DISTRIBUTED SPACE-FREQUENCY CODING IN BROADBAND AD HOC NETWORKS

Wei Zhang[†], Yabo Li^{*}, Xiang-Gen Xia[‡], P. C. Ching[§], and Khaled Ben Letaief[†]

[†]Department of ECE, Hong Kong University of Science and Technology, Hong Kong ^{*} Nortel Networks, Richardson, TX 75082, USA [‡]Department of ECE, University of Delaware, DE 19716, USA [§]Department of EE, The Chinese University of Hong Kong, Hong Kong

ABSTRACT

A distributed space-frequency (SF) coded cooperative technique is proposed for broadband wireless ad-hoc networks. By employing an SF coding at the source node and a circular shift at each relay node, the proposed cooperative diversity technique can exploit the maximum relay diversity and multipath diversity. Pairwise error probability analysis shows that the achievable diversity gain is $\min(ML_1, ML_2)$, where Mis the number of the relay nodes, and L_1 and L_2 are the number of taps of the multipath fading channels from the source node to the relay nodes and from the relay nodes to the destination node, respectively.

Index Terms— Cooperative diversity, multipath channels, relay channels, space-frequency coding.

1. INTRODUCTION

Cooperative diversity or user diversity allows a virtual antenna array to achieve spatial diversity gain in a distributed fashion [1, 2]. A variety of cooperative diversity algorithms have been developed to achieve the full spatial diversity gain, such as repetition coding [2], distributed space-time block coding (D-STBC) [3, 4] and conventional channel coding integrated into cooperation [5]. The pairwise error probability (PEP) of D-STBC has also been analyzed when linear space-time codes are employed among the relays and the results demonstrate that the PEP decays as $\left(\frac{\log P}{P}\right)^M$, where M denotes the number of parallel relay nodes and P is the total transmit power [4].

In broadband wireless cooperative networks, OFDM was applied with STBC to achieve cooperative diversity [6, 7]. But the multipath diversity was not exploited. Recently, two distributed STBC schemes were proposed to exploit the multipath diversity for a cooperative system with two relay nodes [8]. Another recent work using convolutional coding was shown to achieve cooperative and multipath diversity in a two-user cooperative network [9]. However, the schemes in [8] and [9] do not address the general case of more than two cooperative users.

In this paper, we propose a cooperative diversity technique with a distributed SF coding for broadband wireless adhoc networks. The proposed cooperative diversity technique with distributed SF coding can exploit both relay and multipath diversity for any number of relay nodes. The maximum achievable diversity gain is shown to be $\min(ML_1, ML_2)$, where M is the number of relays, L_1 and L_2 are the number of independent paths of the channel from the source to relays and that from the relays to the destination, respectively.

The rest of this paper is organized as follows. In Section 2, the system model of the proposed cooperative technique for broadband wireless ad-hoc networks is described. In Section 3, the PEP is analyzed and the diversity gain is given. In Section 4, a distributed SF coding is proposed for cooperative networks and its achievable diversity gain is shown. In Section 5, simulation results are shown. Finally, we draw our conclusions in Section 6.

2. SYSTEM MODEL

The information is transmitted via two phases as shown in Fig. 1. In the first phase, the source node S broadcasts information to all relay nodes R_i $(i = 1, \dots, M)$. During the second phase, the source node S stops the transmission and all relay nodes R_i $(i = 1, \dots, M)$ retransmit the received signals to the destination node D simultaneously. It is assumed that there are no communications between any two relay nodes and amplify-and-forward protocol is employed at each relay node. The channel impulse response of $S \rightarrow R_i$ is given by

$$h_{\text{SR}_{i}}(\tau) = \sum_{l'=0}^{L_{1}-1} h_{\text{SR}_{i}}(l')\delta(\tau - \mu_{l'})$$
(1)

and the channel $R_i \rightarrow D$ is given by

$$h_{R_{i}D}(\tau) = \sum_{l=0}^{L_{2}-1} h_{R_{i}D}(l)\delta(\tau - \nu_{l}), \qquad (2)$$

This work was partially supported by a research grant awarded by the Hong Kong Research Grant Council and in part by the Air Force Office of Scientific Research (AFOSR) under Grant No. FA9550-05-1-0161 and the National Science Foundation under Grant CCR-0097240 and CCR-0325180.



Fig. 1. Network structure with one source node, one destination node and *M* relay nodes

for any $i = 1, \dots, M$, where μ_l and ν_l denote the *l*th path delay of the channel $S \to R_i$ and the channel $R_i \to D$, respectively. $h_{SR_i}(l)$ and $h_{R_iD}(l)$ represent the Rayleigh fading coefficients of the *l*th path of the channel $S \to R_i$ and the channel $R_i \to D$, respectively, and are both zero-mean complex Gaussian random variables (RVs) with variance δ_l^2 . Suppose that all paths in any channel are independent with each other and the channels between any two nodes are also uncorrelated. Furthermore define

$$\mathbf{h}_{\mathrm{SR}_{i}} = \begin{bmatrix} h_{\mathrm{SR}_{i}}(0) & h_{\mathrm{SR}_{i}}(1) & \cdots & h_{\mathrm{SR}_{i}}(L_{1}-1) \end{bmatrix}^{T} \\ \mathbf{h}_{\mathrm{R}_{i}\mathrm{D}} = \begin{bmatrix} h_{\mathrm{R}_{i}\mathrm{D}}(0) & h_{\mathrm{R}_{i}\mathrm{D}}(1) & \cdots & h_{\mathrm{R}_{i}\mathrm{D}}(L_{2}-1) \end{bmatrix}^{T} ,$$
(3)

where $[\cdot]^{\mathcal{T}}$ denotes the transpose. We assume that the channel power constraint as $\mathbf{E}[\mathbf{h}_{SR_i}^{\mathcal{H}}\mathbf{h}_{SR_i}] = \mathbf{E}[\mathbf{h}_{R_iD}^{\mathcal{H}}\mathbf{h}_{R_iD}] = 1$, where $[\cdot]^{\mathcal{H}}$ and $\mathbf{E}[\cdot]$ stand for Hermitian and statistical average, respectively. To overcome the ISI induced by multipath fading, we adopt OFDM in the cooperative network. We shall assume that each OFDM block has N subcarriers and one OFDM block is transmitted in the following discussion.

At the source node, information bits are firstly mapped to QAM symbols and then a block of N QAM symbols **S** are encoded into a vector $\mathbf{X} = \begin{bmatrix} X_1 & X_2 & \cdots & X_N \end{bmatrix}^T$. Afterwards, the coded vector **X** will be fed to the N-point IDFT block and appended with a cyclic prefix of length L_{SR}^{cp} larger than the maximum path delay of the channel between source node to any relay node. Finally, the OFDM signals are broadcasted to all M relay nodes.

At the *i*th $(i = 1, 2, \dots, M)$ relay node, the received signals from the source node will be amplified and forwarded to the destination node, referred to as amplify-and-forward protocol. Specifically, each relay will process a few simple steps as follows:

- 1. removal of the cyclic prefix of length L_{SR}^{cp} from each received OFDM block;
- 2. circular shift of k_i samples (The choice of k_i will be specified later) and amplified with a factor α ; and
- 3. insertion of a cyclic prefix of length L_{RD}^{cp} larger than the

maximum path delay of the channel between any relay node to the destination node.

At the destination node, the cyclic prefix of length L_{RD}^{cp} will be discarded from the received signals. Thus, the ISI induced by multipath fading channels between relay node *i* and the destination node is eliminated. After passing through an *N*-point DFT, the signals are transformed to frequency domain signals **Z**. Assume that the perfect channel knowledge is available at the destination node, maximum likelihood (ML) detection can then be performed.

3. PAIRWISE ERROR PROBABILITY

Let $\mathbf{Y}_i = [Y_i(1) \ Y_i(2) \ \cdots \ Y_i(N)]^T$ be the received OFDM block at relay node *i* in the frequency domain. It can be given by

$$\mathbf{Y}_i = \sqrt{P_1}(\mathbf{X} \circ \mathbf{H}_{\mathrm{SR}_i}) + \mathbf{W}_i, \qquad (4)$$

where \circ denotes component-wise product, P_1 is the transmit power of the source node, $\mathbf{W}_i \in \mathbb{C}^N$ is the zero-mean complex Gaussian noise at relay node *i* with covariance matrix $\mathbf{E}[\mathbf{W}_i \mathbf{W}_i^{\mathcal{H}}] = N_0 \mathbf{I}_N$, and $\mathbf{H}_{SR_i} \in \mathbb{C}^N$ denotes the frequency response of the channel \mathbf{h}_{SR_i} . The average SNR of the channel between the source node and the *i*th relay node is given by $\gamma_1 = P_1/N_0$.

Note that the circular shift of the signal by k_i samples in time domain at relay *i* is equivalent to the multiplication of f^{k_i} in frequency domain, the signal Y_i is then converted to

$$\mathbf{A}_i = \alpha \cdot \mathbf{f}^{k_i} \circ \mathbf{Y}_i,\tag{5}$$

where $\mathbf{f} = \begin{bmatrix} 1 & e^{-\mathbf{j}2\pi/N} & \cdots & e^{-\mathbf{j}2\pi(N-1)/N} \end{bmatrix}^T$ and the scalar α ensures that the average transmit power of any relay node is P_2 with $\alpha = \sqrt{\frac{P_2}{N_0+P_1}}$. Let $\gamma_2 = P_2/N_0$, then $\alpha = \sqrt{\frac{\gamma_2}{1+\gamma_1}}$. Afterwards, the time domain signals are forwarded to the destination node from all relay nodes simultaneously.

Let $\mathbf{Z} = \begin{bmatrix} Z(1) & Z(2) & \cdots & Z(N) \end{bmatrix}^{\mathcal{T}}$ be the signal vector at the destination node after the *N*-point DFT. Then the relationship between \mathbf{A}_i and \mathbf{Z} for all $i = 1, \cdots, M$ can be regarded as MISO-OFDM and is therefore formulated as

$$\mathbf{Z} = \sum_{i=1}^{M} (\mathbf{A}_i \circ \mathbf{H}_{\mathbb{R}_i \mathbb{D}}) + \mathbf{W}_D,$$
(6)

where $\mathbf{W}_D \in \mathbb{C}^N$ is the zero-mean complex Gaussian noise at the destination node with covariance matrix $\mathbf{E}[\mathbf{W}_D\mathbf{W}_D^{\mathcal{H}}] = N_0\mathbf{I}_N$, and $\mathbf{H}_{\mathsf{R}_i\mathsf{D}} \in \mathbb{C}^N$ denotes the frequency response of the channel $\mathbf{h}_{\mathsf{R}_i\mathsf{D}}$. By substituting (4) and (5) into (6), we obtain

$$\mathbf{Z} = \alpha \sqrt{P_1} \sum_{i=1}^{M} \operatorname{diag}(\mathbf{f}^{k_i} \circ \mathbf{X})(\mathbf{H}_{\mathrm{SR}_i} \circ \mathbf{H}_{\mathrm{R}_i \mathrm{D}}) + \mathbf{V}, \quad (7)$$

where $\mathbf{V} = \alpha \sum_{i=1}^{M} (\mathbf{H}_{R_i D} \circ \mathbf{f}^{k_i} \circ \mathbf{W}_i) + \mathbf{W}_D$ is the zeromean complex Gaussian noise term with covariance matrix $\mathbf{E}[\mathbf{V}\mathbf{V}^{\mathcal{H}}] = N_0 \left(\mathbf{I}_N + \alpha^2 \operatorname{diag}\left(\sum_{i=1}^M |\mathbf{H}_{\mathsf{R}_i\mathsf{D}}|^2 \right) \right).$ Let $L_{min} = \min(L_1, L_2)$ and define

$$G_d = \min_{\forall \mathbf{X} \neq \hat{\mathbf{X}}} \operatorname{rank} \left(\operatorname{diag}(\mathbf{X} - \hat{\mathbf{X}}) \mathbf{G} \right)$$
(8)

where the matrix **G** of size $N \times ML_{min}$ is composed of ML_{min} column vectors, $\{\mathbf{f}^{(\mu_{l'}+\nu_l)N/T+k_i}\}$ for all l = l' = $0, \dots, L_{min} - 1$ and $i = 1, \dots, M$. We then give the following main result.

Theorem 1: For a code design **X** and $k_i (i = 1, \dots, M)$, if $G_d = ML_{min}$, then the PEP is upper bounded by

$$P(\mathbf{C} \to \hat{\mathbf{C}}) < G_c^{-ML_{min}} \left(\frac{\log \rho}{\rho}\right)^{ML_{min}}$$
$$\simeq (G_c \rho)^{-ML_{min}} \text{ as } \rho \to \infty, \quad (9)$$

where G_c is a constant and $\rho \simeq \frac{M\gamma_1\gamma_2}{1+\gamma_1+M\gamma_2}$. The proof of *Theorem 1* is in [11].

It can be seen from the PEP upper bound in (9) that the achievable diversity gain (ADG) is ML_{min} , i.e., the product of the cooperative (relay) diversity M and the multipath diversity $\min(L_1, L_2)$. In order to achieve diversity ML_{min} , in the following, we will design **X** and k_i $(i = 1, 2, \dots, M)$.

4. DISTRIBUTED SPACE-FREQUENCY CODING

4.1. Coding Structure

Suppose that a block of N data symbols **S** are generated from the signal alphabet $\mathcal{A} \in \mathbb{Z}[\mathbf{j}]$ such as QAM at the source node. The SF codeword $\mathbf{X} \in \mathbb{C}^N$ is of the form,

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1^{\mathcal{T}} & \mathbf{X}_2^{\mathcal{T}} & \cdots & \mathbf{X}_J^{\mathcal{T}} \end{bmatrix}^{\mathcal{T}},$$
(10)

where J = N/K and $K = 2^{\lceil \log_2 \tilde{L} \rceil}$. $\lceil \cdot \rceil$ denotes the ceiling operation. Here \tilde{L} is a parameter to adjust the value of K and we shall see later that the choice of K depends on the diversity-complexity tradeoff. The vector $\mathbf{X}_n \in \mathbb{C}^K$ is given by

$$\mathbf{X}_n = \mathbf{\Theta} \mathbf{S}_n, \quad n = 1, 2, \cdots, J, \tag{11}$$

where

$$\Theta = \mathcal{F}_{K}^{\mathcal{H}} \operatorname{diag} \left(\begin{array}{ccc} 1 & \theta & \cdots & \theta^{K-1} \end{array} \right),$$

$$\mathbf{S}_{n} = \left[\begin{array}{ccc} s_{(n-1)K+1} & s_{(n-1)K+2} & \cdots & s_{nK} \end{array} \right]^{\mathcal{T}} (12)$$

$$\mathbf{S} = \left[\begin{array}{ccc} \mathbf{S}_{1}^{\mathcal{T}} & \mathbf{S}_{2}^{\mathcal{T}} & \cdots & \mathbf{S}_{J}^{\mathcal{T}} \end{array} \right]^{\mathcal{T}}.$$
(13)

 \mathcal{F}_K is a $K \times K$ DFT matrix with the (i, j)th entry $[\mathcal{F}_K]_{i,j} =$ $e^{-j2\pi(i-1)(j-1)/K}$ and $\theta = e^{j2\pi/(4K)}$.

One important property of the design of the constellation rotation matrix Θ is that there exists no zero entries in the vector $(\mathbf{X}_n - \hat{\mathbf{X}}_n)$ for any $\mathbf{S}_n - \hat{\mathbf{S}}_n \neq \mathbf{0}$ in (11), which is called signal space diversity or modulation diversity [10].

4.2. Achievable Diversity Gain

In the following, we will examine the achievable diversity gain of the proposed SF coding with the circular shift technique in cooperative networks.

We rewrite **G** of size $N \times ML_{min}$ in (8) as follows,

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1^{\mathcal{T}} & \mathbf{G}_2^{\mathcal{T}} & \cdots & \mathbf{G}_J^{\mathcal{T}} \end{bmatrix}^{\mathcal{T}},$$
(14)

where the submatrices $\mathbf{G}_n, n = 1, 2, \cdots, J$, have size $K \times$ ML_{min} . Suppose that rank(G) = p, then rank(G_n) = $\min(K, p)$.

For any pair of distinct SF codewords X and $\hat{\mathbf{X}}$ related to \mathbf{S} and $\mathbf{\hat{S}}$, respectively, it is certain from (13) that there exists at least one index q $(1 \le q \le J)$ such that $\mathbf{S}_q \neq \mathbf{S}_q$. Then, from (11) it can be shown that there are no zero entries in the length-K vector $(\mathbf{X}_q - \hat{\mathbf{X}}_q)$. Denote a matrix $\boldsymbol{\Upsilon}$ of size $K \times$ ML_1L_2 by $\Upsilon = \text{diag}(\mathbf{X}_q - \hat{\mathbf{X}}_q)\mathbf{G}_q$. From (8) we can then have $G_d = \operatorname{rank}(\mathbf{\Upsilon}) = \operatorname{rank}(\mathbf{G}_q)$. Recall $\operatorname{rank}(\mathbf{G}_n) =$ $\min(K, p)$ for any $n = 1, \dots, J$, we can obtain

$$G_d = \min\left(K, p\right). \tag{15}$$

Note that the rank of G of size $N \times ML_{min}$ is equal to the cardinality of the following set,

$$\mathcal{I} = \left\{ (\mu_{l'} + \nu_l) \frac{N}{T} + k_i, | \quad \forall l = l' = 0, 1, \cdots, L_{min} - 1; \\ i = 1, \cdots, M \right\}$$

i.e., $p = |\mathcal{I}|$, where $|\cdot|$ denotes the cardinality of a set. Assume that $(\mu_{(L_{min}-1)}+\nu_{(L_{min}-1)})\frac{N}{T}<\lfloor N/M\rfloor$ and let

$$k_i = (i-1)\lfloor N/M \rfloor, \ i = 1, \cdots, M,$$
(16)

where $|\cdot|$ denotes the floor operation. We immediately get $p = |\mathcal{I}| = ML_{min}$. If $K \ge ML_{min}$, from (15) we have

$$G_d = ML_{min}.$$

From *Theorem 1*, we see that the proposed SF coding (11) combined with a circular shift (16) can achieve the diversity ML_{min} . It should be mentioned that (16) is not the unique design of k_i . If the knowledge of path delays is known at relay nodes, it is not hard to obtain many choices for k_i such that $p = ML_{min}$.

In a special case when the channels $S \rightarrow R$ and $R \rightarrow D$ are both flat fading channels, our proposed distributed SF coding scheme can obtain the full relay diversity M. It should be mentioned that the distributed space-time coding [4] can also achieve full relay diversity over flat fading channels.

5. SIMULATION RESULTS

The simulated OFDM system has 64-point DFT and the length of the cyclic prefix is $L_{SR}^{cp} = L_{RD}^{cp} = 16$. The sampling frequency is set at 20 MHz. Thus, the effective duration of one



Fig. 2. Symbol error rate performance vs. SNR ρ (dB) in doubly flat fading channels and doubly frequency-selective fading channels. Two and four relay nodes are considered.

OFDM symbol is $T = 3.2\mu s$. The circular shift is $k_i = (i - 1)\lfloor N/M \rfloor$ at the *i*th relay node, $i = 1, 2, \dots, M$. The data symbols are from 4QAM. The channels $S \rightarrow R$ and $R \rightarrow D$ are both frequency-selective fading with two equal power rays with the path delay $\mu = [0, 0.5]\mu s$ and $\nu = [0, 0.5]\mu s$ respectively. Assume that the channel knowledge is perfectly known at the destination node.

Fig. 2 shows the symbol error rate (SER) performance versus SNR of the distributed SF coding over frequency selective fading channels ($S \rightarrow R$ and $R \rightarrow D$) for two and four relay nodes. For a comparison, the flat fading channels are also considered. As the two-ray channel model is used, we use K = 2M in order to achieve the diversity gain ML_{min} . It can be seen from Fig. 2 that the SER performance is improved over frequency-selective fading channels compared to that over flat fading channels for both cases of two and four relay nodes. The best one among the four performance curves in Fig. 2 having the smallest SER is achieved when M = 4over frequency-selective fading channels which has the diversity gain $ML_{min} = 8$ as seen from *Theorem 1*. On the contrary, the worst one is given by M = 2 over flat fading channels which has only diversity $ML_{min} = 2$. The other two curves has the same slope, especially in high SNR. This results from the same diversity gain $ML_{min} = 4$ obtained for M = 4 in flat fading and M = 2 in frequency-selective fading.

6. CONCLUSION

For broadband wireless ad-hoc networks which allows a parallel set of nodes to serve as amplify-and-forward relays (i.e., non-regenerative relays) in two-hop wireless relay networks, we have developed a distributed SF coding scheme that can achieve a diversity gain of $min(ML_1, ML_2)$, where M is the number of relay nodes, and L_1 and L_2 are the number of taps of the multipath fading channels from the source to the relays and from the relays to the destination, respectively.

7. REFERENCES

- A. Sendonaris, E. Erkip and B. Aazhang, "User cooperation diversity – Part I: system description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.
- [2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocals and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.
- [3] R. U. Nabar, H. Bölcskei, and F. W. Kneubühler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Select. Areas Commun.* vol. 22, pp. 1099–1109, Aug. 2004.
- [4] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 3524–3536, Dec. 2006.
- [5] A. Stefanov and E. Erkip, "Cooperative coding for wireless networks," *IEEE Trans. Commun.*, vol. 52, pp. 1470– 1476, Sept. 2004.
- [6] P. A. Anghel and M. Kaveh, "Relay assisted uplink communication over frequency-selective channels," in *Proc. IEEE SPAWC*, June 2003, pp. 125–129.
- [7] T. Miyano, H. Murata, and K. Araki, "Cooperative relaying scheme with space-time code for multihop communications among single antenna terminals," in *Proc. IEEE GLOBECOM*, Dallas, Texas, USA, Nov. 29–Dec.3, 2004. pp. 3763–3767.
- [8] H. Mheidat and M. Uysal, "Equalization techniques for space-time coded cooperative systems," in *Proc. IEEE VTC*, Los Angles, CA, Sep. 26-29, 2004, vol. 3, pp. 1708–1712.
- [9] J. C. H. Lin and A. Stefanov, "Coded cooperation for OFDM systems," in *Proc. IEEE WNCMC*, Maui, Hawaii, June 13-16, 2005, vol. 1, pp. 7–10.
- [10] X. Giraud, E. Boutillon, and J. C. Belfiore, "Algebraic tools to build modulation schemes for fading channels," *IEEE Trans. Inf. Theory*, vol. 43, pp. 938–952, May 1997.
- [11] W. Zhang, Y. Li, X.-G. Xia, P. C. Ching, and K. B. Letaief, "Distributed space-frequency coding for cooperative diversity in broadband wireless ad hoc networks," *IEEE Trans. Wireless Commun.*, submitted for publication.