NETWORK BEAMFORMING USING RELAYS WITH PERFECT CHANNEL INFORMATION

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ABSTRACT

This paper is on beamforming in wireless relay networks with perfect channel information at the receiver and relays. It is assumed that every node in the network has its own power constraint. An amplify-and-forward protocol is used. Relays use not only the channel direction information to form a beam at the receiver but also the channel strength information to adaptively adjust their transmit powers. Our results show that the optimal power used at a relay is not a binary function. It can take any value between zero and its maximal transmit power. Also, surprisingly, this value depends on the quality of all other channels in addition to the relay's own channels.

Index Terms— Wireless relay network, power control, beamforming, amplify-and-forward

1. INTRODUCTION

It is well-known that due to the fading effect, performance of wireless communication is much worse than that of wired communication. For the simplest point-to-point communication system, which has only two users, one transmitter and one receiver, using multiple antennas can improve the capacity and reliability. Space-time coding and beamforming are among the most successful techniques developed for multiple-antenna systems during the last decades. However, in many systems, due to the limited size and processing power, it is not practical to implement multiple antennas especially for small wireless mobile devices. Thus, recently, wireless network communication is attracting more and more attention. A large amount of effort has been given to improve the communication by having different users in a network cooperate. This improvement is conventionally addressed as cooperative diversity and the techniques cooperative schemes.

Many cooperative schemes have been proposed in the literature (e.g. [3–12]). Some assume channel information at the receiver but not the transmitter and relays, for example, the noncoherent amplify-and-forward (AF) protocol in [4, 5] and distributed space-time coding (DSTC) in [6]. Some assume channel information at the receiving side of each transmission, for example, the decode-and-forward protocol in [4, 8] and the coded-cooperation in [9]. Some assume no channel

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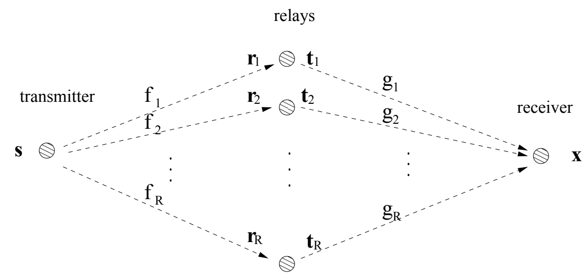

Fig. 1. Wireless relay network.

information at any node, for example, the differential transmission schemes proposed independently in [10–12]. The coherent AF in [5, 7] assumes full channel information at both relays and the receiver. But relays use the channel direction information only to form a beam at the receiver. In all these cooperative schemes, the relays always cooperate on their highest powers. None of the above pioneer work allow relays to adaptively adjust their transmit powers according to channel magnitude information, and this is exactly the concern of this paper. Several related work can be found in [13–17].

For multiple-antenna systems, when there is no channel information at the transmitter, space-time coding can achieve full diversity [1]. If the transmitter has perfect or partial channel information, performance or capacity can be further improved through beamforming (e.g. [2, 18, 19]). In this paper, we show similar improvement achieved by network beamforming over other schemes such as best-relay selection, coherent AF, and DSTC in relay networks.

The paper is organized as follows. In the next section, the relay network model and the main problem are introduced. Section 3 works on the power control problem. We first analytically solve the power control problem of two-relay networks, then give a numerical solution for networks with more relays. Simulated performance of network beamforming and other schemes in two and three-relay networks is also shown. Section 4 contains the conclusion.

2. PROBLEM STATEMENT

We consider a relay network with one transmit-and-receive pair and R relays as depicted in Fig. 1. Every relay has a single antenna which can be used for both transmission and reception. Denote the channel from the transmitter to the ith relay as f_i and the channel from the ith relay to the receiver as g_i. Assume that the ith relay knows f_i and g_i and the receiver knows all channels. The channels can have both fading and path-loss effects. Actually, our results are valid for any channel statistics. We assume that the power constraints at the transmitter and the ith relay are P_0 and P_i, respectively.

We use a two-step AF protocol. During the 1st step, the transmitter sends $\alpha_0\sqrt{P_0}s$ with s the information symbol. With the normalization, $\mathrm{E}|s|^2 = 1$, the average power used at the transmitter is $\alpha_0^2 P_0$. The ith relay receives $r_i = \alpha_0\sqrt{P_0}f_i s + v_i$ with v_i the noise at the ith relay. v_i is assumed to be $\mathcal{CN}(0,1)$. During the 2nd step, the ith relay sends,

$$t_i = \alpha_i\sqrt{\frac{P_i}{1+\alpha_0^2|f_i|^2P_0}}e^{j\theta_i}r_i.$$

The average transmit power of the ith relay can be calculated to be $\alpha_i^2 P_i$. The receiver gets

$$x = \alpha_0\sqrt{P_0}\sum_{i=1}^{R}\alpha_i\sqrt{\frac{P_i}{1+\alpha_0^2|f_i|^2P_0}}f_i g_i e^{j\theta_i}s + \sum_{i=1}^{R}\alpha_i\sqrt{\frac{P_i}{1+\alpha_0^2|f_i|^2P_0}}g_i e^{j\theta_i}v_i + w. \quad (1)$$

w is the noise at the receiver, which is also assumed to be $\mathcal{CN}(0,1)$. According to power constraints, $0 \le \alpha_0, \alpha_i \le 1$.

Our network beamforming design problem is thus the design of $\theta_1, \cdots, \theta_R$ and $\alpha_0, \alpha_1, \cdots, \alpha_R$, such that the error rate of the network communication is the smallest. This is equivalent to the receive SNR maximization. From (1), we can see that an optimal choice of the angles is $\theta_i = -(\arg f_i + \arg g_i)$. Thus, match filters should be used at relays to cancel the channel phases and form a beam at the receiver. We have

$$x = \alpha_0\sqrt{P_0}\sum_{i=1}^{R}\alpha_i\sqrt{\frac{P_i}{1+\alpha_0^2|f_i|^2P_0}}|f_i g_i|s + \sum_{i=1}^{R}\alpha_i\sqrt{\frac{P_i}{1+\alpha_0^2|f_i|^2P_0}}|g_i|e^{-j\arg f_i}v_i + w. \quad (2)$$

What is left is the optimal power control, i.e., the choice of $\alpha_0, \alpha_1, \ldots, \alpha_R$. This is also our main contribution.

3. OPTIMAL RELAY POWER CONTROL

From (2), the instantaneous receive SNR can be calculated to be

$$\frac{\alpha_0 P_0\left(\sum_{i=1}^R \alpha_i \frac{|f_i g_i|\sqrt{P_i}}{\sqrt{1+\alpha_0^2|f_i|^2P_0}}\right)^2}{1+\sum_{i=1}^R \alpha_i^2\frac{|g_i|^2P_i}{1+\alpha_0^2|f_i|^2P_0}}.$$

This is an increasing function of α_0. Therefore, $\alpha_0^* = 1$, i.e., the transmitter should always use its maximal power. The power control problem is thus:

$$\max_{\alpha_1,\ldots,\alpha_R}\psi_R(\alpha_1,\ldots,\alpha_R) \text{ s.t. } 1 \le \alpha_1,\ldots,\alpha_R \le 1,$$

where

$$\psi_R(\alpha_1,\ldots,\alpha_R) = \frac{P_0\left(\sum_{i=1}^R \alpha_i \frac{|f_i g_i|\sqrt{P_i}}{\sqrt{1+|f_i|^2P_0}}\right)^2}{1+\sum_{i=1}^R \alpha_i^2\frac{|g_i|^2P_i}{1+|f_i|^2P_0}}.$$

For networks with two relays, the following algorithm solves this problem.

Algorithm 1.

1. *Define* $\alpha_1' = \frac{|f_1|\sqrt{1+|f_1|^2P_0}}{|g_1|\sqrt{P_1}}\frac{\sqrt{1+|f_2|^2P_0}}{|f_2 g_2|\sqrt{P_2}}\left(1+\frac{|g_2|^2P_2}{1+|f_2|^2P_0}\right)$ *and* $\alpha_2' = \frac{|f_2|\sqrt{1+|f_2|^2P_0}}{|g_2|\sqrt{P_2}}\frac{\sqrt{1+|f_1|^2P_0}}{|f_1 g_1|\sqrt{P_1}}\left(1+\frac{|g_1|^2P_1}{1+|f_1|^2P_0}\right)$. *Calculate* $SNR_1 = \psi_2(1, \min\{1,\alpha_2'\})$ *and* $SNR_2 = \psi_2(\min\{1,\alpha_1'\},1)$.
2. *If* $SNR_1 > SNR_2$*, the optimal solution is* $\alpha_1^* = 1, \alpha_2^* = \min\{1,\alpha_2'\}$*, otherwise,* $\alpha_1^* = \min\{1,\alpha_1'\}, \alpha_2^* = 1$.

Proof. It is sufficient to prove the following items.

1. Either $\alpha_1^* = 1$ or $\alpha_2^* = 1$.
2. Given $\alpha_1 = 1$, ψ_2 achieves its maximum at $\min\{1,\alpha_2'\}$.
3. Given $\alpha_2 = 1$, ψ_2 achieves its maximum at $\min\{1,\alpha_1'\}$.

First we show that $\psi_2(\sqrt{a}\alpha_1, \sqrt{a}\alpha_2)$ is an increasing function of a for $a > 0$. Define $X = P_0\left(\frac{\alpha_1|f_1 g_1|\sqrt{P_1}}{\sqrt{1+|f_1|^2P_0}} + \frac{\alpha_2|f_2 g_2|\sqrt{P_2}}{\sqrt{1+|f_2|^2P_0}}\right)^2$ and $Y = \alpha_1^2\frac{|g_1|^2P}{1+|f_1|^2P_0} + \alpha_2^2\frac{|g_2|^2P}{1+|f_2|^2P_0}$. Thus, $\psi_2(\sqrt{a}\alpha_1, \sqrt{a}\alpha_2) = aX/(1+aY)$ and $\partial\psi_2(\sqrt{a}\alpha_1,\sqrt{a}\alpha_2)/\partial a = X/(1+aY)^2 > 0$. Assume that $\alpha_1^* < 1$ and $\alpha_2^* < 1$. Without loss of generality, assume that $\alpha_1^* \ge \alpha_2^*$. It is obvious that $\alpha_1^* > 0$. Define $a = 1/\alpha_1^{*2}$. We have $a > 1$. Therefore,

$$\psi_2(1, \sqrt{a}\alpha_2^*) = \psi_2(\sqrt{a}\alpha_1^*, \sqrt{a}\alpha_2^*) > \psi_2(\alpha_1^*, \alpha_2^*).$$

This contradicts the assumption that (α_1^*, α_2^*) is optimal. Thus the 1st item is proved.

Due to the symmetry of ψ_2, we only need to prove one of 2 and 3. We prove 3 here. At $\alpha_2 = 1$, we can show straightforwardly that $\partial\psi_2/\partial\alpha_1 = 0 \Leftrightarrow \alpha_1 = \alpha_1'$. It is also easy to show that $\partial\psi_2/\partial\alpha_1 > 0$ when $\alpha_1 < \alpha_1'$ and $\partial\psi/\partial\alpha_1 < 0$ when $\alpha_1 > \alpha_1'$. Therefore, given that $\alpha_2 = 1$, ψ reaches its maximum at $\alpha_1 = \min\{1,\alpha_1'\}$. □

3.1. Generalization to Networks with R Relays

In this section, we discuss the power control problem for networks with any relays. Denote $\mathbf{x} = \begin{bmatrix}\alpha_1 & \cdots & \alpha_R\end{bmatrix}^t$, where A^t is the transpose of A. It is easy to prove that

$$\partial\psi_R/\partial\alpha_j = 0 \Leftrightarrow$$

$$\alpha_j = \phi_j(\mathbf{x}) = \frac{|f_j|\sqrt{1+|f_j|^2P_0}}{|g_j|\sqrt{P_j}}\frac{1+\sum_{i\ne j}\frac{\alpha_i^2|g_i|^2P_i}{1+|f_i|^2P_0}}{\sum_{i\ne j}^R\frac{\alpha_i|f_i g_i|\sqrt{P_i}}{\sqrt{1+|f_i|^2P_0}}}.$$

Also, $\partial\psi_R/\partial\alpha_j > 0$ when $\alpha_j < \phi_j(\mathbf{x})$ and $\partial\psi_R/\partial\alpha_j < 0$ when $\alpha_j > \phi_j(\mathbf{x})$. Thus, given $\alpha_1, \ldots, \alpha_{j-1}, \alpha_j, \ldots, \alpha_R$, the optimal solution is $\alpha_j = \min\{1, \phi_j(\mathbf{x})\}$. We use the following iterative algorithm to solve these equations.

Algorithm 2.

1. *Set the maximal number of iterations* $iter$ *and the threshold* $thre$. *Set count to be 0 and* $\mathbf{x}_{pre}$ *to be* $\mathbf{x}_0$, *which is the initial value of* $\mathbf{x}$.
2. *Calculate* $\mathbf{x} = \min\{\mathbf{1}, \Phi(\mathbf{x}_{pre})\}$, *where* $\mathbf{1}$ *is the vector of all 1s and* $\Phi = \left[\begin{array}{ccc} \phi_1 & \cdots & \phi_R \end{array}\right]^t$.
3. *Add count by 1. If* $count > iter$ *or the Frobenius norm of* $\mathbf{x} - \mathbf{x}_{pre}$ *is less than* $thre$, *stop, otherwise,* $\mathbf{x}_{pre} = \mathbf{x}$ *and go back to Step 2.*

3.2. Discussion and Simulation

It is natural to expect the relay power to undergo an on-or-off scenario: a relay uses its maximal power if its channels are good enough and otherwise not to cooperate. Our result shows otherwise. The optimal power used at a relay can be any value between zero and its maximal power. In many situations, a relay should use its partial power, whose value is determined not only by its own channels but also all others. This is because every relay has two effects on the transmission. For one, it helps the transmission by forwarding the information, while for the other, it harms the transmission by forwarding noise. Its power has a non-linear effect on the powers of both the signal and noise, which makes the optimization solution not an on-or-off one, not a decoupled one, and, in general, not even a differentiable function of channel coefficients.

Networks with an aggregate power constraint P on relays were analyzed in [15]. In this case, with the same notation in Section 2, $P_i = P$ and $\sum_{i=1}^R \alpha_i^2 = 1$. The optimal solution is $\alpha_i = \frac{1}{c}\frac{|f_i g_i|\sqrt{1+|f_i|^2 P_0}}{|f_i|^2 P_0 + |g_i|^2 P + 1}$, where $c = \sqrt{\sum_{j=1}^R \frac{|f_j g_j|^2 (1+|f_j|^2 P_0)}{(|f_j|^2 P_0 + |g_j|^2 P + 1)^2}}$. We can see that α_i is a function of its own channels f_i, g_i only and an extra coefficient c, which is the same for all relays. Therefore, this power allocation can be done distributively at relays with the extra knowledge of one single coefficient c, whose value can be broadcasted by the receiver. In our case, every relay has a separate power constraint. This is a more practical assumption in sensor networks etc. since every sensor or wireless device has its own battery power limit. The power control solutions of the two cases are totally different. Distributive schemes for our power control result are under investigation.

If relay selection is used and only one relay is allowed to cooperate, it can be proved easily that we should choose the relay with the highest

$$h_i = h(f_i, g_i) = \frac{P_i |f_i g_i|^2}{1 + |f_i|^2 P_0 + |g_i|^2 P_i}.$$

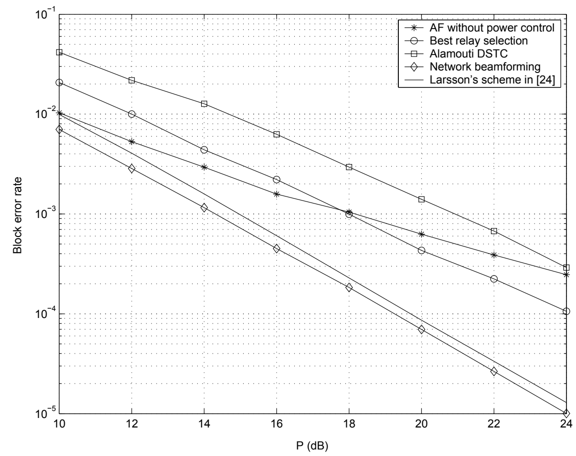

Fig. 2. Performance of two-relay networks

We call h the relay-selection function. While all relays are allowed to cooperate, the concepts of the best relay and relay-selection function are not clear. Since the power control problem is a coupled one, it is hard to measure how much contribution a relay has. It is worth mentioning that in network beamforming, there can be more than one relay using its maximal power. Actually, it is not hard to see that if at one time all channels are *good*, every relay should use its maximal power.

In Fig. 2, we compare simulated block error rates of network beamforming to best-relay selection, Larsson's scheme, DSTC, and coherent AF without power control in two-relay networks. We assume Rayleigh fading channels, i.e., f_i and g_i are i.i.d. $\mathcal{CN}(0, 1)$. Also, $P_0 = P_i = P$. The information symbol s is modulated as BPSK. We can see that network beamforming outperforms all other schemes. It is about 0.5dB and 5dB better than Larsson's scheme and best-relay selection, respectively. It is 7dB better than Alamouti DSTC, which needs no channel information at relays. Coherent AF with no power control (every relay uses its maximal power) has diversity 1, the best-relay selection and DSTC have diversity slightly less than 2, while network beamforming and Larsson's scheme achieve diversity 2. Fig. 3 shows simulated performance three-relay networks. Similar diversity results are obtained. Network beamforming is 1.5dB and 6dB better than Larsson's scheme and best-relay selection, respectively.

4. CONCLUSION

In this paper, we propose the new idea of network beamforming based on AF to achieve both diversity and array gain. Unlike many previous works in cooperative diversity, in this scheme, relays use not only the channels' phase information but also their magnitude. Match filters are applied at relays to cancel the channel phase effect and form a coherent beam at the receiver, in the meanwhile, optimal power control is performed based on the channel magnitude to decide the relay

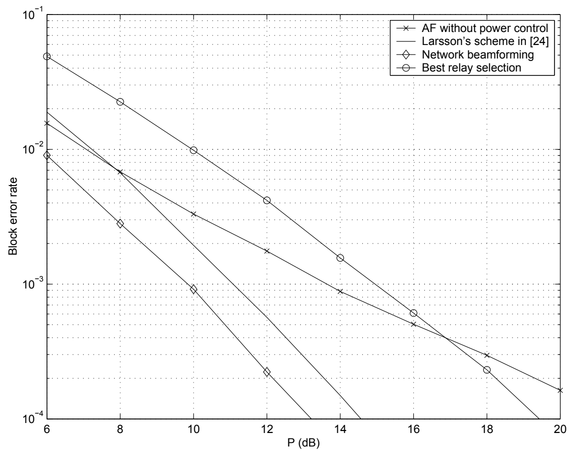

Fig. 3. Performance of three-relay networks

powers. Analytical result is obtained for two-relay networks and an iterative numerical method is provided for networks with more relays. Our results show that the power used at a relay depends on not only its own channels nonlinearly but also all other channels in the network. In general, it is not even a differentiable function of channel coefficients. Simulation with Rayleigh fading channels show that this scheme achieves the maximal diversity while AF without power control only achieves diversity 1. Compared to the best-relay-selection scheme, network beamforming is about 5-6dB better. We should note that only AF is considered here. For decode-and-forward, the result may be different.

5. REFERENCES

[1] H. Jafarkhani, *Space-Time Coding: Theory and Practice*. Cambridge Academic Press, 2005.

[2] A. Hottinen, O. Trikkonen, and R. Wichman, *Multi-Antenna Transciever Techniques for 3G and Beyond*. John Willey, 2003.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part I: System description," *IEEE Trans. Comm.*, vol. 51, pp. 1927–1938, Nov. 2003.

[4] R. U. Nabar, H. Bolcskei, and F. W. Kneubuhler, "Fading relay channels: Performance limits and space-time signal design," *IEEE Jour. Sele. Areas in Comm.*, pp. 1099 – 1109, Aug. 2004.

[5] H. Bolcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling laws in MIMO relay networks," *IEEE Trans. Wireless Comm.*, pp. 1433–1444, June 2006.

[6] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Comm.*, pp. 3524-3536, Dec. 2006.

[7] A. F. Dana and B. Hassibi, "On the power-efficiency of sensory and ad-hoc wireless networks," *IEEE Trans. Info. Theo.*, pp. 2890 – 2914, July 2006.

[8] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless network," *IEEE Trans. Info. Theo.*, vol. 49, pp. 2415–2425, Oct. 2003.

[9] M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: space-time transmission and iterative decoding," *IEEE Trans. Sig. Proc.*, pp. 362–371, Feb. 2006.

[10] Y. Jing and H. Jafarkhani, "Distributed differential space-time coding in wireless relay networks," *Accepted in IEEE Trans. Comm.*, 2006.

[11] T. Kiran and B. S. Rajan, "Partial-coherent distributed space-time codes with differential encoder and decoder," in *Proc. of IEEE Int. Symp. Info. Theo.*, 2006.

[12] F. Oggier and B. Hassibi, "A coding strategy for wireless networks with no channel information," in *Prof. of Allerton Conf.*, 2006.

[13] E. G. Larsson and Y. Cao, "Collaborative transmit diversity with adaptive resource and power allocation," *IEEE Comm. Letters*, pp. 511–513, June 2005.

[14] Y. Cao, B. Vojcic, and M. Souryal, "User-cooperative transmission with channel feedback in slow fading environment," in *Proc. of IEEE Vehicular Tech. Conf. 2004-Fall*, pp. 2063–2067, 2004.

[15] P. Larsson, "Large-scale cooperative relaying network with optimal combining under aggregate relay power constraint," in *Proc. of Future Telecomm. Conf.*, 2003.

[16] P. A. Anghel and M. Kaveh, "On the performance of distributed space-time coding systems with one and two non-regenerative relays," *IEEE Trans. Wireless Comm.*, pp. 682–692, Mar. 2006.

[17] N. Ahmed, M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "Outage minimization with limited feedback for the fading relay channel," *IEEE Trans. Comm.*, pp. 659–699, Apr. 2006.

[18] A. Narula, M. L. Lopez, M. D. Trott, and G. Wornell, "Efficient use of side information in multiple-antenna data transmission over fading channels," *IEEE Jour. Sele. Areas in Comm.*, pp. 1423–1436, Oct. 1998.

[19] S. A. Jafar and A. J. Goldsmith, "Transmit optimization and optimality of beamforming for multiple antenna systems with imperfect feedback," *IEEE Trans. Wireless Comm.*, pp. 1165–1175, July 2004.