A MULTICHANNEL COOPERATIVE SCHEME FOR WIRELESS NETWORKS AND PERFORMANCE CHARACTERIZATION

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ABSTRACT

A cooperative random access protocol, namely ALLIANCES, was recently proposed for resolving collisions in wireless networks. In [1], we proposed an multichannel extension of ALLIANCES that in addition to cooperation diversity can exploit multipath diversity, and thus improve throughput at high traffic load and reduce packet delays. In this paper, we propose an improvement on [1] that makes more efficient use of available bandwidth and thus can achieve high throughput at all traffic loads. Furthermore, we present analytic performance characterization that provides insight on the relationship between achievable diversity and parameters like collision order, number of relays, channel length and number of carriers per subchannel.

***Index Terms*— cooperation, wireless networks, multichannel, collision resolution**

I. INTRODUCTION

ALOHA-type random access protocols for wireless networks suffer throughput penalty and under-utilization of channel resources due to collisions between network users. In the event of a collision the collided packets are totally discarded. Network-assisted diversity multiple access (NDMA) was proposed in [6] to extract information from collided packets instead of just discarding them; however, it requires the channel coefficients to be uncorrelated between adjacent slots, which is rather unrealistic. In [3], a scheme named ALLIANCES was proposed to overcome this drawback. It relies on cooperation diversity as well as time diversity, where the cooperation diversity is introduced through the use of relays. Once the BS detects a collision, the system enters a cooperative transmission epoch (CTE) to resolve it. During the CTE, a set of nodes designated as non-regenerative relays retransmit the signal that they received during the collision slot. If a chosen relay happens to be a source node it will simply retransmit its own packet. Based on the initially collided packets and the signals forwarded by the relays, the BS formulates a MIMO problem, the solution of which yields the original packets. It was shown in [3] that cooperation diversity helps reduce BER and improve throughput. The initial ALLIANCES was developed for a flat fading channel. In wideband communications, channels are usually frequency selective. Although frequency selective fading is difficult to deal with, if appropriately exploited it can be turned into a source of diversity.

In [1], we proposed a multi-channel extension of [3] that can exploit multipath diversity as well as cooperation diversity. The total bandwidth was divided into separable subchannels. Users can transmit a fixed number of packets over multiple but different subchannels, which are selected in a random fashion. Cooperative transmissions, as in ALLIANCES, are used to resolve collisions. To minimize average packet processing time, the BS allocates all available subchannels to resolve collisions over one subchannel at a time, starting from the highest order and moving towards the lowest order collision. However, having users transmit a fixed number of packets can lead to under-utilization of bandwidth under light traffic load.

In this paper, we propose an improved scheme, where the number of packets to be transmitted is selected in an adaptive fashion based on the traffic load. The proposed scheme, as compared to that of [1] maintains high throughput at all traffic loads. We also present diversity analysis that provides helpful guidelines for BER reduction.

Notation - $\mathbf{I}_N$ denotes an identity matrix of size $N \times N$; $\otimes$ denotes Kronecker product; $Diag(x_1, x_2, \ldots)$ is a diagonal matrix with diagonal elements $x_1, x_2, \ldots$; $diag(\mathbf{A})$ denotes a row vector formed by the diagonal elements of matrix $\mathbf{A}$; $vec(\mathbf{A})$ denotes the column vector formed by stacking vertically the columns of matrix; $||.||_F$ denotes Frobenius norm; $E\{\cdot\}$ is the expectation operation.

II. A BANDWIDTH EFFICIENT VERSION OF THE SCHEME OF [1]

As in [1], we consider a slotted small-scale multi-access wireless system. Each node communicates only with the BS, and obtains feedback from it via a separate control channel specifying whether the packet was transmitted successfully. The communication channel is assumed to be frequency selective, and the channel taps are constant over one slot but may vary between different slots. It consists of M separable subchannels, denoted by $C_m (m = 0, ..., M-1)$. A user becomes active only if there are p or more $(1 \le p \le M)$ packets in its buffer. A user can transmit simultaneously p packets using different subchannels, so that each packet occupies one subchannel for its transmission.

When the number of active users is large, p should be small in order to avoid a large number of collisions. Otherwise, p should be large to exploit all available bandwidth. We propose the following adaptive approach for selecting p, that involves minimal control overhead. Based on the traffic load, and also other available criteria, such as BER and channel state information, the base station will take one of the following three actions at the end of each slot: increase p by 1, decrease p by 1, or keep p unchanged. Then, the BS will broadcast its decision via the control channel to all users using one bit at the end of a slot,

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i.e., “0” for decrease p by 1; “1” for increase p by 1; nothing for keeping p the same as in previous slot. Other issues like collision resolution and relay selection will be done in the same way as in [1].

Simulation Results - Although a simple idea, the proposed scheme can significantly improve the throughput of [1]. Let us consider a network of $J = 32$ users. The traffic load λ is defined as the average number of users that are active in the network during a specific time slot. To investigate the network performance under certain traffic load λ, we use a Bernoulli model. In each trial, all users are statistically the same, and each one becomes active with probability λ/J. The throughput is defined as the average number of packets that are successfully transmitted in a time slot under traffic load λ. We consider a frequency selective channel with $L = 3$ taps, chosen independently according to Jake’s model [7]. The number of OFDM carriers is 64. A zero-forcing decoder is used.

In single-channel ALLIANCES, for a packet containing b bits, the capacity is 1 packet, or b bits per slot. By dividing the fixed bandwidth to M subchannels, the channel capacity is still same, and a packet carries b/M information bits. In order to compare the throughput under different number of subchannels, we normalize the maximum throughput to 1. Throughput of 1 corresponds to the channel capacity b bits/slot under the single-channel case. Fig. 1 shows the normalized throughput for different values of p, and $M = 4$ subchannels. One can see that we need a small p to sustain high throughput at high traffic load, and a large p to sustain high throughput at low traffic load. In Fig. 2, we show the throughput of the adaptive-p scheme, where traffic load is the only criteria to select p. One can see that the resulting throughput is high at all traffic loads. Also, the throughput corresponding to $M = 4$ is higher than that of $M = 2$ and $M = 1$ for all traffic loads.

III. MATHEMATICAL FORMULATION

As in [1], let us consider that the physical layer is an F-carrier OFDM system, where the carriers are divided into groups of N carriers each, i.e., $C_0, \ldots, C_{M-1}$ with $N = F/M$. For simplicity and without loss of generality we assume that F/M is an integer. Let $h_{ij}(m; n)$, $m = 0, ..., L-1$ denote the L channel taps between nodes i and j during slot n. We will assume that L is the length of the longest taps among all internode channels. The F-point discrete Fourier Transform (DFT) of $h_{ij}(m; n)$ is:

$$H_{ij}(k; n) = \sum_{m=0}^{L-1} h_{ij}(m; n) e^{-j\frac{2\pi}{F}km}, \; k = 0, ..., F-1 \quad (1)$$

A packet consists of B OFDM symbols. Let $\mathbf{x}_i^m(n)$ be a $B \times N$ matrix denoting the packet sent by user i over subchannel m, in slot n. Each row of that matrix contains an OFDM symbol before modulation.

The effect of the channel over the k-th carrier is just a multiplication by the carrier gain $H_{ij}(k; n)$. In the absence of collision and after demodulation, the received packet at the BS equals:

$$\mathbf{y}_d^m(n) = \mathbf{x}_i^m(n)\mathbf{H}_{id}^m(n) + \mathbf{w}_d^m(n) \quad (2)$$

where $\mathbf{H}_{id} = diag[H_{id}(mN; n), ..., H_{id}((m+1)N-1; n)]$ $(N \times N)$, and $\mathbf{w}_d^m(n)$ is a $B \times N$ matrix denoting noise at the BS over C_m.

Now, suppose that a collision of order K_m occurs on subchannel C_m in slot n. Let us focus on the cooperative collision resolution, to be denoted as CTE$_m$.

Suppose that node r is selected as the j-th relay ($j = 1, \ldots, \hat{K}_m - 1$) during the CTE slot $n + k$ ($\hat{K}_m \geq K_m$). Note that k may be different than j, since according to [1], multiple relays can be used in the same slot. The value of k is determined by the availability of subchannels and the subchannel allocation scheme. Without loss of generality, let us assume that among the $\hat{K}_m - 1$ nodes, the first η nodes are source relays, and the next l nodes are non-source relays. It holds $\eta + l + 1 = \hat{K}_m$.

Let us form a matrix, $\mathbf{Z}$, $(B \times \hat{K}_m N)$, whose first block column is the packet received at the BS during the collision slot, and subsequent blocks are packets from relay transmissions received at the BS during CTE$_m$, i.e., $\mathbf{Z} = [\mathbf{y}_d^m(n), \mathbf{z}_{r_1 d}^{m,\ell_1}(n + k_1), \ldots, \mathbf{z}_{r_\eta d}^{m,\ell_\eta}(n+k_\eta), \ldots]$, where $\mathbf{y}_d^m(n)$ is the collision signal, and $\mathbf{z}_{r_i d}^{m,\ell_i}(n + k_i)$ is the signal received from relay r_i on subchannel C_{ℓ_i} during slot $n + k_i$.

It holds:

$$\mathbf{Z} = \mathbf{X}^m\mathbf{H} + \mathbf{W} \quad (3)$$

where **(i)** $\mathbf{X}^m$ is a $(B \times K_m N)$ matrix based on the packets of users that collided over C_m, i.e., $\mathbf{X}^m = [\mathbf{x}_{i_1}^m(n), \mathbf{x}_{i_2}^m(n), \ldots, \mathbf{x}_{i_{K_m}}^m(n)]$; **(ii)** $\mathbf{H}$ is a $(K_m N \times \hat{K}_m N)$ matrix structured as follows. It elements are all diagonal matrices of size $N \times N$, or zero. The first block column consists of matrices $\mathbf{H}_{i_1 d}^m(n), \mathbf{H}_{i_2 d}^m(n), \ldots, \mathbf{H}_{i_{K_m} d}^m(n)$. Each of the remaining block columns corresponds to the transmission of some relay. If the relay is a source node, then the corresponding block column is a stack of matrices $\mathbf{0}, \ldots, \mathbf{0}, \mathbf{H}_{i_j d}^\ell(n + k), \mathbf{0}, \ldots, \mathbf{0}$, where the position of the non-zero block element is the same as that of the source $\mathbf{x}_{i_j}^m(n)$ in $\mathbf{X}^m$. Here k is the CTE slot during which that particular relay transmitted, and ℓ is the index of the subchannel used by the relay to retransmit. If the relay is a non-source node, the corresponding block column is a stack of the matrices: $\mathbf{H}_{i_1 r}^m(n)\mathbf{c}_r(n+k)\mathbf{H}_{rd}^\ell(n+k), \ldots, \mathbf{H}_{i_{K_m} r}^m(n)\mathbf{c}_r(n+k)\mathbf{H}_{rd}^\ell(n+k)$. Here $\mathbf{c}_r(n+k)$ is a $N \times N$ diagonal matrix whose elements scale the relay signal at each carrier in order to maintain constant transmission power over all carriers; **(iii)** $\mathbf{W}$ is a $(B \times \hat{K}_m N)$ matrix based on the noise at the BS during the collision slot, and each subsequent retransmission. The first block column equals $\mathbf{w}_d^m(n)$, i.e., the noise over C_m at the BS. The form of each subsequent columns depends on whether that column corresponds to a transmission from a source relay or a non-source relay. If it corresponds to source relay it equals $\mathbf{w}_d^m(n + k)$. Otherwise, it equals $\mathbf{w}_r^m(n)\mathbf{c}_r(n + k)\mathbf{H}_{rd}^\ell(n + k) + \mathbf{w}_d^m(n + k)$, where r is the non-source relay.

Example- Suppose that there are totally 2 subchannels and a 3-fold collision occurred over C_0 at slot n. In slot $n+1$, there is 1 source relay, i.e., i_1, using C_1, and 1 non-source relay, r, using C_0. The matrices $\mathbf{Z}$, $\mathbf{H}$ and $\mathbf{W}$ are:

$$\mathbf{Z} = [\mathbf{y}_d^0(n), \mathbf{z}_{i_1 d}^{0,1}(n + 1), \mathbf{z}_{rd}^{0,0}(n + 1)] \quad (4)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{i_1 d}^0(n) & \mathbf{H}_{i_1 d}^1(n+1) & \mathbf{H}_{i_1 r}^0(n)\mathbf{c}_r(n+1)\mathbf{H}_{rd}^0(n+1) \\ \mathbf{H}_{i_2 d}^0(n) & \mathbf{0} & \mathbf{H}_{i_2 r}^0(n)\mathbf{c}_r(n+1)\mathbf{H}_{rd}^0(n+1) \\ \mathbf{H}_{i_3 d}^0(n) & \mathbf{0} & \mathbf{H}_{i_3 r}^0(n)\mathbf{c}_r(n+1)\mathbf{H}_{rd}^0(n+1) \end{bmatrix} \quad (5)$$

$$\mathbf{W} = [\mathbf{w}_d^0(n), \mathbf{w}_d^1(n+1), \mathbf{w}_r^0(n)\mathbf{c}_r(n+1)\mathbf{H}_{rd}^0(n+1) + \mathbf{w}_d^0(n+1)] \quad (6)$$

IV. PAIRWISE ERROR PROBABILITY

We will make the following *assumptions*: (A1) for a fixed n the channel taps $h_{ij}(m;n)$ in (1) are i.i.d. zero-mean, circularly symmetric complex Gaussian random variables with variance σ_a^2. Furthermore, to highlight the cooperation diversity advantage we will assume that the channel stays constant from slot to slot, thus the dependence on the slot index n will not be shown. (A2) the power of transmitted symbols $\mathbf{X}$ in (3) is σ_x^2. (A3) elements of the matrix $\mathbf{w}_r(n+k)$, i.e., the noise at the BS and relays, are uncorrelated, complex, zero-mean white Gaussian with variance σ_w^2. (A4) the diagonal elements of $\mathbf{c}_r(n+k)$ are chosen so that the average power for each carrier is kept equal to σ_x^2. (A5) relays are not re-used during the same CTE.

Based on the above assumptions, $\mathbf{c}_r(n+k) = c\mathbf{I}_N$, where $c = \sigma_x/(K_m\sigma_x^2\sigma_a^2 L + \sigma_w^2)^{1/2}$.

Let us rewrite (3) as follows:

$$\begin{aligned}\tilde{\mathbf{Z}} &= \sigma_x\tilde{\mathbf{X}}\mathbf{H}\mathbf{R}_w^{-1/2} + \mathbf{W}\mathbf{R}_w^{-1/2} \\ &= \sigma_x\tilde{\mathbf{X}}\mathbf{H}\mathbf{R}_w^{-1/2} + \tilde{\mathbf{W}}\end{aligned} \quad (7)$$

where $\mathbf{R}_w$ is the covariance matrix of $\mathbf{W}$ in (3); $\tilde{\mathbf{X}}$ is a unit variance version of $\mathbf{X}$; $\tilde{\mathbf{W}}$ is a unit variance version of $\mathbf{W}$.

Based on (A1), the cross-correlation of the channel gains equals:

$$E\{H_{i_l d}(k_1)H_{i_l d}^*(k_2)\} = \sigma_a^2 \sum_{m=0}^{L-1} e^{-j\frac{2\pi}{F}(k_1-k_2)m} \quad (8)$$

Using (A1), (A3) and (8), we can show that $\mathbf{R}_w$ is a $(\hat{K}_m N \times \hat{K}_m N)$ diagonal matrix of the form: $\mathbf{R}_w = Diag(\mathbf{R}_{w_1}, \mathbf{R}_{w_2})$, where $\mathbf{R}_{w_1} = \sigma_w^2\mathbf{I}_{(1+\eta)N}$, and corresponds to collision and source relay retransmissions, and $\mathbf{R}_{w_2} = \sigma_w^2\tilde{c}\mathbf{I}_{lN}$, and corresponds to non-source relay transmissions, with $\tilde{c} = 1 + c^2\sigma_a^2 L$.

Let us express $\mathbf{H}$ as $\mathbf{H} = [\mathbf{F}_1|\mathbf{F}_2]$, where $\mathbf{F}_1$ $(K_m N \times (\eta+1)N)$ contains the columns of $\mathbf{H}$ that correspond to collision slot and retransmissions by source relays, and $\mathbf{F}_2$ $(K_m N \times lN)$ contains the columns of $\mathbf{H}$ that correspond to retransmission by non-source relays (see (5) for an example). Groups $\mathbf{F}_1$ and $\mathbf{F}_2$ are independent from each other. Due to assumption (A5) the columns of $\mathbf{F}_2$ are uncorrelated, while there is correlation between the columns of $\mathbf{F}_1$ as they correspond to retransmissions of source nodes. Let us further express $\mathbf{F}_2$ as $\mathbf{F}_2 = c\mathbf{H}_1\mathbf{H}_2$, where

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{H}_{i_1 r_1}^m & \dots & \mathbf{H}_{i_1 r_l}^m \\ \vdots & \dots & \vdots \\ \mathbf{H}_{i_{K_m} r_1}^m & \dots & \mathbf{H}_{i_{K_m} r_l}^m \end{bmatrix} \quad (9)$$

and

$$\mathbf{H}_2 = Diag(\mathbf{H}_{r_1 d}^{\ell_1}, \dots, \mathbf{H}_{r_l d}^{\ell_l}) \quad (10)$$

The pairwise error probability of $\tilde{\mathbf{X}}$ being transmitted and $\hat{\tilde{\mathbf{X}}}$ being recovered, conditioned on K_m, l, η satisfies [4]:

$$\begin{aligned} P(\tilde{\mathbf{X}}, \hat{\tilde{\mathbf{X}}}|K_m, l, \eta) &\le E\{\exp(-\frac{\sigma_x^2}{4}||(\tilde{\mathbf{X}} - \hat{\tilde{\mathbf{X}}})\mathbf{H}\mathbf{R}_w^{-1/2}||_F^2)\} \\ &= E\{E\{\exp(-\frac{\sigma_x^2}{4}||(\tilde{\mathbf{X}} - \hat{\tilde{\mathbf{X}}})\mathbf{F}_2\mathbf{R}_{w_2}^{-1/2}||_F^2)\}|\mathbf{H}_2\} \\ &\times E\{\exp(-\frac{\sigma_x^2}{4}||(\tilde{\mathbf{X}} - \hat{\tilde{\mathbf{X}}})\mathbf{F}_1\mathbf{R}_{w_1}^{-1/2}||_F^2)\} \end{aligned} \quad (11)$$

It holds:

$$\begin{aligned} &E\{\exp(-\frac{\sigma_x^2}{4}||(\tilde{\mathbf{X}} - \hat{\tilde{\mathbf{X}}})\mathbf{F}_1\mathbf{R}_{w_1}^{-1/2}||_F^2)\} \\ &= E\{-\frac{\sigma_x^2}{4\sigma_w^2}\mathbf{f}_1^H(\mathbf{I}_{(\eta+1)N} \otimes \mathbf{R}_\Delta)\mathbf{f}_1\} \\ &= [\det(\mathbf{I} + \frac{\sigma_x^2}{4\sigma_w^2}\mathbf{R}_{f_1}(\mathbf{I}_{(\eta+1)N} \otimes \mathbf{R}_\Delta))]^{-1} \\ &= \prod_{i=1}^{I_1}(1 + \frac{\sigma_x^2}{4\sigma_w^2}\lambda_i)^{-1} \end{aligned} \quad (12)$$

where $\mathbf{f}_1 = vec(\mathbf{F}_1)$; $\mathbf{R}_\Delta = (\tilde{\mathbf{X}} - \hat{\tilde{\mathbf{X}}})^H(\tilde{\mathbf{X}} - \hat{\tilde{\mathbf{X}}})$; $\mathbf{R}_{f_1}$ is the covariance of $\mathbf{f}_1$; and λ_i, I_1 are, respectively, the eigenvalues and rank of $\mathbf{R}_{f_1}(\mathbf{I}_{(\eta+1)N} \otimes \mathbf{R}_\Delta)$.

Also, it holds:

$$\begin{aligned} &E\{\exp(-\frac{\sigma_x^2}{4}||(\tilde{\mathbf{X}} - \hat{\tilde{\mathbf{X}}})\mathbf{F}_2\mathbf{R}_{w_2}^{-1/2}||_F^2)|\mathbf{H}_2\} \\ &= [\det(\mathbf{I} + \frac{\sigma_x^2}{4\sigma_w^2}\mathbf{R}_{f2}(\mathbf{I}_{lN} \otimes \mathbf{R}_\Delta))]^{-1} \\ &= [\det(\mathbf{I} + \frac{\sigma_x^2}{4\sigma_w^2}\frac{c^2}{\tilde{c}}\mathbf{R}_{h_1}(\mathbf{H}_2\mathbf{H}_2^H \otimes \mathbf{R}_\Delta))]^{-1} \\ &= \prod_{i=1}^{I_2}(1 + \frac{\sigma_x^2}{4\sigma_w^2}\frac{c^2}{\tilde{c}}\tilde{\lambda}_i)^{-1} \end{aligned} \quad (13)$$

where $\mathbf{R}_{f2}$ is the covariance of $vec(\mathbf{F}_2\mathbf{R}_{w_2}^{-1/2})$; $\mathbf{R}_{h_1}$ is the covariance of $\boldsymbol{h}_1 = vec(\mathbf{H}_1)$; and $\tilde{\lambda}_i$, I_2 are respectively the eigenvalues and rank of $\mathbf{R}_{h_1}(\mathbf{H}_2\mathbf{H}_2^H \otimes \mathbf{R}_\Delta)$. Also, $\mathbf{R}_{f2} = c^2(\mathbf{R}_{w_2}^{-1/2}\mathbf{H}_2^T \otimes \mathbf{I})\mathbf{R}_{h_1}(\mathbf{H}_2^*\mathbf{R}_{w_2}^{-1/2} \otimes \mathbf{I})$.

The following properties were used: $vec(\mathbf{AB}) = (\mathbf{B}^T \otimes \mathbf{I})vec(\mathbf{A})$; $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC} \otimes \mathbf{BD})$; $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$.

Let us go back to (11) and assume high SNR (i.e., σ_x^2/σ_w^2 is a large number). Then we can write $c^2/\tilde{c} = \frac{1}{\sigma_a^2 L(K_m+1)}$. Noting that λ_i and $\tilde{\lambda}_i$ are positive, we get:

$$\begin{aligned} P(\tilde{\mathbf{X}}, \hat{\tilde{\mathbf{X}}}|K_m, l, \eta) &\le \gamma^{-I_1-I_2}(\frac{1}{\sigma_a^2 L(K_m+1)})^{-I_2}(\prod_{i=1}^{I_1}\lambda_i)^{-1} \\ &\int \prod_{i=1}^{I_2}\tilde{\lambda}_i^{-1} f(\tilde{\lambda}_1, \dots, \tilde{\lambda}_{I_2})d\tilde{\lambda}_1 \dots \tilde{\lambda}_{I_2} \end{aligned} \quad (14)$$

where $\gamma = \sigma_x^2/4\sigma_w^2$ is used here as the SNR, and $f(\tilde{\lambda}_1, \dots, \tilde{\lambda}_{I_2})$ is the joint probability density function of $\tilde{\lambda}_1, \dots, \tilde{\lambda}_{I_2}$. An expression for the latter can be found in [2].

The diversity order of the system [4] is $I_1 + I_2$.

We can always find a row permutation matrix, $\mathbf{P}$, so that the covariance of $\mathbf{P}\mathbf{f}_1$, is of the form $Diag(\mathbf{R}_{i_1 d}, ..., \mathbf{R}_{i_{K_m} d}, \mathbf{0})$. If the source i_j served as a relay using a subchannel $m_j \neq m$ for its retransmission, then $\mathbf{R}_{i_j d}$ is the covariance matrix of vector

$[diag(\mathbf{H}_{i_j d}^{m}), diag(\mathbf{H}_{i_j d}^{m_j})]$, and its rank is $\min(2N, L)$; otherwise, $\mathbf{R}_{i_j d}$ is the covariance matrix of the vector $[diag(\mathbf{H}_{i_j d}^{m})]$, and its rank is $\min(N, L)$. Suppose that there are ξ $(\leq \eta)$ source relays that switch to different subchannels during CTE$_m$. It holds:

$$rank(\mathbf{R}_{f_1}) = \xi \min(2N, L) + (K_m - \xi) \min(N, L) \quad (15)$$

Finally,

$$\begin{aligned} I_1 &\leq \min(rank(\mathbf{R}_{f_1}), rank(\mathbf{I}_{(\eta+1)N} \otimes \mathbf{R}_\Delta)) \\ &= \min(\xi \min(2N, L) + (K_m - \xi) \min(N, L), (\eta+1)N r_\Delta) \end{aligned} \quad (16)$$

where $r_\Delta = rank(\mathbf{R}_\Delta)$.

In a similar fashion, $rank(\mathbf{R}_{h_1}) = K_m l \min(N, L)$. Also, $rank(\mathbf{H}_2\mathbf{H}_2^H \otimes \mathbf{R}_\Delta) = rank(\mathbf{H}_2\mathbf{H}_2^H) rank(\mathbf{R}_\Delta) = rank(\mathbf{H}_2\mathbf{H}_2^H) r_\Delta$. Thus:

$$I_2 \leq \min(K_m l \min(N, L), rank(\mathbf{H}_2\mathbf{H}_2^H) r_\Delta) \quad (17)$$

Let us look at the maximum possible diversity, which can be achieved if the data have been coded so that $\mathbf{R}_\Delta$ is full-rank, i.e., $r_\Delta = K_m N$ (note that it is only a sufficient condition). Then, given N and L, the maximum of I_1, is $I_1^{\max} = \xi \min(2N, L) + (K_m - \xi) \min(N, L)$.

Assuming that $\mathbf{H}_2\mathbf{H}_2^H$ is also full-rank, the maximum value for I_2 is $I_2^{\max} = K_m l \min(N, L)$. The assumption on $\mathbf{H}_2\mathbf{H}_2^H$ requires that none of the carriers which are used by the relays give zero gain.

Based on (16) and (17), the maximum achievable diversity order of the virtual MIMO problem of (7) can be found as $I_1^{\max} + I_2^{\max}$.

Discussions of diversity results - (Case 1) If $N \geq L$ it holds: $I_1^{\max} = K_m L$ and $I_2^{\max} = K_m l L$, and thus the maximum diversity order is $K_m L(l+1)$. (Case 2) If $N < L < 2N$, then $I_1^{\max} + I_2^{\max} = \xi(L - N) + K_m N(l+1)$. (Case 3) If $L \geq 2N$ then, $I_1^{\max} + I_2^{\max} = \xi N + K_m N(l+1)$.

One can see that only in cases 2 and 3 the switching of source relays to a different channel to retransmit may increase diversity. Also, only the source relays that switch to a different subchannel contribute to diversity. However, the increase is very limited because N is small in those cases. Among all N and L possibilities, the highest diversity is achievable under case 1, where the multipath diversity offered by the channel is fully exploited.

In all cases the maximum diversity order increases with l. Although we could use $\hat{K}_m > K_m$ to improve BER, a large number of non-source relays might require a long CTE, which increases delays. The PEP expression in (14) can be used as a guide by the BS to determine how many relays are needed to maintain a certain BER, given some delay constraint.

Similarly with STC-OFDM systems in [5], the diversity in multichannel ALLIANCES is related to R_Δ, but there are no simple expressions for I_1 and I_2. The retransmissions of non-source relays in multichannel ALLIANCES can be treated as a traditional Space-Time Coded OFDM (STC-OFDM) system, and the result of $I_2^{\max}$ matches that in [5].

V. CONCLUSIONS

We presented a simple modification of multichannel ALLIANCES that resulted in throughput improvement at all traffic loads. We also conducted error analysis, which shows how the maximum achievable diversity can be affected by the number of non-source relays, and the number of carriers per subchannel as compared to the channel length. The maximum diversity order in multipath channels is higher than that in flat fading channels, and it increases with the number of non-source relays.

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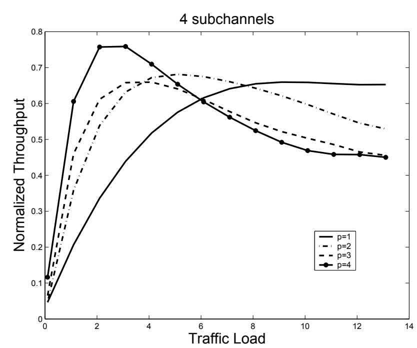

Fig. 1. Normalized throughput for fixed p

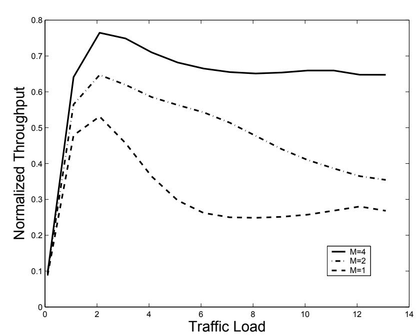

Fig. 2. Normalized throughput for adaptive p