -\frac{[-\epsilon(\tilde{\tau}_l, \tilde{P}_i, r_l) + \epsilon_0] f_{\tilde{g}}(\tilde{\tau}_l)}{\int_{\tilde{\tau}_i}^{\tilde{\tau}_{i+1}} [\partial \epsilon(\tilde{g}, \tilde{P}_i, r_l)/\partial P] f_{\tilde{g}}(\tilde{g}) d\tilde{g}}, & i = l; \\ \frac{[-\epsilon(\tilde{\tau}_l, \tilde{P}_i, r_{l-1}) + \epsilon_0] f_{\tilde{g}}(\tilde{\tau}_l)}{\int_{\tilde{\tau}_i}^{\tilde{\tau}_{i+1}} [\partial \epsilon(\tilde{g}, \tilde{P}_i, r_{l-1})/\partial P] f_{\tilde{g}}(\tilde{g}) d\tilde{g}}, & i = l-1; \\ 0, & \text{otherwise.} \end{cases} \quad (15)$$

Notice that calculating the optimal $\tilde{\tau}_l^*$ here depends not only on $\tilde{\lambda}^*$ but also on the previous $\tilde{\tau}_{l-1}^*$ and the next $\tilde{\tau}_{l+1}^*$. This prevents one from obtaining a closed-form expression for $\tilde{\tau}_l^*$. Therefore can be obtained numerically through a two-dimensional search which is computationally affordable.

Algorithm 1: Off-line Power-Efficient Quantization

[S1.0] Let δ denote **the** small tolerance, ε small step size, and $\tilde{\tau}_L^{\max} > 0$ the maximum value for the highest quantization threshold (e.g., a value bringing the probability of the highest region close to 0).

[S1.1] Initialize $\tilde{\lambda}$ as a small positive number and $\tilde{\tau}_L = \tilde{\tau}_L^{\max}$; then calculate $\{\tilde{\tau}_l\}_{l=2}^L$ via solving (14). If $C2$ is not satisfied for some τ_l, $\tau_l = \tau_{l+1}$. If the obtained solution is feasible, go to (S3.2); otherwise decrease $\tilde{\tau}_L = \tilde{\tau}_L - \varepsilon$ and repeat (S3.1).

[S1.2] Based on the closed-form in (8), calculate the average rate $\bar{r} = \sum_{l=1}^L (F_{\tilde{g}}(\tilde{\tau}_{l+1}) - F_{\tilde{g}}(\tilde{\tau}_l)) r_l$. Check $C1$ and if $|\bar{r} - r_0|/r_0 < \delta$ then *stop*; otherwise, calculate $\triangle\tilde{\lambda} := (\bar{r} - r_0)c$ (parameter c is an adaptive penalty parameter [2]), update the multiplier $\tilde{\lambda} = \tilde{\lambda} + \triangle\tilde{\lambda}$, and go back to *[S1.1]*.

Notice that the main computational complexity burden in Algorithm 1 pertains to the calculation of the optimal thresholds in step 1, which requires a two-dimensional search. Once the optimal $\tilde{\lambda}^*$ as well as $\{\tilde{\tau}_l^*\}_{l=2}^L$ are calculated, the optimal quantizer $Q_t(\cdot)$ can be readily determined as

$$\mathbf{c}_t = Q_t(\mathbf{h}) := \arg_i \left\{ g\left[1 - \min_{\mathbf{u}\in\mathcal{U}} d_{ch}^2(\mathbf{u}, \mathbf{h}/\|\mathbf{h}\|)\right] \in [\tilde{\tau}_i, \tilde{\tau}_{i+1}) \right\}. \quad (16)$$

Then using the AMC mode index $\mathbf{c}_t$, the optimal rate and power allocation are obtained via (11) and (12).

3.3. On-line Feedback and Adaptation of Transmitters

Based on the optimal beamforming and resource allocation policy, we outline next the on-line algorithm executed by the FC and the sensors per channel realization.

Algorithm 2: For each channel realization $\mathbf{h}$:

[S2.1] The FC obtains $\mathbf{c}_u = Q_u(\mathbf{h})$ and $\mathbf{c}_t = Q_t(\mathbf{h})$ using, respectively, (5) and (16), and broadcasts the aggregate codeword $\mathbf{c} = [\mathbf{c}_u; \mathbf{c}_t]$ to the sensors.

[S2.2] Each sensor m transmits according to the mth entry of the optimal beamforming vector indexed by $\mathbf{c}_u$, and loads the optimal power and rate allocation indexed by $\mathbf{c}_t$.

Notice that the quantized optimal beamforming and resource allocation configurations must be revealed to both the FC and sensors during the training phase. In step *[S2.2]* the optimal transmit-power $\tilde{P}_{\mathbf{c}_t}$ is calculated at the sensors by solving (10).

Table 1. Average transmit-power (in dB_W) for (P, I, Q, and S)-CSIT schemes. (Reference case: $M = 4$, $r_0 = 2.5$, $\epsilon_0 = 10^{-3}$, $L = 4$, $r_l = [0, 1, 3, 5]$, $N_u = 16$, $E_s/N_0 = 1$; in other CASES, only one indicated parameter is changed w.r.t. the reference case.)

CASE	P-CSIT	Q-CSIT	S-CSIT	Open-loop
Reference Case	8.6	10.7	13.8	30.5
$r_0 = 1.75$	6.0	7.7	11.8	28.4
$r_0 = 3.0$	10.3	12.6	15.2	31.8
$M = 6$	5.0	6.9	11.2	28.7
$E_s/N_0 = 3$	3.9	5.9	9.1	25.7
$\epsilon_0 = 10^{-4}$	10.1	12.1	16.7	33.4
$L = 6$	8.5	9.8	13.8	30.5
$r_l = [0, 1, 2.6, 4]$	8.7	10.5	13.2	29.9

4. SIMULATIONS

In this section, we present numerical results of the transmit-power consumed by the sensors based Q-CSIT. The system parameters are selected to satisfy $E_s/N_0 = 1$. The simple cases tested include four sensors with fading links adhering to (as1). Unless otherwise specified, we suppose that each sensor supports $L = 4$ AMC modes implementing M-QAM modulations with transmission rates: $r_l = 0,\ 1,\ 3,\ 5\ bits/symbol$. In all simulations, we set the BER requirement to $\epsilon_0 = 10^{-3}$ and the codebook of beamforming vector has size $N_u = 16$.

For variable rate requirements, Table 1 lists the average transmit-power in dB_W for our Q-CSIT design. For comparison and illustration purposes we consider three additional CSIT scenarios: (i) perfect (P-) CSIT (the sensors implement optimal spatial beamforming and power and rate allocation based on perfect deterministic knowledge of $\mathbf{h}$, clearly the design based on P-CSIT is a benchmark for our Q-CSIT design); (ii) spatial (S-) CSIT (the sensors implement optimal spatial beamforming based on the perfect $\mathbf{h}/\|\mathbf{h}\|$ but do not implement temporal power allocation across time); and (iii) open-loop system (with no feedback, the sensors transmit with homogeneous rate, power and beamformer). From Table 1, we have the following interesting observations: (i) Q-CSIT strategy can achieve power efficiency close to the benchmark P-CSIT based one (only $1 \sim 3dB$ loss); (ii) Q-CSIT based strategy clearly outperforms the optimal S-CSIT scheme although the latter requires P-CSIT of $\mathbf{h}$ while the former only requires of few bits of feedback (this is basically due to the fact that S-CSIT scheme does not exploit the temporal diversity of fading channel); and, (iii) power consumed by the open-loop design is $20 \sim 25\ dB$ higher than that of our closed-loop Q-CSIT design (this certifies that as expected, CSIT can largely reduce power requirements, and thus considerably increase the *lifespan* of the WSN).

The previous results correspond to the case where the required feedback to form the Q-CSIT codeword is $B = \lceil \log_2(4) + \log_2(16) \rceil = 6$ bits per channel realization. Results in Table 2 gauges how sensitive is the power performance with respect to (w.r.t.) the number of feedback bits. The main conclusion of the listed results is that our Q-CSIT does not require a high number of feedback bits to perform close to the P-CSIT benchmark, moreover we also see how the first and second increments of L and N_u bring the largest power savings and how when $L = N_u = \infty$, Q-CSIT=P-CSIT.

5. CONCLUSIONS

In a WSN entailing coherent sensor communications with a fusion center, we minimized the average transmit-power subject to average

Table 2. Average transmit-power (in dB_W) as (L, N_u) varies.

(L, N_u)	(1,16)	(2,16)	(4,16)	(4,4)	(4,512)	(∞, ∞)
P-CSIT	9.8	9.4	8.6	8.6	8.6	8.3
Q-CSIT	14.2	11.7	10.7	11.7	9.6	8.3

rate and BER requirements when quantized (Q-) CSIT is available though a limited rate feedback link. We separated the main design in two subproblems: (i) spatial quantization and beamforming, and (ii) rate/power quantization and allocation. By exploiting the parallelism between the coherent WSN setup and a distributed MISO system, we relied on non-linear programming tools to solve the programs at hand and derived the corresponding power-efficient channel quantization and adaptive transmission policies. Numerical results confirmed that our Q-CSIT based solution attain power efficiency surprisingly close to the perfect CSIT based benchmark, outperform schemes which only exploits spatial diversity and offer significant power savings relative to open loop systems that do not exploit CSIT

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