ITERATIVE FREQUENCY DOMAIN EQUALIZATION FOR OFDM OVER DOUBLY SELECTIVE CHANNELS

Yao Yu, Shu-hung Leung and Fei Lu

Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong
Tel:(+852)27887784, Email:EEEUGSHL@cityu.edu.hk

ABSTRACT

In this paper, we present an iterative frequency domain equalization method for orthogonal frequency division multiplexing (OFDM) over doubly selective channels. We approximate the channel by the oversampling basis expansion model (OBEM) because it can increase the model accuracy for the systems with high mobility. However, the OBEM will create more serious inter-carrier interference (ICI) than the nonoversampling BEM. We propose a new iterative method to suppress the ICI effects in the frequency domain. The channel frequency response matrix (CFRM) is divided into two parts: a main band matrix as target CFRM and an out-of-band matrix as interference CFRM. At each iteration, the interference signals caused by interference CFRM are cancelled from the received signal and the target CFRM deduces the equalizer to obtain the signal estimate. Simulation results show that our proposed method outperforms the linear MMSE equalizer and has lower complexity.

Index Terms— Equalizer, iterative method.

1. INTRODUCTION

OFDM has been considered as one of the key technologies for the next generation wireless communication systems because it has high spectral efficiency and can convert a frequency selective fading channel into several nearly flat fading subchannels. However, the ICI induced by the time selectivity of doubly selective (time and frequency selective) channels destroys the orthogonality between OFDM subcarriers and thus degrades the system performance and increases the complexity of equalizers. Therefore, the equalization for OFDM over doubly selective channels has drawn much attention recently.

In the literature, the equalizers designed for OFDM over doubly selective channels can be categorized into two classes: non-iterative equalizers and iterative equalizers. For non-iterative equalizers, the linear minimum mean square error equalizer (LMMSE) [1] can provide the better performance than most of non-iterative equalizers. On the other hand, iterative equalizers [2] based on small-size MMSE estimation and soft-decision feedback can yield better performance and requires lower complexity than the LMMSE equalizer.

For some iterative methods [3], the Doppler resolution of the BEM model is $1/(NT_s)$ (it is referred to as non-oversampling BEM (NOBEM)), where N is the block size of OFDM and T_s is OFDM sampling period. However, the BEM with this resolution suffers from bordering effect that causes degradation in model accuracy [4] [5]. In order to improve the model accuracy, oversampling BEM (OBEM), which has the Doppler resolution of $1/(KT_s)$, where K is a multiple of N, is preferred [4] [5]. Unfortunately, the OBEM will introduce more serious ICI in the frequency domain than the NOBEM. It is observed that the elements of the channel frequency response matrix (CFRM) of OBEM are no longer confined within a narrow band. As a result, most of iterative equalizers for OFDM in the frequency domain have performance degradation for OBEM. However, the OBEM can provide diversity capability to increase the detection performance.

In the paper, we propose a new iterative OFDM equalizer in the frequency domain for OBEM. We divide the CFRM of OBEM into two matrices: target CFRM that contains the main band of the CFRM and interference CFRM that contains the out-of-band elements of the CFRM. We develop an iterative equalization procedure to remove the ICI caused by the interference CFRM from the received signal and perform equalization and signal estimation using target CFRM. Experimental results show that this approach can provide significant performance gain and complexity reduction over the LMMSE. Also, the performance can be further improved by increasing the width of the banded matrix.

Notations: The small and capital letters in bold denote vectors and matrices. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and all-zero matrix as $\mathbf{0}_N$. $Re\{\cdot\}$, and the superscripts $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote the real part, conjugation, transposition, and Hermitian respectively. The statistical expectation and the probability of a random variable are denoted by $E\{\cdot\}$ and $Pr\{\cdot\}$. $\mathbf{F}$ denotes the N-point unitary DFT matrix and $diag\{\mathbf{x}\}$ is a diagonal matrix with $\mathbf{x}$ on the diagonal.

The work described in this paper was fully supported by a grant from CityU (7001851)

2. SYSTEM MODEL

The doubly selective channels in mobile wireless communications modeled by OBEM are expressed as [4] [5]:

$$h(n;\tau) = \sum_{l=0}^{L-1} \delta(\tau - l) \sum_{q=-Q/2}^{Q/2} h_{q,l} e^{j2\pi qn/K}$$

where L is the number of paths. Each channel tap is modeled as the sum of $Q+1$ complex time-varying exponential basis. The relation between Q (Q is an even number) and K should satisfy: $Q \geq \lceil 2f_{nom}K \rceil$, where $\lceil x \rceil$ is the ceiling of x, and f_{nom} is the maximum normalized Doppler frequency. The received signal in the time domain can be expressed as

$$y(n) = \sum_{l=0}^{L-1} \sum_{q=-Q/2}^{Q/2} h_{q,l} e^{j2\pi qn/K} x(n-l) + v(n)$$

where $x(n)$ and $v(n)$ are respectively the transmitted data and additive noise in the time domain. For an OFDM block of size N, the received signal vector can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{1}$$

where $\mathbf{y} = [y(0), \ldots, y(N-1)]^T$, $\mathbf{x} = [x(0), \ldots, x(N-1)]^T$, $\mathbf{v} = [v(0), \ldots, v(N-1)]^T$, and $\mathbf{H}$ is the channel impulse response matrix and can be formulated as

$$\mathbf{H} = \sum_{q=-Q/2}^{Q/2} \mathbf{D}_q \mathbf{H}_q \tag{2}$$

where $\mathbf{D}_q = diag\{[e^{j2\pi q \times 0/K}, \ldots, e^{j2\pi q \times (N-1)/K}]^T\}$

$$\mathbf{H}_q = \begin{bmatrix} h_{q,0} & 0 & & h_{q,L-1} & \cdots & h_{q,1} \\ \vdots & \ddots & \ddots & & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & h_{q,L-1} \\ h_{q,L-1} & & & \ddots & \ddots & \\ & \ddots & & & \ddots & 0 \\ 0 & & h_{q,L-1} & \cdots & \cdots & h_{q,0} \end{bmatrix}$$

After DFT transformation on (1), the received signal vector in the frequency domain is given by

$$\mathbf{r} = \mathbf{F}\mathbf{H}\mathbf{F}^H\mathbf{s} + \mathbf{F}\mathbf{v} = \mathbf{H}_f\mathbf{s} + \mathbf{v}_f \tag{3}$$

where $\mathbf{s}$ and $\mathbf{v}_f$ are respectively the transmitted data vector and the noise in the frequency domain, and $\mathbf{H}_f = \mathbf{F}\mathbf{H}\mathbf{F}^H$ is the CFRM. For time-selective channels, the CFRM is neither a diagonal matrix nor a narrowly banded matrix, but a full matrix. Substituting (2) into (3), we obtain

$$\begin{aligned} \mathbf{r} &= \sum_{q=-Q/2}^{Q/2} \mathbf{F}\mathbf{D}_q\mathbf{F}^H\mathbf{F}\mathbf{H}_q\mathbf{F}^H\mathbf{s} + \mathbf{v}_f \\ &= \sum_{q=-Q/2}^{Q/2} \tilde{\mathbf{D}}_q\tilde{\mathbf{H}}_q\mathbf{s} + \mathbf{v}_f \end{aligned} \tag{4}$$

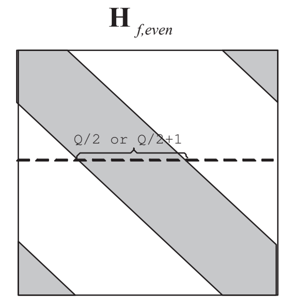

Fig. 1. The structure of $H_{f,even}$

where $\tilde{\mathbf{D}}_q = \mathbf{F}\mathbf{D}_q\mathbf{F}^H$ and $\tilde{\mathbf{H}}_q = \mathbf{F}\mathbf{H}_q\mathbf{F}^H$. $\tilde{\mathbf{H}}_q$ is a diagonal matrix since $\mathbf{H}_q$ is a circulant matrix.

Assume $K = 2N$. For even q, $\tilde{\mathbf{D}}_q$ is a circulant matrix defined as $\mathbf{Z}_m$ with $m = q/2$, whose first column is $[(N+m)_{modulo\ N} + 1]^{th}$ column of $\mathbf{I}_N$. $\tilde{\mathbf{D}}_q$ is nevertheless a full matrix for odd q. Consequently, the CFRM $\mathbf{H}_f$ can be divided into two parts: $\mathbf{H}_{f,even}$ obtained from even-index coefficients $\{h_{q,l} \mid q = \ldots, -2, 0, 2, \ldots\}$, and $\mathbf{H}_{f,odd}$ from odd-index coefficients $\{h_{q,l} \mid q = \ldots, -1, 1, \ldots\}$. $\mathbf{H}_{f,even}$ is a banded matrix with the width $Q/2$ (if $Q_{modulo\ 4} \neq 0$) or $Q/2+1$ (if $Q_{modulo\ 4} = 0$), as shown in Fig.1. However, $\mathbf{H}_{f,odd}$ is a full matrix with most of energy around its main diagonals.

3. PROPOSED ITERATIVE EQUALIZER

We define a banded CFRM with the width of $2M+1$ not less than that of $\mathbf{H}_{f,even}$ as the target CFRM, then the out-of-band interference only comes from $\mathbf{H}_{f,odd}$. Therefore, we divide $\mathbf{H}_{f,odd}$ into two parts: $\mathbf{H}_b$, the banded matrix with the width of $2M+1$ and $\mathbf{H}_{ICI}$, the residual part of $\mathbf{H}_{f,odd}$. We define $\mathbf{H}_{target} = \mathbf{H}_{f,even} + \mathbf{H}_b$, then (4) becomes

$$\mathbf{r} = \mathbf{H}_{target}\mathbf{s} + \mathbf{H}_{ICI}\mathbf{s} + \mathbf{v}_f \tag{5}$$

where $\mathbf{H}_{ICI}\mathbf{s}$ is the interference in the received signal. We modify (5) to obtain a new iteration relation as follows and depict it in Fig. 2.

$$\begin{aligned} \mathbf{r}_{target} &= \mathbf{r} - \mathbf{H}_{ICI}\bar{\mathbf{s}} \\ &= \mathbf{H}_{target}\mathbf{s} + \mathbf{H}_{ICI}(\mathbf{s} - \bar{\mathbf{s}}) + \mathbf{v}_f \end{aligned} \tag{6}$$

where $\bar{\mathbf{s}} = E\{\mathbf{s}\}$. $\mathbf{s}$ and $\mathbf{v}_f$ are assumed to be independent and identically distributed (i.i.d). So their covariance matrices are $\mathbf{C}_s = diag\{[c_s(0), c_s(1), \ldots, c_s(N-1)]\} = \sigma_s^2\mathbf{I}_N$, where $c_s(k)$ is the variance of $s(k)$, and $\mathbf{C}_v = \sigma_{v_f}^2\mathbf{I}_N$. We also assume the noise is zero mean and independent of $\mathbf{s}$. Based on (6), a low-complexity MMSE-iterative algorithm is described as follows.

Step 1) Initially, the mean of $\mathbf{s}$ is set equal to zero and $\mathbf{C}_s = \mathbf{I}_N$. The elements $\{c_s(k) \mid k = 0, \ldots, N-1\}$ on the diagonal of $\mathbf{C}_s$ will be updated at each iteration.

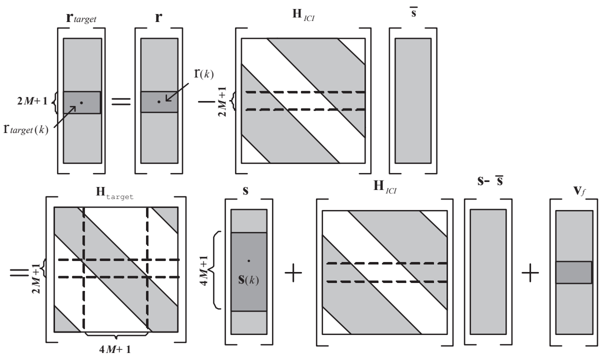

Fig. 2. The new iteration model

Step 2) We set $\mathbf{C}_{ICI,k} = \mathbf{H}_{ICI,k}\mathbf{C}_s\mathbf{H}_{ICI,k}^H$, $a_k = k-M$ and $b_k = k+M$ (modulo N), where $\mathbf{H}_{ICI,k}$ contains $2M+1$ rows of the matrix $\mathbf{H}_{ICI}$ from a_k to b_k, then the soft estimate of $s(k)$ is given by the following equations.

$$\mathbf{r}_{target,k} = \mathbf{r}_k - \mathbf{H}_{ICI,k}\bar{\mathbf{s}}$$

$$\mathbf{R}_k = \mathbf{G}_k\mathbf{C}_s\mathbf{G}_k^H + \mathbf{C}_{ICI,k} + \sigma_{v_f}^2\mathbf{I}_{2M+1}$$

$$\mathbf{w}_k = \mathbf{R}_k^{-1}\mathbf{g}_k c_s(k)$$

$$\hat{s}(k) = \bar{s}(k) + \mathbf{w}_k^H(\mathbf{r}_{target,k} - \mathbf{G}_k\bar{\mathbf{s}})$$

where $\mathbf{G}_k$ contains from the a_k^{th} to the b_k^{th} row of $\mathbf{H}_{target}$, $\mathbf{r}_{target,k} = [r_{target}(a_k), \ldots, r_{target}(b_k)]^T$, $\mathbf{r}_k = [r(a_k), \ldots, r(b_k)]^T$ and $\mathbf{g}_k$ is the k^{th} (modulo N) column of $\mathbf{G}_k$. $\mathbf{w}_k$ is the MMSE equalizer coefficients to find the estimate of $s(k)$.

In (6), $\mathbf{H}_{ICI}(\mathbf{s}-\bar{\mathbf{s}})$ is the interference to the target signals considered as another independent noise term. Therefore, the MMSE solution of $\mathbf{w}_k$ must include the covariance matrix of the additional noise term $\mathbf{C}_{ICI,k}$ in the inverse matrix.

Step 3) We consider BPSK signals. The *a priori* and *a posteriori* log-likelihood-ratios (LLRs) are computed as [2]

$$\begin{aligned}
L[s(k)] &= \ln\frac{Pr\{s(k)=1\}}{Pr\{s(k)=-1\}} \\
L[s(k) \mid \hat{s}(k)] &= \ln\frac{Pr\{s(k)=1 \mid \hat{s}(k)\}}{Pr\{s(k)=-1 \mid \hat{s}(k)\}} \\
&= L[s(k)] + \triangle L[s(k)] \\
&= 4Re\{\hat{s}(k)\}/(1-\mathbf{g}_k^H\mathbf{w}_k) \\
\bar{s}(k)_{new} &= \tanh(L[s(k) \mid \hat{s}(k)]/2) \\
c_s(k)_{new} &= 1-\bar{s}(k)_{new}^2
\end{aligned}$$

Step 4) Go to step 2) until reaching the preset number of iterations or the difference between the detected symbols of two successive iterations falls below the preset threshold.

4. COMPLEXITY ANALYSIS

The complexity of our approach is mainly determined by the computations of the MMSE equalizer in the proposed algorithm.

1) Generally, the inversion of an $(2M+1)\times(2M+1)$ matrix $\mathbf{R}_k$ requires $O((2M+1)^3)$ operations, but we can reduce it to $O((2M+1)^2)$ by exploiting the common submatrices within $\mathbf{R}_k$ and $\mathbf{R}_{k+1}$ as [6].

2) We consider the computation of $\mathbf{C}_{ICI,k}$. We adopt two ways to reduce the complexity. First, we simplify $\mathbf{C}_s$ to an identity matrix multiplying the average variance of the symbols. Consequently, we only need to compute $\mathbf{H}_{ICI,k}\mathbf{H}_{ICI,k}^H$ for each symbol before the iterative equalization. Moreover, we find that the elements of $\mathbf{H}_{ICI,k}\mathbf{H}_{ICI,k}^H$ are overlapped for successive k. Actually, the elements of $\mathbf{H}_{ICI,k}\mathbf{H}_{ICI,k}^H$ for all the symbols are distributed within the band of $\mathbf{H}_{ICI}\mathbf{H}_{ICI}^H$ over $4M+1$ diagonals. Because $\mathbf{H}_{ICI}\mathbf{H}_{ICI}^H$ is a Hermitian matrix, only the elements on $2M+1$ diagonals are required to compute.

$\mathbf{H}_{ICI}$ can be rewritten as

$$\mathbf{H}_{ICI} = \bar{\mathbf{Z}} \odot \mathbf{H}_{f,odd}$$

where $\bar{\mathbf{Z}} = \mathbf{U} - \sum_{m=-Q/2}^{Q/2}\mathbf{Z}_m$ and $\mathbf{U}$ is an $N\times N$ unit matrix. By using OBEM, we have

$$\mathbf{H}_{ICI}\mathbf{H}_{ICI}^H = \sum_{q1,odd}\sum_{l1}\sum_{q2,odd}\sum_{l2} h_{q1,l1}h_{q2,l2}^*\mathbf{T}_{q1,q2,l1,l2}$$

where

$$\mathbf{T}_{q1,q2,l1,l2} = [\bar{\mathbf{Z}} \odot (\mathbf{F}\mathbf{D}_{q1}\mathbf{Z}_{l1}\mathbf{F}^H)][\bar{\mathbf{Z}}^H \odot (\mathbf{F}\mathbf{Z}_{l2}^H\mathbf{D}_{q2}^H\mathbf{F}^H)]$$

$\mathbf{T}_{q1,q2,l1,l2}$ can be precomputed offline and scaled by different OBEM coefficients. Therefore, the computation of $2M+1$ diagonals of $\mathbf{H}_{ICI}\mathbf{H}_{ICI}^H$ requires $O([(Q/2\pm1)^2L^2(2M+1)N/2])$ CM operations.

In conclusion, if the number of iterations is *P*, $O([(Q/2\pm1)^2L^2(2M+1)N/2]/P + (2M+1)^2N)$ operations are required for each iteration.

5. SIMULATION RESULTS AND DISCUSSION

In this section, we compare the performance of the proposed equalizer with the LMMSE equalizer in terms of BER performance versus SNR. We consider the OFDM system with $N = 128$ subcarriers and BPSK signals. OBEM is used to simulate the doubly selective channels with $L = 4$ and $N/4$. The coefficients are approximated by least square fit of Zheng and Xiao's model in [7]. We show the simulation results for different normalized Doppler shift $f_{nom} = 0.005, 0.01, 0.02$ in Fig.3 to Fig.5. The corresponding numbers of OBEM bases are $Q = 4, 6, 12$. Furthermore, we show the effect of the band size M on BER performance of the proposed algorithm. The simulation results show that the performance and complexity of the proposed approach are dependent on f_{nom} and the selection of *M*. This new iterative equalizer can outperform the LMMSE with a relatively small *M*. The performance can be further improved by increasing M for high mobility. This shows that our iterative method can make use of the ICI of OBEM to improve the performance.

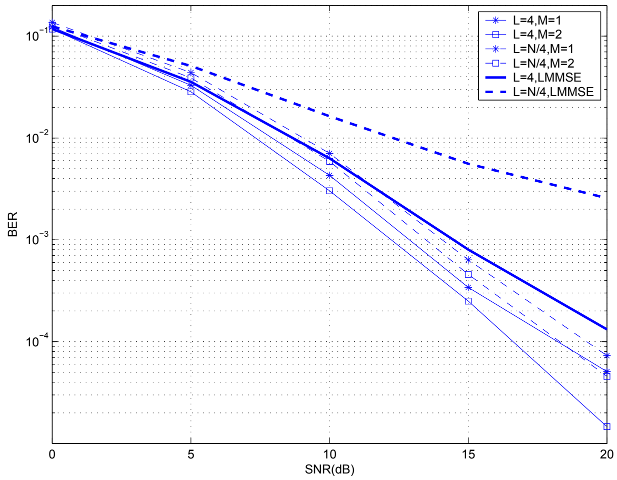

Fig. 3. BER versus SNR for $f_{nom} = 0.005$ after 5 iterations

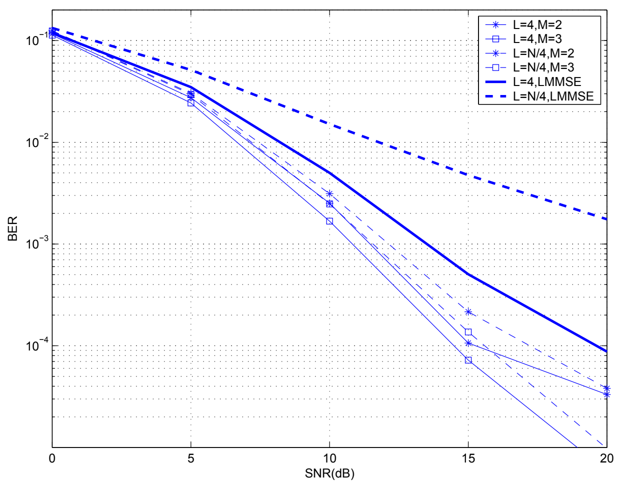

Fig. 4. BER versus SNR for $f_{nom} = 0.01$ after 5 iterations

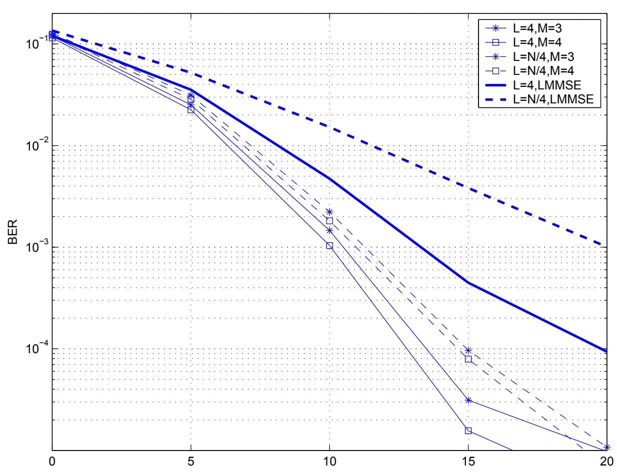

Fig. 5. BER versus SNR for $f_{nom} = 0.02$ after 5 iterations

6. CONCLUSIONS

In this paper, we propose an iterative MMSE equalization in the frequency domain for OFDM systems over doubly selective channels modeled by OBEM. Though the OBEM increases the ICI, the new method can enhance the performance by increasing the band size of the target matrix and suppressing the interference. This property makes OBEM not only provide better accuracy in modeling the channel and also give good performance in detection. The simulation results demonstrate that our proposed algorithm outperforms the LMMSE with lower complexity.

7. REFERENCES

[1] I. Barhumi, G. Leus and M. Moonen, "Equalization for OFDM over doubly selective channels," *IEEE Trans. Signal Processing*, vol. 54, pp. 1445-1458, April 2006.

[2] P. Schniter, "Low-complexity equalization of OFDM in doubly selective channels," *IEEE Trans. Signal Processing*, vol. 52, pp. 1002-1011, April 2004.

[3] B. K. Ng and D. Falconer, "A novel frequency domain equalization method for single-carrier wireless transmissions over doubly-selective fading channels," *IEEE GLOBECOM*, vol. 1, pp. 237-241, Dec. 2004.

[4] G. Leus, "On the estimation of rapidly time-varying channels," *EUSIPCO 2004*, pp. 2227-2230, Sep. 2004.

[5] T. Cui, C. Tellambura and Y. Wu, "Low-complexity pilot-aided channel estimation for OFDM systems over doubly-selective channels," *IEEE ICC*, vol. 3, pp. 1980-1984, May 2005.

[6] M. Tuchler, A. Singer, and R. Koetter, "Minimum mean square error equalization using a priori information," *IEEE Trans. Signal Processing*, vol. 50, pp. 673C683, Mar. 2002.

[7] Y. R. Zheng and C. Xiao, "Improved models for the generation of multiple uncorrelated Rayleigh fading waveforms ," *IEEE Communications Letters*, vol. 6, pp. 256-258, June 2002.