BiZDFE: Bidirectional Zero-Based Decision Feedback Equalizers

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ABSTRACT

In this paper, a new bidirectional decision feedback equalizer (BiDFE) is proposed. The novelty of the proposed structure is that the proposed equalizer design is based on channel zeros, which is different from traditional BiDFE designs. The proposed bidirectional zero-based decision feedback equalizer (BiZDFE) efficiently exploits the time reversal operation, which results in that zeros of the equivalent channel seen from equalizer are the reciprocals of the actual channel zeros. It is shown that BiZDFE is more efficient in fading channel equalization than traditional BiDFE and minimum mean square error-decision feedback equalizer (MMSE-DFE).

Index Terms — *BiDFE; Channel Equalization; Zero Based Equalizers; Time-reversal; BiZDFE*

1 INTRODUCTION

Over the last three decades, the use of *Decision feedback equalizer* (DFE) has been reported in digital communication applications to mitigate ISI [1, 2]. DFE provides a good compromise between performance and complexity, delivering a much better performance than a linear equalizer at a much lower complexity than that of the optimum detector—the maximum likelihood sequence estimation (MLSE).

Time-reversal DFE technique has drawn research interest since [3], for the reason that the time reversal operation results in equivalent channel zeros seen from equalizer as the reciprocals of the actual channel zeros (the equivalent channel impulse response is the time-reverse of the actual channel impulse response [3]). BiDFE has two branches [4], one DFE branch is used for normal time sequence equalization, the other DFE branch is used for time-reversal sequence equalization. As channel diversity is exploited, BiDFE has better performance over traditional DFE [4, 5]. Here we consider the DFE where both feedforward filter (FFF) and feedback filter (FBF) are implemented using FIR filters based on channel zeros, as FIR filters are more robust in filter implementation [6].

2 SYSTEM MODEL AND TIME-REVERSAL OPERATION

For single-user transmissions, the discrete-time base-band model is a time-invariant single-input single-output (SISO) system described by [7],

$$y(n) = y_s(n) + w(n)$$
$$y_s(n) = \sum_{k=0}^{L} x(n-k)h(k) \tag{1}$$

Note that the input i.i.d. transmitted communication signal {x(n)} is independent of the additive white Gaussian noise {w(n)}. The channel transfer function can be expressed in a cascade form as the multiplication of first order polynomials given by,

$$H(z^{-1}) = \sum_{k=0}^{L} h(k)z^{-k} = \prod_{k=1}^{L}\left(1-b(k)z^{-1}\right) \tag{2}$$

where {h(k)} represent the channel impulse response and $\{b(k)\}$ are channel zeros. Before we proceed with further discussion, the following assumptions on the channel are noted:

1) Channel zeros can be estimated accurately by using exiting techniques, such as methods in [7, 8], or obtained from the channel's zero distribution information as in wireless communication channels [9]. Furthermore, no zeroes exists on the unit circle.

2) The channel order L is known.

Reference [3] notes that by reversing the observed signal $\{y(n)\}$ before equalization, the equivalent channel impulse response changes to the time-reversal of the actual channel impulse response. The time-reversed channel transfer function can be expressed as,

$$\begin{aligned}\overline{H}(z^{-1}) &= h(0) + h(1)z + \cdots + h(L)z^{L} \\ &= \prod_{i=1}^{L}(1-b(i)z) = c\prod_{i=1}^{L} z(1-b(i)^{-1}z^{-1})\end{aligned} \tag{3}$$

where constant $c = (-1)^L \prod_{i=1}^{L} b(i)$ can be directly derived from the channel zeros. From (2) and (3), we can see that the zeros of the time-reversal channel in (3) are the reciprocal of the original zeros of the channel in (2) operating in the traditional mode.

In order to recover the transmitted signal, which is corrupted by multi-path effect and additive noise, a DFE can be used to equalize the time-reversal signals. In parallel, a traditional DFE can also be adopted for equalization with the normal sequence (i.e. non time reversed).

3 PROPOSED BiZDFE

It has been shown in [10], that the over-all FFF transfer function of the channel-zero based DFE can be expressed as,

$$F(z^{-1}) = \prod_{i=1,|b(i)|>1}^{L} \frac{(1-b(i)^{-1}z^{-1})}{(1-b(i)z^{-1})} \qquad (4)$$

And (4) can be implemented in FIR form with a suitable delay corresponding to each zero *b(i)*. While the transfer function of the FBF is,

$$B(z^{-1}) = 1 - \prod_{i=1,|b(i)|>1}^{L}(1-b(i)^{-1}z^{-1}) \prod_{i=1,|b(i)|<1}^{L}(1-b(i)z^{-1}) \qquad (5)$$

The DFE equalizer output can be expressed as,

$$y_1(n) = \sum_{j=-Lf}^{0} f_j y_{n-j} + \sum_{j=1}^{Lb} b_j \tilde{v}_{n-j} \qquad (6)$$

where $\{f_j\}$ are the tap coefficients of the FFF, *Lf* is the length of the FFF; $\{b_j\}$ are the tap gains of the FBF, *Lb* is the length of FBF. $\{\tilde{v}(n)\}$ is an estimate of the n[th] information symbols. The assumption of accurate detection of the previous symbol is necessary and is often satisfied in DFE implementations.

We noted that one branch of BiDFE is used to equalize normal time sequence; this is as discussed above. The second branch of the DFE is for equalizing the time-reversed sequence. The input signal $\{y(n)\}$ is reversed in time-domain before feedings into the second branch of the DFE. Subsequently, the DFE output is time reversed again to obtain the equalized signal $\{y_2(n)\}$, as shown in the lower branch in Fig. 1.

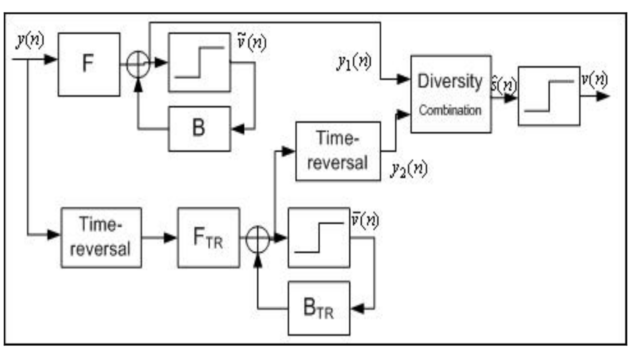

Fig. 1 The Structure of BiDFE in [3-5]

The transfer functions of the FFF and FBF of time-reversal DFE can be obtained in a similar manner to (4) and (5),

$$F_{TR}(z^{-1}) = \frac{1}{c} \prod_{i=1,|b(i)|<1}^{L} \frac{(1-b(i)z^{-1})}{(1-b(i)^{-1}z^{-1})} \qquad (7)$$

$$B_{TR}(z^{-1}) = 1 - \prod_{i=1,|b(i)|>1}^{L}(1-b(i)^{-1}z^{-1}) \prod_{i=1,|b(i)|<1}^{L}(1-b(i)z^{-1}) \qquad (8)$$

The equalized signals of the two branches are combined together before used at the input of the decision device shown in Fig.1.

From (5) and (8), we can see that the coefficients of the FBF of the normal model DFE and the time-reversal DFE are same. As seen from (4), the coefficients of the FFF of DFE (F in Fig 1) depends on the maximum phase zeros of the communication channel; and from (7), the coefficients of the FFF of time-reversal DFE (FTR in Fig 1) depends the minimum phase zeros of the original communication channel. This is the major difference between FFFs of the upper branch DFE and the lower branch time-reversal DFE. When the communication channel is a maximum phase channel or has more maximum phase zeros than minimum phase zeros, FTR=1, thus the structure of the time-reversal DFE in Fig 1 would be simpler than that of normal zero-based DFE proposed in [3-5]. Note however, that impulse responses of both F and FTR of Figure 1 are maximum phase sequences, and thus implementing them requires long filters having excessive time delays. An elegant architecture that reduces the complexity of BiDFE realization is described below.

A novel structure, which deploys four time-reversal operators (two in each branch), is proposed for the bi-directional operation. It is different from the normal BiDFE [3-5]. The block structure of the proposed BiZDFE is illustrated in Fig. 2. BiZDFE also has two branches, the upper branch is for normal time sequence equalization. Compared with Fig. 1, in Fig. 2 two more time-reversal operators are inserted on either sides of FFF. In the lower branch, the FFF is placed before the time-reversal operator. These are the essential differences from the normal BiDFE in Fig. 1 and it will be shown that such modifications greatly reduces the equalizer length in implementation.

It can be easily shown that $\tilde{F}(z^{-1}) = F(z)$ and in order to achieve:

$$y(z)F_{TR}(z^{-1}) = y(z)\tilde{F}_{TR}(z) \qquad (9)$$

we need $\tilde{F}_{TR}(z^{-1}) = F_{TR}(z)$. Using above relations, feed-forward filters of BiZDFE, $\tilde{F}(z^{-1})$ and $\tilde{F}_{TR}(z^{-1})$ can be derived by changing z^{-1} to z in (4) and (7), respectively. Note that impulse responses of both feed-forward filters of Figure 2 are minimum phase sequences, thus their implementation is efficient. $\tilde{B}(z^{-1})$ and $\tilde{B}_{TR}(z^{-1})$ of BiZDFE are same as $B(z^{-1})$ and $B_{TR}(z^{-1})$ of BiDFE.

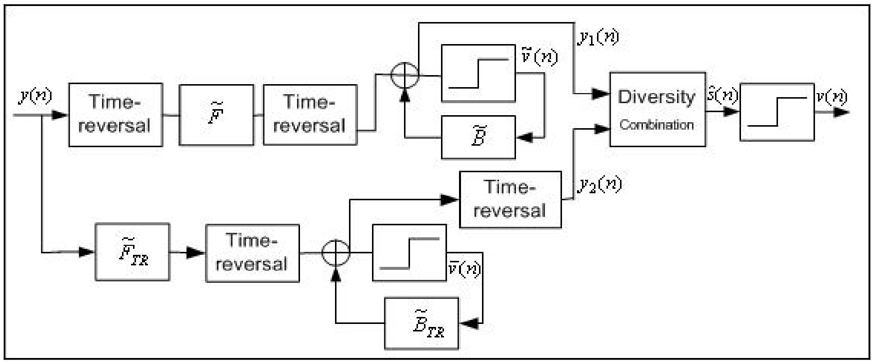

Fig. 2 The new structure of the proposed BiZDFE

4 SIMULATION RESULTS

In this section, 3 different channels are used for simulations. The input signal is selected as 16-QAM signal with zero-mean. Symbol error rate (SER) is used as the criterion to evaluate the equalizer performances. We denote the BiZDFE using structure in Fig.1 as BiZDFE I, and the BiZDFE using proposed structure in Fig.2 as BiZDFE II.

The performance of two BiZDFEs are compared with that of traditional BiDFE in [5]. The BiDFE is designed under an MMSE criterion based on channel coefficients. That is the BiDFE corresponding to BiZDFE in Fig. 1, is implemented using two MMSE-DFEs in the two branches. Comparative results are shown in the following.

The first channel is $h_1 = [1\ \text{-}0.9\ \ 0.385\ 0.771]$, which is a widely cited channel, for example in [7] and references therein. The channel consists of two maximum-phase zeros: 0.75 + 0.85i and 0.75-0.85i, and one minimum-phase zero at -0.6. SER comparison results are shown in Fig.3.

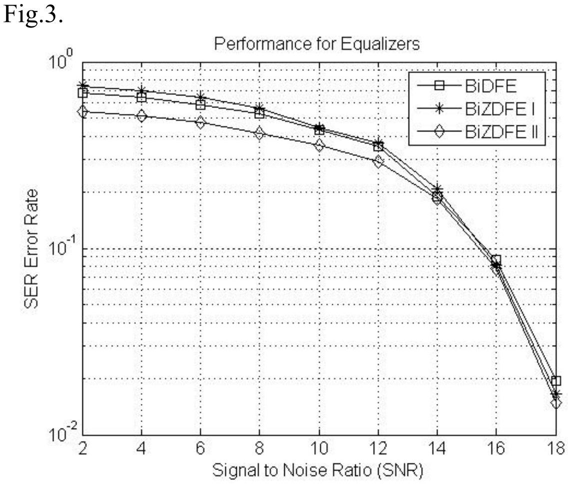

Fig. 3 Performance of BiZDFE in mixed-phase channel

For the mixed phase channel, as seen from Fig. 3, the proposed BiZDFE has better performance than BiDFE, although length of the FFF of BiZDFE is shorter than the length of FFF of BiDFE. Furthermore, the use of diversity combining [3], has provided the BiZDFE with the best performance among the three equalizers.

The second channels is a maximum phase channel having a channel impulse response given by: $h_2 = [0.6\ \ 0.1\ \ \text{-}0.729\ \ 1.285]$. It has three maximum-phase zeros at $[\text{-}1.6667, 0.75 + 0.85\text{i}\ 0.7\ \text{-}\ 0.85\text{i}]$. SER results are shown in Fig.4.

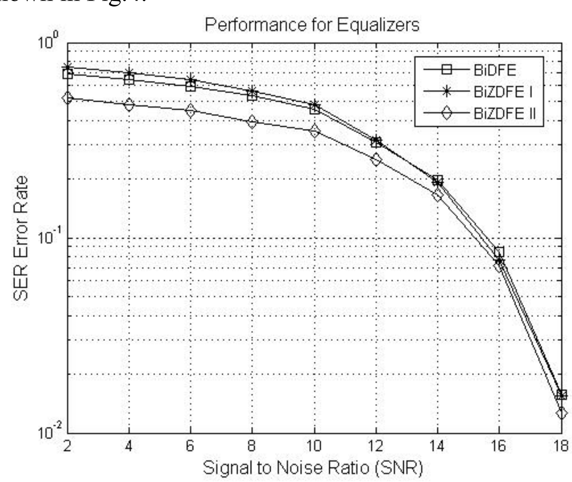

Fig. 4 Performance of BiZDFE in maximum phase channel

For maximum phase channel, the FFF of time-reversal zero-based DFE can be realized using a very short filter via equation (7). On the other hand, the FFF of normal zero based DFE is very long. From Fig. 3 and Fig. 4, we can see the performance of BiZDFE gets better when the number of maximum-phase zeros of the channel is increasing. Fig. 5 and Fig. 6, show the equalization performance of the proposed BiZDFE II.

The third example is the minimum-phase channel with impulse response given by $h_3 = [1.285\ \ \text{-}0.729\ \ 0.1\ 0.6]$. All channel zeros are inside the unit circle. SER results of the equalizers are shown in Fig.7. It can be seen that in minimum-phase channels the

MMSE-BiDFE performs marginally better than the BiZDFE at low SNR. However, at SNR > 10dB the BiZDFE I shows superior results. This is because, when the channel is minimum phase from equation (4) we get F=1 and the top branch of Fig 1 can be very efficiently implemented.

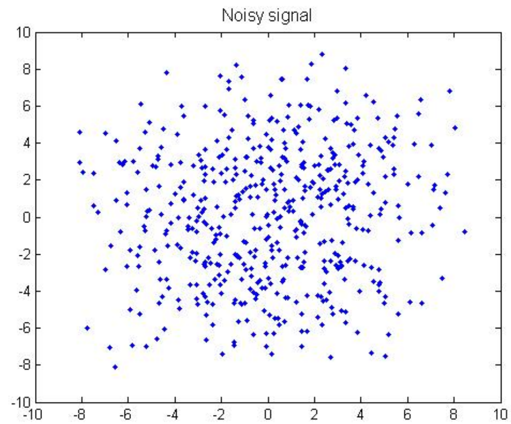

Fig. 5 The received noisy signal for maximum phase channel SNR=18dB.

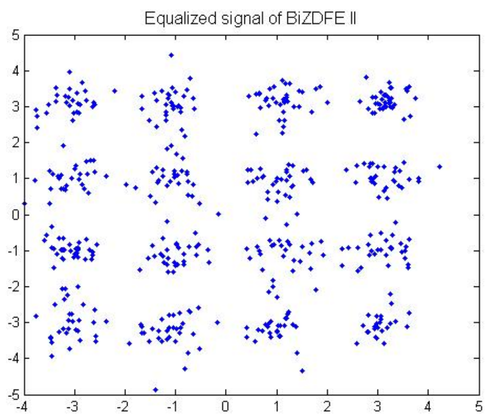

Fig. 6 Equalized signal from BiZDFE II output for maximum phase channel.

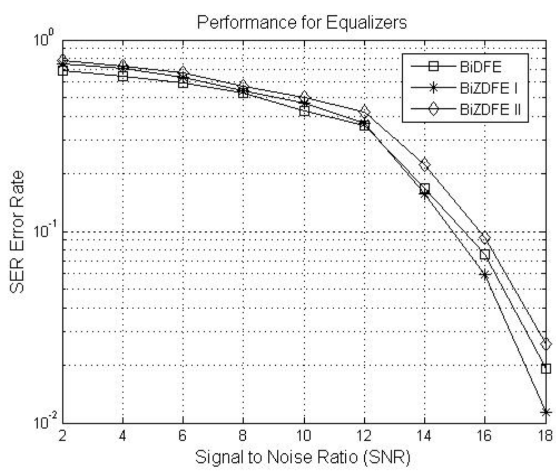

Fig. 7 Performance of BiZDFE in minimum-phase channel

Overall, the SER results suggest that BiZDFE II architecture in Fig. 2 provides very good performances. In addition, the equalizer in Fig. 2 can be efficiently realized using very short filters. Thus, the shown results are quite encouraging in block based channel equalization, as the use of short equalizing filters provide performance closer to those achieved using other longer equalizers. Such a property is very useful in hardware designs such as in FPGA implementations which aims at reducing the number of register usage.

5 SUMMARY

In this paper, a novel zero-based bidirectional decision feedback equalizer is proposed. The proposed BiZDFE makes full use of time-reversal operation, which results in an equivalent channel with zeros which are the reciprocal of the actual channel zeros. Performance of the proposed equalizer is compared with traditional equalizers in minimum phase, mixed-phase and maximum phase communication channels. The results show that the proposed BiZDFE has very good performance, at the same time providing large advantage in hardware implementation by reducing filter lengths.

6 REFERENCES

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