SENSITIVITY OF ACHIEVABLE RATES FOR THE RELAY CHANNEL. APPLICATION TO RELAYING WITH CHANNEL ESTIMATION ERROR

Abdellatif Zaidi and Luc Vandendorpe

Communication and remote sensing Lab. (TELE)
Université Catholique de Louvain (UCL) Louvain-La-Neuve 1348, Belgium
Abdellatif.Zaidi@ensta.org,Vandendorpe@tele.ucl.ac.be

ABSTRACT

This paper investigates the sensitivity of the achievables rates for the full-duplex relay channel to small additive disturbances on channel links. The focus is on two relaying strategies—-the decode-and-forward (DF) mode and the compress-and-forward (CF) mode. We use Fisher Information and De-Bruijn's identity to assess the decrease in the corresponding rates due to small additive contaminating noise. Analysis sheds light on the respective sensitivity levels of these schemes and hence, provides insights onto the choice of appropriate relaying strategies in the situations where some trade-off between transmission rate and sensitivity is needed. Next, we show that these results can be used to emphasize the effect of channel estimation error on relaying transmissions. An important (somehow intuitive) observation at this stage is that transmission through the direct link (i.e., relay is off) may improve upon both decode-and-forward and compress-and-forward schemes, when the channel is "bad enough". Finally, a lower bound on the capacity of a relay channel under channel estimation error is obtained by combining well known relaying strategies, each over the appropriate SNR range. Analysis is supported by some examples.

Index Terms— Relay channel, cooperative systems, channel sensitivity, capacity bounds.

1. INTRODUCTION

Relaying transmissions model problems where one or more relays help one or more pair of terminals communicate. The simplest form of such transmissions is the one with one transmitter-receiver pair, commonly known as the relay channel (RC) [1]. Though introduced since relatively long, the capacity region of the general RC is still unknown and, in fact, the most thorough analysis to date was provided in 1979 by Cover and El Gamal [2]. In [2], the authors derived the capacity region under certain restrictive conditions for the relay channel (being physically degraded). They also developed two coding strategies for general RC. The first strategy achieves the rate in [2, Theorem 4] and is now commonly known as *decode-and-forward* (DF) [3]. The second strategy achieves the rate in [2, Theorem 6] and is now often called *compress-and-forward* (CF) (or, equivalently, *estimate-and-forward* or *quantize-and-forward* [4]). There have been then the now popular *amplify-and-forward* (AF) [5, 6] and its variation *scale-and-forward* (SF) [4]. These coding strategies gained considerable interest in the last few years, due to the potential use of cooperative coding in a variety of applications such as as a multihop wireless network and a sensor network.

Much of the earlier research on relaying strategies concentrated on the comparison of these schemes in terms of achievable rates (e.g., see [4]). They all came with the conclusion that none of these schemes truly extract the potential benefits of cooperating nodes and that, in many cases, efficient coding should rely on mixed strategies. This paper focuses on relaying strategies from another standpoint: sensitivity to small additive disturbances (or perturbations). The aim is to determine which of these schemes is less/more sensitive and whether the most efficient scheme (in terms of achievable rate) has the smallest/biggest sensitivity. The question of sensitivity to small additive disturbance (or, roughly speaking, to noise) has at least two motivations. First, from a practical point-of-view, it is clear that as transmission over wireless channels often suffers from random fluctuations in signal level known as fading and from co-channel interference, one should consider not only the potentially achievable transmission rate but also robustness to noise and interference in the design of relay communication systems. Second, from an information-theoretical point-of-view, the study of sensitivity to noise is important since it permits to predict how transmission rate decreases in presence of small perturbations. In addition, this framework readily applies to relaying with channel estimation error.

The remaining of this paper is organized as follows. In section 2, we use Fisher Information and De-Bruijn's identity to assess the decrease of the rates achievable by both DF and CF, for the general relay channel (not necessarily Gaussian). We model the noise on each link as the sum of a dominant (possibly non Gaussian) noise and a relatively-weak Gaussian contaminating noise. Section 3 specializes the results of Section 2 to the Gaussian case. In section 4, we emphasize the impact of channel estimation error on the achievable rates for the full-duplex Gaussian RC, in the case where the channel coefficients are known only partially. An achievable rate region is derived based on a combination of DF, CF and transmission through the direct link (DL). Section 5 concludes the paper.

2. SENSITIVITY OF ACHIEVABLE RATES

Consider the general relay channel (RC) depicted in Fig. 1, where the source terminal sends message W (from set $\mathscr{W}$, i.e., $W \in \mathscr{W}$) to the destination through a relay terminal. So, the relay receives no specific information and only assists the destination decode message W. Throughout this paper, we use S, R and D to refer to the source, relay and destination terminals, respectively. We assume that time is discrete and at time t, S and R send $X_1(t)$ and $X_2(t)$, respectively and R and D receive $Y_2(t)$ and $Y = Y_3(t)$, respectively. The source signal $X_1(t)$ is function of the message $W \in \mathscr{W}$ and the relay signal $X_2(t)$ is function of past relay's inputs $Y_2^{t-1} \triangleq Y_2(1), Y_2(2), \cdots, Y_2(t-1)$. We also suppose that the three links S $\rightarrow$ R, S $\rightarrow$ D and R $\rightarrow$ D are characterized by channel coefficients h_{12}, h_{13} and h_{23}, respectively. Channel coefficients are assumed to be nonrandom at this level.

Next, we assume that channel noise on each link is the sum of a dominant, possibly non-Gaussian, noise (denoted by Z_2 for R and by Z_3 for D) and a relatively weak Gaussian contaminating noise

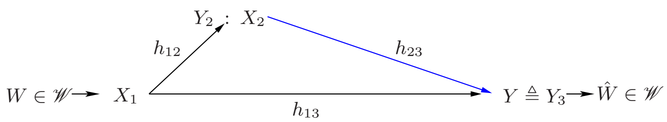

Fig. 1. The general Relay Channel (RC)

(denoted by $\theta_2 V_2$ for R and by $\theta_3 V_3$ for D). The two noise components may model ambient noise and small channel variations or any additive weak interference, respectively. With these notations, the received signals are given by

$$\widetilde{Y}_2(t) = h_{12}X_1(t) + Z_2(t) + \theta_2 V_2(t) = Y_2(t) + \theta_2 V_2(t),$$
$$\widetilde{Y}(t) = h_{13}X_1(t) + h_{23}X_2(t) + Z_3(t) + \theta_3 V_3(t) = Y(t) + \theta_3 V_3(t),$$

where Z_2 and Z_3 are assumed to be independent of each other and also independent of V_2 and V_3. We suppose without loss of generality that $\mathbb{E}[V_2] = \mathbb{E}[V_3] = 0$ and $\mathbb{E}[V_2^2] = \mathbb{E}[V_3^2] = 1$. Also, let $\theta = (\theta_2, \theta_3)^T$. The relay is allowed to operate in either DF or CF modes. The aim is to emphasize the sensitivity of (the rate achievable by) these schemes to the weak perturbations $\theta_2 V_2$ and $\theta_3 V_3$.

2.1. Decode-and-Forward (DF)

Assume the relay operates in DF. In the classical case where the is no perturbation (i.e., $\theta = 0$), DF achieves any rate up to [2, Theorem 4]

$$R_{\mathrm{DF}}(0) = \max_{p(x_1,x_2)} \min \Big\{ I(X_1X_2;Y), I(X_1;Y_2|X_2) \Big\}, \quad (2)$$

where maximization is over all joint distributions $p(x_1, x_2)$. Now, when $\theta \neq 0$, the perturbations result in rate loss with respect to the nominal rate $R_{\mathrm{DF}}(0)$. This loss can be measured by the rate difference $R_{\mathrm{DF}}(0) - R_{\mathrm{DF}}(\theta)$, i.e., by the tow mutual information differences

$$\mathcal{I}_{\mathrm{DF}}^{(1)}(\theta) = I(X_1;Y_2|X_2) - I(X_1;\widetilde{Y}_2|X_2), \quad (3a)$$
$$\mathcal{I}_{\mathrm{DF}}^{(2)}(\theta) = I(X_1X_2;Y) - I(X_1X_2;\widetilde{Y}). \quad (3b)$$

An interpretation of (3) in the light of the superposition block Markovian encoding in [2] where the source terminal sends *fresh information* on top of *refinement information*, is that, here, $\mathcal{I}_{\mathrm{DF}}^{(1)}$ characterizes the rate loss in transmitting the fresh information (from S to R) and $\mathcal{I}_{\mathrm{DF}}^{(2)}$ characterizes the rate loss in transmitting the refinement information (from S and R to D), due to the perturbation. So, $\mathcal{I}_{\mathrm{DF}}^{(1)}$ basically depends on the quality of the S $\to$ R channel, and $\mathcal{I}_{\mathrm{DF}}^{(2)}$ basically depends on those of the R $\to$ D and S $\to$ D channels.

Using the "Information Chain Rule" and the "Entropy Chain Rule" [7], this "information loss" can be related to entropy, as

$$\mathcal{I}_{\mathrm{DF}}^{(1)}(\theta) = (H(X_1X_2\widetilde{Y}_2) - H(X_1X_2Y_2)) - (H(X_2\widetilde{Y}_2) - H(X_2Y_2)), \quad (4a)$$

$$\mathcal{I}_{\mathrm{DF}}^{(2)}(\theta) = (H(X_1X_2\widetilde{Y}) - H(X_1X_2Y)) - (H(\widetilde{Y}) - H(Y)) \quad (4b)$$

Now, recall De-Bruijn's identity which relates entropy $H(\cdot)$ to Fisher information $J(\cdot)$.

Lemma 1 *(De Bruijn's Identity [7, Theorem 16.6.2])*
Let X be any random variable with finite variance and density $f(x)$. Let Z be an independent normally distributed random variable with zero mean and unit variance. Then

$$\frac{\partial}{\partial t} H(X + \sqrt{t}Z) = \frac{1}{2} J(X + \sqrt{t}Z). \quad (5)$$

Using the multivariate version of (5) and some algebra to evaluate the entropy of random vectors $(X_1, X_2, \widetilde{Y}_2) = (X_1, X_2, Y_2) + (0, 0, \theta_2)V_2$, $(X_2, \widetilde{Y}_2) = (X_2, Y_2) + (0, \theta_2)V_2$, $(X_1, X_2, \widetilde{Y}) = (X_1, X_2, Y) + (0, 0, \theta_3)V_3$ and $\widetilde{Y} = Y + \theta_3 V_3$ in (4), we end up with the following expressions (kind of Taylor expansion in θ) for the DF rate loss:

$$\mathcal{I}_{\mathrm{DF}}^{(1)}(\theta) = \frac{|\theta_2|^2}{2}\{\mathrm{Tr}(J(X_1X_2Y_2)) - \mathrm{Tr}(J(X_2Y_2))\} + o(|\theta_2|^2), \quad (6a)$$

$$\mathcal{I}_{\mathrm{DF}}^{(2)}(\theta) = \frac{|\theta_3|^2}{2}\{\mathrm{Tr}(J(X_1X_2Y)) - \mathrm{Tr}(J(Y))\} + o(|\theta_3|^2), \quad (6b)$$

where $\mathrm{Tr}(\cdot)$ and $\mathrm{Var}(\cdot)$ denote the trace operator and the variance, respectively.
From (6) we see that DF can be characterized by a pair $\gamma_{\mathrm{DF}} = (\gamma_{\mathrm{DF}}^{(1)}, \gamma_{\mathrm{DF}}^{(2)})$ of sensitivity coefficients where we define

$$\gamma_{\mathrm{DF}}^{(1)} \triangleq \lim_{\|\theta\| \to 0} \frac{\mathcal{I}_{\mathrm{DF}}^{(1)}(\theta)}{|\theta_2|^2} \text{ and } \gamma_{\mathrm{DF}}^{(2)} \triangleq \lim_{\|\theta\| \to 0} \frac{\mathcal{I}_{\mathrm{DF}}^{(2)}(\theta)}{|\theta_3|^2}. \quad (7)$$

These two coefficients measure system sensitivity in the transmission of fresh information (to R, by S) and refinement information (to D, by both S and R), respectively. In particular, comparison of $\gamma_{\mathrm{DF}}^{(1)}$ and $\gamma_{\mathrm{DF}}^{(2)}$ allows to determine to the quality of which link DF is most sensitive. This is clearly useful in resource management (think of rate/power allocation) and also in system design.

Remark 1: Eq. (7) means that for small values of θ, we have

$$R_{\mathrm{DF}}(\theta) = \max_{p(x_1,x_2)} \min \{ I(X_1;Y_2|X_2) - \gamma_{\mathrm{DF}}^{(1)}|\theta_2|^2 + o(|\theta_2|^2),$$
$$I(X_1X_2;Y) - \gamma_{\mathrm{DF}}^{(2)}|\theta_3|^2 + o(|\theta_3|^2)\}, \quad (8)$$

where $\gamma_{\mathrm{DF}}^{(1)}$ and $\gamma_{\mathrm{DF}}^{(2)}$, as defined by (7), are given by

$$\gamma_{\mathrm{DF}}^{(1)} = \frac{1}{2}\mathrm{Tr}(J(X_1X_2Y_2)) - \frac{1}{2}\mathrm{Tr}(J(X_2Y_2)), \quad (9a)$$
$$\gamma_{\mathrm{DF}}^{(2)} = \frac{1}{2}\mathrm{Tr}(J(X_1X_2Y)) - \frac{1}{2}\mathrm{Tr}(J(Y)). \quad (9b)$$

Hence, the sensitivity coefficient γ_{DF} does not depend on the strength of the perturbation (since defined as a derivative of $\mathcal{I}_{\mathrm{DF}}$ w.r.t. this perturbation). Rather, it can be interpreted as a measure of the *intrinsic* "robustness" of DF to small additive channel perturbations.

2.2. Compress-and-Forward (CF)

Assume now the relay operates in CF. If $\theta = 0$, the CF strategy has R forwarding a quantized and compressed version $\widehat{Y}_2$ of its channel output to D. The compression uses Wyner-Ziv coding [8], i.e., it exploits the destination's side information Y. This approach lets one achieve any rate up to [2, Theorem 6]

$$R_{\mathrm{CF}}(0) = \max I(X_1; Y\widehat{Y}_2|X_2) \quad (10)$$

subject to the constraint $I(X_2;Y) \geq I(Y_2;\widehat{Y}_2|X_2Y)$ where maximization is over all joint distributions of the form $p(x_1, x_2, y, y_2, \hat{y}_2) = p(x_1)p(x_2)p(y, y_2|x_1, x_2)p(\hat{y}_2|y_2, x_2)$. Now, if the perturbation is non zero (i.e., $\theta \neq 0$), Wyner-Ziv coding sees a noisy side information (since signal Y at D is corrupted by perturbation $\theta_3 V_3$) and also a noisy signal to be compressed (since signal Y_2 at R is corrupted by perturbation $\theta_2 V_2$). Let the quantized version of signal $\widetilde{Y}_2$ be $\widetilde{\widehat{Y}}_2 = \widehat{Y}_2 + e$, where e is a small error (with variance σ_e^2) due to noisy quantization. We proceed as above and measure the rate loss due to the perturbation by the difference $\mathcal{I}_{\mathrm{CF}}(\theta) = R_{\mathrm{CF}}(0) - R_{\mathrm{CF}}(\theta)$. We obtain

$$R_{\mathrm{CF}}(\theta) = R_{\mathrm{CF}}(0) - \gamma_{\mathrm{CF}}(|\theta_3|^2 + \sigma_e^2) + o(|\theta_3|^2 + \sigma_e^2), \quad (11a)$$

$$\gamma_{\mathrm{CF}} = \frac{1}{2}\mathrm{Tr}(J(X_1 X_2 \widehat{Y}_2 Y)) - \frac{1}{2}\mathrm{Tr}(J(X_2 \widehat{Y}_2 Y)). \quad (11b)$$

Remark 2: At this stage, it is interesting to compare CF and DF sensitivity coefficients γ_{DF} and γ_{CF}, for the same small additive disturbance (i.e., given θ). For that, one has to compute Fisher information in (9) and (11). This is not obvious in general (see Section 3 for comparison in the Gaussian case). However, viewing Fisher Information as a measure of the accuracy in estimating the involved signals, we can see that CF is less sensitive than DF at high transmission rate. The reason is that γ_{CF} in (11b) can be viewed as the error in estimating the input X_1 if one observes $(X_2, \widehat{Y}_2, Y)$, which is of course smaller than the error obtained by estimating both X_1 and X_2 from Y only, as in (9b). A similar argument shows that CF is more sensitive than DF at low rate.

3. SENSITIVITY OF ACHIEVABLE RATES FOR THE GAUSSIAN RC

Consider again the channel shown in Fig. 1. Let the noises Z_2 and Z_3 be Gaussian with unit variance and $\mathbb{E}[X_i^2] = P_i$, $i = 1, 2$. We concentrate on the evaluation of DF and CF sensitivities to (the quality of) the S $\rightarrow$ R channel (R $\rightarrow$ D and S $\rightarrow$ D are assumed to be perturbation-free) and denote by SNR the corresponding signal-to-noise ratio at the relay. We now assume that the channel is known at R within some mean-square error. More specifically, we break h_{12} into $\hat{h}_{12}$ and $\tilde{h}_{12}$, where $\mathbb{E}[h_{12}] = \hat{h}_{12}$ and $\mathbb{E}[\tilde{h}_{12}] = 0$. Intuitively, we view $\hat{h}_{12}$ as the measurement (or estimation) of the channel at R and $\tilde{h}_{12}$ as the zero-mean measurement (or estimation) error, assumed to be normally distributed, i.e., $\tilde{h}_{12} \sim N(0, \delta_{12}^2)$.

The framework of Section 2 may apply here, as perturbation $\theta_2 V_2$ may model the error in measuring channel coefficient h_{12} (disregarding for the moment the fact that this error noise is potentially non Gaussian; this problem is handled in Section 4).

Fig. 2 depicts the evolution of the coefficients γ_{DF} and γ_{CF}, calculated assuming joint Gaussian distributions in (9) and (11b). Note that, in order to model the fact that R can accurately estimate the channel at high SNR, we let the error δ_{12}^2 vary inversely proportionally to SNR. More specifically, both $|h_{12}|^2$ and δ_{12}^2 vary in Fig. 2: As $|h_{12}|^2$ increases from -10 dB to 10 dB, δ_{12}^2 varies from $10^{-3}|h_{12}|^2$ at large SNR to $10^{-1}|h_{12}|^2$ at small SNR. We observe that: 1) DF sensitivity firmly increases with the transmission rate (even though the estimation error then becomes smaller). However, CF sensitivity increases only slightly. This is due to the fact that, when it operates in CF, the relay needs only quantize signal $\widetilde{Y}_2$. Thus, small estimation error has only limited impact on CF, as small signal variations do not cause $\widetilde{Y}_2$ to fall outside the quantization cell (however, error noise may have a larger impact if channels R $\rightarrow$ D and S $\rightarrow$ D are known only partially, since this may cause a mismatch between Wyner-Ziv encoder (at R) and decoder (at D)). We also observe that 2) DF is more (resp. less) sensitive than CF at high (resp. low) transmission rate, which conforms remark 2 in Section 2.

4. FULL-DUPLEX GAUSSIAN RC UNDER CHANNEL ESTIMATION ERROR

In this section, we investigate the effect of channel estimation error on the achievable rates for a Full-Duplex Gaussian RC. We show that the superiority of both DF and CF over simple transmission over the direct link (DL) (i.e., relay off) may be questioned if the channel is severe. Then, we build upon this observation to derive bounds on the capacity region of the GRC under channel estimation error.

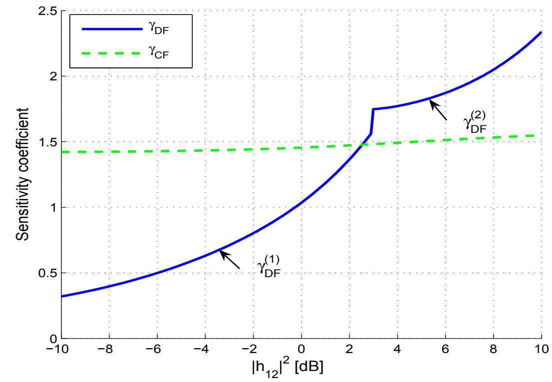

Fig. 2. Sensitivity of DF and CF to small perturbations on the S $\rightarrow$ R channel, due to channel estimation error. $P_1 = P_2 = 5$ dB.

4.1. Effect of estimation error on the Full-Duplex GRC

Consider the GRC considered in Section 3,

$$\tilde{y}_2 = \hat{h}_{12}x_1 + \tilde{h}_{12}x_1 + z_2 \quad (12a)$$

$$\tilde{y} = \hat{h}_{13}x_1 + \hat{h}_{23}x_2 + \tilde{h}_{13}x_1 + \tilde{h}_{23}x_2 + z_3, \quad (12b)$$

where $\tilde{h}_{12}, \tilde{h}_{23}$ and $\tilde{h}_{13}$ stand for estimation errors and have variances $\delta_{12}^2, \delta_{23}^2$ and δ_{13}^2, respectively. From an information theoretical standpoint, the estimation error causes the GRC to deviate from the classical description for at least three reasons. First, treating the error terms $\tilde{h}_{12}x_1$, $\tilde{h}_{23}x_2$ and $\tilde{h}_{13}x_1$ as noise is clearly suboptimal, as these terms may potentially convey "information" about channel input, since they are statistically dependent on both X_1 and X_2. Second, it is not clear whether one could find a combination (R,S,D) that can exploits this dependence and hence, the capacity of the resulting channel (even when the original channel is degraded) is unknown. Third, even if the estimation error is sub-optimally relegated to noise, this noise remains non-conventional, in that it is possibly non-Gaussian (for the rate-maximizing input distribution) and above all dependent on both X_1 and X_2.

4.2. Bounds on capacity

We obtain an upper bound by simply using the "max-flow-min-cut" Theorem [7, Theorem 14.10.1] or [2, Theorem 4] and conditioning on vector $h = (h_{12}, h_{23}, h_{13})^T$. Let $C(x) \triangleq 0.5\log_2(1+x)$.

Proposition 4.1 *(Upper bound) The capacity of the Full-Duplex Gaussian RC with estimation error (12) is upper bounded by*

$$\begin{aligned} C^{out} &= \max_{p(x_1,x_2)} \min\{I(X_1X_2; \widetilde{Y}|h), I(X_1; \widetilde{Y}\widetilde{Y}_2|X_2h)\} \\ &= \max_{\beta} \min \Big\{C(|\hat{h}_{13}|^2 P_1 + |\hat{h}_{23}|^2 P_2 + 2\sqrt{\beta|\hat{h}_{13}|^2|\hat{h}_{23}|^2 P_1 P_2} \\ &\quad + \delta_{13}^2 P_1 + \delta_{23}^2 P_2), C((1-\beta)(|\hat{h}_{12}|^2 + |\hat{h}_{13}|^2 + \delta_{12}^2 + \delta_{13}^2)P_1)\Big\}. \end{aligned}$$

Next, we obtain a lower bound by replacing the error noise in (12) by Gaussian noise with the same variance and combining the rate regions achievable by DF, CF and DL (this will be justified below).

Proposition 4.2 *(Achievable rate region) The capacity of the Full-Duplex GRC with estimation error (12) is lower bounded by*

$$C^{in} = \max_{0 \le \beta \le 1} \{C_1^{in}(\beta), C_2^{in}, C_3^{in}\}, \quad (13)$$

where

$$C_1^{in}(\beta) = \min\left\{C(\frac{|\hat{h}_{13}|^2 P_1 + |\hat{h}_{23}|^2 P_2 + 2\sqrt{\beta|\hat{h}_{13}|^2|\hat{h}_{23}|^2 P_1 P_2}}{1+\delta_{13}^2 P_1 + \delta_{23}^2 P_2}),\right.$$
$$\left. C(\frac{(1-\beta)|\hat{h}_{12}|^2 P_1}{1+\delta_{12}^2 P_1})\right\}, \quad (14a)$$

$$C_2^{in} = C(\frac{|\hat{h}_{13}|^2 P_1}{1+\delta_{13}^2 P_1+\delta_{23}^2 P_2} + \frac{|\hat{h}_{12}|^2 P_1}{1+\sigma_w^2+\delta_{12}^2 P_1}), \quad (14b)$$

$$C_3^{in} = C(\frac{|\hat{h}_{13}|^2 P_1}{1+\delta_{13}^2 P_1}). \quad (14c)$$

The rate $C_1^{\text{in}}(\beta)$ is obtained by lower-bounding $I(X_1; \widetilde{Y}_2|X_2)$ and $I(X_1X_2; \widetilde{Y})$ using techniques which are similar in nature to those used in [9] to bound the capacity region of a multiple access channel (MAC) (However, caution should be exercised here since, by opposition to MAC inputs in [9], X_1 and X_2 are correlated). The rate C_2^{in} is obtained by evaluating the achievable rate $\max_\rho I(X_1; \widetilde{Y}\widetilde{\widehat{Y}}_2|X_2)$ with the choice of input distribution s.t. $\mathbb{E}[X_1X_2] = \rho\sqrt{P_1P_2}$ and $\widetilde{\widehat{Y}}_2 = Y_2 + Z_w$, where $Z_w \sim \mathcal{N}(0, \sigma_w^2)$ and σ_w^2 is the "compression noise" satisfying $I(X_2; \widetilde{Y}) = I(\widetilde{Y}_2; \widetilde{\widehat{Y}}_2|X_2\widetilde{Y})$. The rate C_3^{in} corresponds to DL. Note that the additional compression noise due to the estimation error is $\sigma_e^2 = \sigma_w^2 - (1 + (|\hat{h}_{12}|^2 + |\hat{h}_{13}|^2)P_1)/|\hat{h}_{23}|^2 P_2$.

4.3. Discussion

Fig. 3 plots the bounds C^{in} and C^{out} and the rates achievable by DF (C_1^{in}), CF (C_2^{in}) and DL (C_3^{in}), versus the SNR in the link S $\rightarrow$ R. We observe that:

1. when the channel is perfectly known (curves in solid line), CF always gives a rate gain over DL. So, in this case turning the relay off (i.e., operating in DL) inevitably results in rate loss. However, in presence of estimation error (curves in dashed line), this no longer holds since DL may improve upon CF when the channel is "bad enough" (i.e., small values of $|h_{12}|$).
2. the best rate achievable with a mixed strategy that uses DF, CF and DL (i.e., the one that achieves the lower bound C^{in}) consists in keeping the relay off at very small SNR, then have the relay operate in CF for small-to-medium SNR and finally, when the channel becomes "sufficiently good", use DF.
3. DF is more efficient (in terms of transmission rate) than CF at high rate and (unfortunately) it is also more sensitive. The same remark is valid for CF at low rate. This is because, the more one scheme benefits from channel knowledge (e.g., DF when R is close to S) the more it is vulnerable to small variations in this channel. A trade-off rate/sensitivity is needed for non-demanding rate applications.

Remark 3: Note that the fact that, by opposition to the case when the channel is known, DL may potentially improve upon both DF and CF in presence of channel estimation error reveals one of main limitations of cooperative communication in real situations: error propagation from one node to another.

5. CONCLUSION

In this paper, we investigated the sensitivity of two relaying strategies—the decode-and-forward (DF) scheme and the compress-and-forward (CF) scheme, to small additive disturbances. We used Fisher Information and De-Bruijn's Identity to assess the decrease in the corresponding rates. Analysis sheds light on the "robustness" of these two schemes to small channel variations and provides insights onto the choice of appropriate relaying strategies that meet a certain trade-off between transmission rate and sensitivity, in real situations. Next, we used these result to emphasize the impact of channel estimation error on a Full-Duplex Gaussian relay channel and derive lower and upper bounds on its capacity.

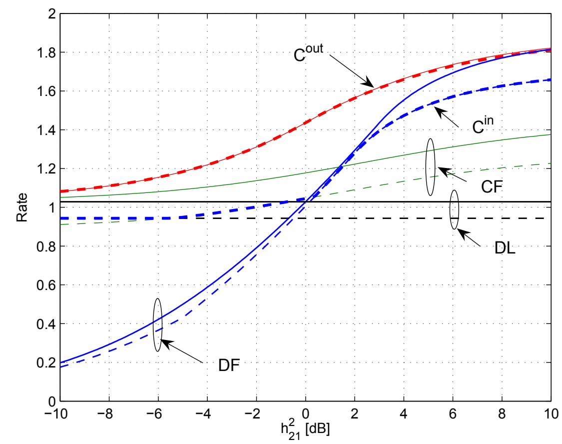

Fig. 3. Lower bound C^{in}, upper bound C^{out}, and rates achievable by DF, CF and DL for Full-Duplex GRC with (dashed) and without (solid) channel estimation error, versus the SNR in the link S $\rightarrow$ R. The variance δ_{12}^2 of the estimation error varies inversely proportionally to SNR as in Section 3. $P_1 = P_2 = 5$ dB.

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