ACHIEVABLE RATE FOR GAUSSIAN MULTIPLE RELAY CHANNELS WITH LINEAR RELAYING FUNCTIONS

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ABSTRACT

The achievable rate of a Gaussian multiple relay channel with linear relaying functions is derived here. With *Linear relaying*, relays transmit (on every channel use) a causal linear combination of their past received inputs. In this paper, the optimum temporal covariance of the signal transmitted by the source is derived and linear relaying functions proposed. Results show that *Linear relaying* outperforms all known relaying techniques for relay channels and generates the tightest lower bound on the capacity of the multiple relay channel.

Index Terms— Multiple relay channel, linear relaying functions.

1. INTRODUCTION

The wireless multiple relay channel (MRC) consists of a single source-destination pair aided in its communication by $N > 1$ wireless relays. This architecture is shown to improve capacity (and reliability) of wireless channels when properly allocating network resources [1].

Capacity of MRC is still an open problem for the field of Information Theory. However, it has been analyzed and bounded by means of the achievable rates of the four known forwarding techniques for relay channels (and their superposition and time-sharing), namely: *1*) *Decode-and-forward*. It was presented in [2] for one relay node and extended to MRC in [1]. Constructed through block-Markov encoding and forward decoding, it is shown to be spectrally inefficient due to signal regeneration at the relay node. *2*) *Amplify-and-forward*, where relay nodes transmit an amplified version of the signal received from the source. Its main drawback is the noise amplification in the low SNR regime [3]. *3*) *Partial decoding*, where relay nodes just decode and retransmit a part of the source message. Proposed in [2] for one relay, it is extended to synchronous and asynchronous MRC in [4], showing significant capacity gains. It is the highest known achievable rate for the MRC. Finally, *4*) *Compress-and-forward*. It was also introduced in [2], and consists of relays retransmitting a compressed version of their received input. It is extended to the MRC in [1], and claimed to be spectrally inefficient in the high SNR regime in [5].

The aim of this paper is to contribute a new (and tighter) achievable rate for the MRC, based upon *Linear Relaying* at the relay nodes [6]. With *Linear Relaying*, relays send (on every channel use) a linear combination of their past received signals[1] (referred to as linear relaying function). In such a set up, two parameters can be optimized in order to achieve higher capacity: *i*) the linear relaying functions at the relay nodes, and *ii*) the temporal covariance of the signal transmitted by the source. Our contribution analyzes a MRC with transmit and receive channel knowledge at all network nodes, and overall energy constraint. It derives the optimum temporal covariance matrix for the source signal, and proposes suboptimum linear relaying functions that outperform all previously known relaying schemes. Indeed, results show a capacity increase of up to 1 bps/Hz with respect to well known *amplify-and-forward* and *decode-and-forward* architectures.

The remainder of this paper is organized as follows: Section 2 introduces signal model and defines the achievable rate. In Section 3, we derive the optimum covariance at the source and in Section 4 we propose the suboptimum relaying functions. Finally, Section 5 depicts numerical results and Section 6 summarizes conclusions.

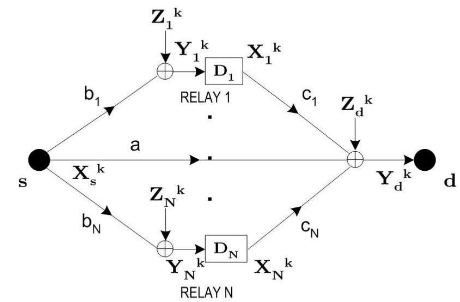

Fig. 1. Multiple relay channel with linear relaying functions.

2. SIGNAL MODEL AND CAPACITY

We consider a wireless MRC with a source node s, a destination node d, and a set of N relay nodes[2] (See Fig. 1). Wireless channels among nodes are *time-invariant*, *memoryless*, and modelled with a complex coefficient, where a denotes the complex channel between source and destination, and b_i and c_i the complex channel from source to relay i, and relay i to destination, respectively.

During κ uses of the channel, the source transmits a sequence of random symbols $\mathbf{X}_s^\kappa = \left[\mathrm{X}_s^1, \cdots, \mathrm{X}_s^\kappa\right]^T \in \mathrm{C}^\kappa$, which is received at both relays and destination under additive noise. The received sequence at relay $i \in \{1, \cdots, \mathrm{N}\}$ is written as:

$$\mathbf{Y}_i^\kappa = b_i \cdot \mathbf{X}_s^\kappa + \mathbf{Z}_i^\kappa, \quad (1)$$

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[1] Notice that *amplify-and-forward* is just a particular case of *Linear Relaying*.

[2] All network nodes have only one antenna

being $\mathbf{Z}_i^\kappa = \left[\mathrm{Z}_i^1, \cdots, \mathrm{Z}_i^\kappa\right]^T \in \mathrm{C}^\kappa$ the noise sequence. As mentioned earlier, the set of relays aids the source-destination communication by retransmitting a causal linear combination of their received input. Hence, relay $i \in \{1, \cdots, \mathrm{N}\}$ transmits a sequence of κ symbols $\mathbf{X}_i^\kappa = \left[0, \mathrm{X}_i^2, \cdots, \mathrm{X}_i^\kappa\right]^T \in \mathrm{C}^\kappa$, where

$$\begin{aligned} \mathbf{X}_i^\kappa &= \mathbf{D}_i \cdot \mathbf{Y}_i^\kappa \\ &= \mathbf{D}_i \cdot \left(b_i \cdot \mathbf{X}_s^\kappa + \mathbf{Z}_i^\kappa\right), \end{aligned} \tag{2}$$

where $\mathbf{D}_i \in \mathrm{C}^{\kappa\times\kappa}$ is the linear relaying matrix of relay i, defined strictly lower triangular to preserve causality in the system (i.e., $\left[\mathbf{D}_i\right]_{p,q} = 0$ for $p \le q$, with p row and q column). Then, considering the signal transmitted by source and relays, the received sequence at the destination node reads

$$\mathbf{Y}_d^\kappa = \left(a \cdot \mathbf{I} + \sum_{i=1}^N b_i c_i \mathbf{D}_i\right) \cdot \mathbf{X}_s^\kappa + \left(\mathbf{Z}_d^\kappa + \sum_{i=1}^N c_i \mathbf{D}_i \mathbf{Z}_i^\kappa\right). \tag{3}$$

The achievable rate $\mathcal{C}$ for such a linear relaying scheme equals the mutual information between the sequences transmitted and received by the source and destination, respectively [7]:

$$\mathcal{C} = \lim_{\kappa\to\infty} \max_{p_{\mathbf{X}_s^\kappa}, \mathcal{D}} \frac{1}{\kappa} \cdot I\left(\mathbf{X}_s^\kappa; \mathbf{Y}_d^\kappa\right) \tag{4}$$

with $\mathcal{D} = [\mathbf{D}_1, \cdots, \mathbf{D}_N]$. In this paper, maximization above is solved for Gaussian channels with an overall power constraint: *i*) hereafter, $\mathbf{Z}_s^\kappa$, $\mathbf{Z}_i^\kappa$ and $\mathbf{Z}_d^\kappa$ are assumed zero-mean, circularly symmetric, complex AWGN with power σ_o^2. Moreover, *ii*) the total transmitted power in the network during the κ uses of the channel is constrained to $\kappa\mathrm{P}$, i.e,

$$\mathrm{tr}\left\{\mathrm{E}\left\{\mathbf{X}_s^\kappa \left(\mathbf{X}_s^\kappa\right)^H\right\}\right\} + \sum_{i=1}^N \mathrm{tr}\left\{\mathrm{E}\left\{\mathbf{X}_i^\kappa \left(\mathbf{X}_i^\kappa\right)^H\right\}\right\} \le \kappa\mathrm{P} \tag{5}$$

where $\mathrm{E}\{\cdot\}$ denotes expectation and $\mathrm{tr}\{\cdot\}$ the matrix trace. Making use of the signal model in (2), the above constraint reduces to $\mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} \le \kappa\mathrm{P}$, where $\mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \in \mathrm{C}_+^{\kappa\times\kappa}$ is the covariance of $\mathbf{X}_s^\kappa$ (semidefinite positive) and

$$\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right) = \mathbf{\Sigma}_{\mathrm{X}_s^\kappa}\left(\mathbf{I} + \sum_{i=1}^N |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i\right) + \sigma_o^2 \sum_{i=1}^N \mathbf{D}_i^H \mathbf{D}_i. \tag{6}$$

Following Information Theory arguments in [6], definition (4) is maximized in AWGN when the signal transmitted by the source is Gaussian. Hence, the achievable rate (in [bps/Hz]) of the system remains

$$\begin{aligned} \mathcal{C} = \lim_{\kappa\to\infty} \max_{\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}} \frac{1}{\kappa} \cdot \log_2\left(\det\left(\mathbf{I} + \mathbf{H}_e \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \mathbf{H}_e^H\right)\right) \\ \text{s.t. } \mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} \le \kappa\mathrm{P}\ ,\ \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \succeq 0 \end{aligned} \tag{7}$$

where

$$\mathbf{H}_e = \frac{1}{\sqrt{\sigma_o^2}}\left(\mathbf{I} + \sum_{i=1}^N |c_i|^2 \mathbf{D}_i \mathbf{D}_i^H\right)^{-\frac{1}{2}}\left(a \cdot \mathbf{I} + \sum_{i=1}^N b_i c_i \mathbf{D}_i\right). \tag{8}$$

Maximization in (7) is not a convex optimization problem, and thus, no closed form expressions may be obtained. However, for fixed κ and fixed set of relaying functions $\mathcal{D}$, optimization is convex on $\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}$ (as shown in [5] for an optimization problem of the class of (7)) and, hence, the optimum source covariance matrix (conditioned on $\mathcal{D}$) can be found. The following section is devoted to the solution of this maximization.

3. SOURCE COVARIANCE OPTIMIZATION

The achievable rate of a MRC with a fixed set of relaying functions $\mathcal{D} = \{\mathbf{D}_1, \cdots, \mathbf{D}_N\} \in \mathrm{C}^{\kappa\times\kappa}$ is derived in Theorem 1.

Theorem 1 *Consider κ uses of the channel. Given a fixed set of relaying functions $\mathcal{D}$, with SVD-Decomposition $\mathbf{H}_e = \mathbf{U}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{V}^H$ and $\mathbf{\Lambda} = \mathrm{diag}([\lambda_1, \cdots, \lambda_\kappa])$:*
i) The achievable rate for the Gaussian multirelay channel reads

$$\begin{aligned} \mathcal{C}(\kappa, \mathcal{D}) &= \max_{\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}} \frac{1}{\kappa} \cdot \log_2\left(\det\left(\mathbf{I} + \mathbf{H}_e \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \mathbf{H}_e^H\right)\right) \\ &\quad \text{s.t. } \mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} \le \kappa\mathrm{P}\ ,\ \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \succeq 0 \\ &= \frac{1}{\kappa} \cdot \sum_{n=1}^\kappa \log_2\left(1 + \lambda_n \psi_n\right) \end{aligned} \tag{9}$$

where

$$\psi_n = \left[\frac{1}{\mu \cdot (1 + \phi_n)} - \frac{1}{\lambda_n}\right]^+,\ \phi_n = \left[\sum_{i=1}^N \mathbf{V}^H |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i \mathbf{V}\right]_{n,n} \tag{10}$$

$$\sum_{n=1}^\kappa \left[\frac{1}{\mu} - \frac{1 + \phi_n}{\lambda_n}\right]^+ = \kappa\mathrm{P} - \mathrm{tr}\left\{\sigma_o^2 \sum_{i=1}^N \mathbf{D}_i^H \mathbf{D}_i\right\}. \tag{11}$$

ii) Such rate is achievable with covariance matrix $\mathbf{\Sigma}_{\mathrm{X}_s^\kappa} = \mathbf{V}\mathbf{\Psi}\mathbf{V}^H$, being $\mathbf{\Psi} = \mathrm{diag}([\psi_1, \cdots, \psi_\kappa])$.

Proof: Demonstration follows two steps; first, we write the Lagrangian function and the KKT conditions for the problem (9). Later, we show that the covariance matrix that diagonalizes $\mathbf{H}_e$ (i.e, $\mathbf{\Sigma}_{\mathrm{X}_s^\kappa} = \mathbf{V}\mathbf{\Psi}\mathbf{V}^H$ with $\mathbf{\Psi}$ diagonal) satisfies KKT conditions, hence yielding to the optimum solution. The Lagrangian for optimization (9) is expressed as [8]:

$$\begin{aligned} \mathcal{L}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathbf{\Omega}, \mu\right) &= \log\left(\det\left(\mathbf{I} + \mathbf{H}_e \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \mathbf{H}_e^H\right)\right) \\ &+ \mathrm{tr}\left\{\mathbf{\Omega}\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}\right\} - \mu\left(\mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} - \kappa\mathrm{P}\right), \end{aligned} \tag{12}$$

where μ and matrix $\mathbf{\Omega}$ are the Lagrange multipliers for the first and second constraints, respectively. The KKT conditions for the problem are

$$\begin{aligned} &i)\ \mu\left(\mathbf{I} + \sum_{i=1}^N |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i\right) - \mathbf{\Omega} = \\ &\qquad \mathbf{H}_e^H\left(\mathbf{I} + \mathbf{H}_e \mathbf{\Sigma}_{\mathrm{X}_s^\kappa} \mathbf{H}_e^H\right)^{-1} \mathbf{H}_e \\ &ii)\ \mu\left(\mathrm{tr}\left\{\mathbf{P}\left(\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}, \mathcal{D}\right)\right\} - \kappa\mathrm{P}\right) = 0 \\ &iii)\ \mathrm{tr}\left\{\mathbf{\Omega}\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}\right\} = 0 \end{aligned} \tag{13}$$

Now, we show that for $\mathbf{\Sigma}_{\mathrm{X}_s^\kappa} = \mathbf{V}\mathbf{\Psi}\mathbf{V}^H$ the KKT conditions (necessary and sufficient for optimality) hold. First, let us multiply both sides of equality in condition *i*) in (13) by $\mathbf{V}^H$ at the left side and by $\mathbf{V}$ at the right, and introduce diagonalizing $\mathbf{\Sigma}_{\mathrm{X}_s^\kappa}$ in KKT's. The first condition turns into

$$\begin{aligned} &i)\ \mu\left(\mathbf{I} + \sum_{i=1}^N \mathbf{V}^H |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i \mathbf{V}\right) - \mathbf{V}^H \mathbf{\Omega} \mathbf{V} = \\ &\qquad \mathbf{\Lambda}^{\frac{1}{2}}\left(\mathbf{I} + \mathbf{\Lambda}^{\frac{1}{2}} \mathbf{\Psi} \mathbf{\Lambda}^{\frac{1}{2}}\right)^{-1} \mathbf{\Lambda}^{\frac{1}{2}} \end{aligned} \tag{14}$$

Since the right hand side of equality is a diagonal matrix, KKT condition *i*) holds if and only if

$$\mathbf{\Omega} = \mu \left(\mathbf{I} + \sum_{i=1}^{N} |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i \right) - \mathbf{V}\mathbf{P}\mathbf{V}^H, \qquad (15)$$

where $\mathbf{P}$ is diagonal. Furthermore, $\mathbf{\Omega}$ as defined in (15), satisfies KKT condition *iii*) if

$$\mathbf{P} = \mu \cdot \mathrm{diag} \left(\mathbf{I} + \sum_{i=1}^{N} \mathbf{V}^H |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i \mathbf{V} \right). \qquad (16)$$

Therefore, defining $\mathbf{\Phi} = \mathrm{diag}(\sum_{i=1}^{n} \mathbf{V}^H |b_i|^2 \mathbf{D}_i^H \mathbf{D}_i \mathbf{V})$, KKT's reduce to

$$i)\ \mu \cdot (\mathbf{I} + \mathbf{\Phi}) = \mathbf{\Lambda}^{\frac{1}{2}} \left(\mathbf{I} + \mathbf{\Lambda}^{\frac{1}{2}} \mathbf{\Psi} \mathbf{\Lambda}^{\frac{1}{2}} \right)^{-1} \mathbf{\Lambda}^{\frac{1}{2}} \qquad (17)$$

$$ii)\ \mu \left(\mathrm{tr} \left\{ \mathbf{\Psi} (\mathbf{I} + \mathbf{\Phi}) + \sum_{i=1}^{N} \sigma_o^2 \mathbf{D}_i^H \mathbf{D}_i \right\} - \kappa \mathrm{P} \right) = 0$$

Solving equation *i*) in (17) over $\mathbf{\Psi}$ we derive that

$$\psi_n = \left[\frac{1}{\mu \cdot (1 + \phi_n)} - \frac{1}{\lambda_n} \right]^+ \qquad (18)$$

which introduced in *ii*) allows to compute μ from

$$\sum_{n=1}^{\kappa} \left[\frac{1}{\mu} - \frac{1 + \phi_n}{\lambda_n} \right]^+ = \kappa \mathrm{P} - \mathrm{tr} \left\{ \sum_{i=1}^{N} \sigma_o^2 \mathbf{D}_i^H \mathbf{D}_i \right\}. \qquad (19)$$

Hence we have shown that $\mathbf{\Sigma}_{\mathrm{X}_s^\kappa} = \mathbf{V}\mathbf{\Psi}\mathbf{V}^H$ satisfies KKT and, thus, it is optimum. Plugging it into the determinant in (9) concludes the proof.

4. SUBOPTIMUM LINEAR RELAYING FUNCTIONS

In previous section, we have derived the optimum covariance matrix given a set of relaying functions $\mathcal{D}$. In the following, we address the problem of designing linear relaying matrices. First, from capacity definition in (7), the achievable rate of the system remains

$$\mathcal{C} = \lim_{\kappa \to \infty} \max_{\mathcal{D}} \mathcal{C}(\kappa, \mathcal{D}). \qquad (20)$$

As mentioned earlier, the above problem is not convex and thus, no closed form expression on $\mathcal{D}$ can be obtained. Moreover, due to the limit, it is not numerically tractable either. Thus, here we propose a suboptimum set $\mathcal{D}_{\mathrm{so}} = \{\mathbf{D}_1, \cdots, \mathbf{D}_N\} \in C^{\kappa \times \kappa}$ that outperforms known achievable rates for the MRC. Let us consider a relaying scheme where all relay nodes use a weighted version of the same linear relaying function:

$$\mathbf{D}_i = \eta_i \mathbf{D}_o(\beta), \qquad (21)$$

where $\mathbf{D}_o$ is defined in terms of an arbitrary parameter $\beta \in [0,1]$, which is later optimized (see below). The weighting factors η_i allow relay nodes to introduce beamforming gain in the system. In this case, beamforming is carried out considering the equivalent source-to-destination, through relays, channel gains, i.e., $h_i = b_i \cdot c_i$. Hence, defining $\mathbf{h} = [h_1, \cdots, h_N]$, we propose

$$\eta_i = \epsilon \cdot \frac{h_i^*}{|\mathbf{h}|}, \qquad (22)$$

where $\epsilon = e^{j \cdot \arg(a)}$ in order to coherently add the signals from source and relays. Next, we define $\mathbf{D}_o(\beta)$

$$[\mathbf{D}_o(\beta)]_{i,j} = \begin{cases} \sqrt{\frac{\beta\kappa}{\kappa-1} \frac{\mathrm{P}}{\sigma_o^2}} & i = j+1;\ 1 \le j \le \kappa - 1 \\ 0 & \text{elsewhere.} \end{cases} \qquad (23)$$

where we straightforward notice that $\mathrm{tr}\left\{\sum_{i=1}^{N} \sigma_o^2 \mathbf{D}_i^H \mathbf{D}_i\right\} = \beta\kappa\mathrm{P}$, and that $\mathbf{D}_o(\beta)$ is a generalized amplify-and-forward scheme extended to $\kappa > 2$. Introducing the proposed linear relaying functions in $\mathbf{H}_e$ definition (8), we obtain:

$$\mathbf{H}_e(\beta) = \frac{1}{\sqrt{\sigma_o^2}} \left(\mathrm{I} + \left(\sum_{i=1}^{N} |c_i|^2 |\eta_i|^2 \right) \mathbf{D}_o(\beta) \mathbf{D}_o^H(\beta) \right)^{-\frac{1}{2}} \times$$
$$(|a| \cdot \mathrm{I} + |\mathbf{h}| \cdot \mathbf{D}_o(\beta)) e^{j \cdot \arg(a)}. \qquad (24)$$

No closed form expression can be derived for the singular values of $\mathbf{H}_e(\beta)$, which have to be evaluated numerically. However, assume that $\lambda_n(\beta)$ is the $(n \cdot \kappa)^{\text{th}}$ eigenvalue of $\mathbf{H}_e(\beta)\mathbf{H}_e(\beta)^H$ when $\kappa \to \infty$, with $n \in [0,1]$. Then, considering $\kappa \to \infty$ and optimizing over arbitrary value β, the achievable rate with the proposed suboptimum set $\mathcal{D}_{so}$ can be computed from Theorem 1 as:

$$\mathcal{C}_{so} = \max_{0 \le \beta \le 1} \int_0^1 \log_2 (1 + \lambda_n(\beta) \psi_n(\beta))\, dn, \qquad (25)$$

where the integral comes out from the sumatory at the limit, and $\psi_n(\beta) = \left[\frac{1}{\mu(\beta) \cdot (1 + \phi_n(\beta))} - \frac{1}{\lambda_n(\beta)} \right]^+$ with

$$\phi_n(\beta) = \left[\left(\sum_{i=1}^{N} |b_i|^2 |\eta_i|^2 \right) \mathbf{V}^H(\beta) \mathbf{D}_o^H(\beta) \mathbf{D}_o(\beta) \mathbf{V}(\beta) \right]_{n\kappa, n\kappa},$$

$$\int_0^1 \left[\frac{1}{\mu(\beta)} - \frac{1 + \phi_n(\beta)}{\lambda_n(\beta)} \right]^+ dn = (1 - \beta)\mathrm{P}.$$

5. NUMERICAL RESULTS

The achievable rate of a Gaussian multiple relay channel (GMRC) with the linear relaying functions described in section 4 is evaluated here. We assume all channel coefficients within the network as zero-mean, unitary-power, complex Gaussian random variables (i.e, Rayleigh distributed fading), and we plot the expected value of the achievable rate, averaged over the channel distribution. Maximization over β in (25) is carried out using exhaustive search.

For comparison, we plot the *max-flow-min-cut* upper bound (computed in [4]) and the achievable rates of previously known relaying techniques for the GMRC[3]: *i*) *Partial Decoding*, analyzed in [4] and shown to achieve[4]:

$$\mathcal{C}_{\mathrm{PD}} = \max_{1 \le n \le N} \max_{(\eta,\beta)} \min \left\{ \mathcal{R}\left(\left(|a|^2 + \beta(1-\eta) \textstyle\sum_{i=1}^{n} |c_i|^2 \right) \frac{\mathrm{P}}{\sigma_o^2} \right), \right.$$
$$\left. \mathcal{R}\left(|b_n|^2 \beta\eta \frac{\frac{\mathrm{P}}{\sigma_o^2}}{1 + |b_n|^2 (1-\beta) \frac{\mathrm{P}}{\sigma_o^2}} \right) + \mathcal{R}\left(|a|^2 (1-\beta) \frac{\mathrm{P}}{\sigma_o^2} \right) \right\},$$

[3]We do not include the *compress-and-forward* technique, developed in [1] for the MRC, since there is no extension to Gaussian channels in the literature. We define $\mathcal{R}(x) = \log_2(1 + x)$.

[4]We consider the relay nodes ordered following $|b_1| \ge \cdots \ge |b_N|$.

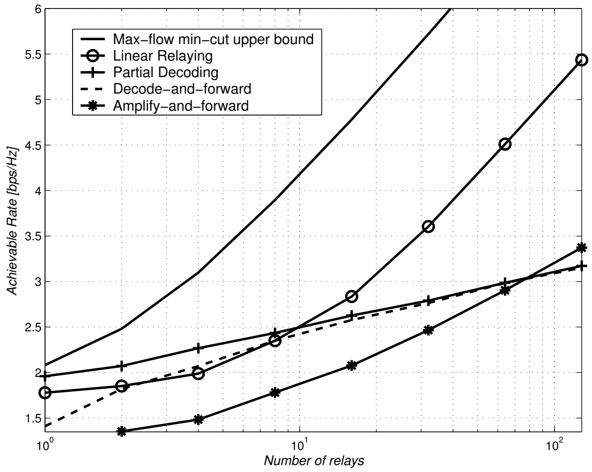

Fig. 2. Expected achievable rate with linear relaying functions. Transmit SNR = 5 dB

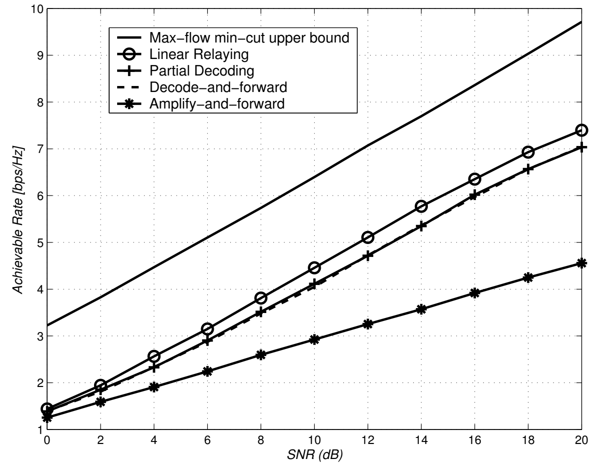

Fig. 3. Expected achievable rate with linear relaying functions. Number of relays N = 16.

ii) *Decode-and-forward*, analyzed in [1] and extended to Gaussian channels by using equation above with $\beta = 1$, i.e,

$$\mathcal{C}_{\mathrm{DF}} = \max_{1\leq n\leq N} \max_{\eta} \min \left\{ \mathcal{R}\left(|b_n|^2 \eta \frac{\mathrm{P}}{\sigma_o^2} \right), \mathcal{R}\left(\left(|a|^2 + (1-\eta) \sum_{i=1}^{n} |c_i|^2 \right) \frac{\mathrm{P}}{\sigma_o^2} \right) \right\},$$

iii) the *Amplify-and-forward* scheme in [6], originally proposed for the single relay channel and extended in our contribution to the multiple relay set up as follows:

$$\begin{aligned} \mathbf{\Sigma}_{\mathrm{X}_s^2}^{\mathrm{AF}} &= 2\alpha\mathrm{P} \begin{pmatrix} \beta & \sqrt{\beta(1-\beta)} \\ \sqrt{\beta(1-\beta)} & 1-\beta \end{pmatrix} \\ \mathbf{D}_i^{\mathrm{AF}} &= \begin{pmatrix} 0 & 0 \\ d_i & 0 \end{pmatrix} \end{aligned}$$

with $d_i = \epsilon \cdot \frac{h_i^*}{|\mathbf{h}|} \sqrt{\frac{2(1-\alpha)\mathrm{P}}{2\alpha\beta\mathrm{P}|b_i|^2+\sigma_o^2}}$ (See [6] for details). Plugging $\mathbf{D}_i^{\mathrm{AF}}$ in (8), the achievable rate for the amplify-and-forward scheme is obtained:

$$\mathcal{C}_{\mathrm{AF}} = \max_{\alpha,\beta} \frac{1}{2} \cdot \log_2 \left(\det \left(\mathbf{I} + \mathbf{H}_e^{\mathrm{AF}} \mathbf{\Sigma}_{\mathrm{X}_s^2}^{\mathrm{AF}} \left(\mathbf{H}_e^{\mathrm{AF}} \right)^H \right) \right).$$

Fig. 2 depicts the achievable rates for all relaying techniques versus the number of total relay nodes, for a transmit SNR of 5 dB (i.e., $P/\sigma_o^2 = 5$ dB). We first notice that the proposed *linear relaying* technique clearly outperforms *amplify-and-forward* and *decode-and-forward* schemes, with higher capacity of up to 1 bps/Hz. Moreover, although suboptimum, the functions proposed in (21) represent the tightest known bound on the capacity of the GMRC for high enough number of relays (i.e., $N \geq 10$). Nevertheless, *partial decoding* still outperforms linear relaying when the number of relays is low. Furthermore, both the *max-flow-min-cut* bound and linear relaying grow logarithmically (and in the same slope) with the number of relays. Fig. 3 shows the capacity of linear relaying versus the total transmit SNR. N = 16 relays are considered in simulation. First, it is shown that *linear relaying* generate the tightest known bound on the capacity of the GMRC. Moreover, the distance between *linear relaying* and the previously known techniques increases in the high SNR regime, with a 50% more rate than the *amplify-and-forward* scheme proposed in [6].

6. CONCLUSIONS

This paper derives, for the Gaussian multiple relay channel with *Linear relaying*, the optimum temporal covariance matrix at the source node and proposes new linear relaying functions that obtain the greatest known achievable rates. Indeed, it shows that *linear relaying* with optimum covariance matrix (following Thereom 1) clearly outperforms well-known *amplify-and-forward* and *decode-and-forward* schemes, and that, for high enough number of relays, it presents gains up to 1 bps/Hz with respect to *partial decoding* techniques.

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