SUBSPACE IDENTIFICATION METHOD FOR RAYLEIGH CHANNEL ESTIMATION

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ABSTRACT

In this paper, we propose a new pilot-aided channel estimator. Among the existing approaches, some are based on adaptive algorithms, but they are outperformed by methods where the channel is modeled by an AR or an ARMA process. In that case, estimating the model parameters from noisy observations and selecting the model orders are challenging problems. To avoid them, we propose to view the channel estimation as a realization issue. By taking advantage of the subspace methods for identification, the proposed estimator provides the system matrices in the state-space representation of the channel directly from the output observations. At that stage, the channel process can be estimated using a Kalman filter. This method has the advantage of being non-iterative and avoiding an *a priori* model for the channel.

Index Terms— Identification, Rayleigh channels, Kalman filtering.

1. INTRODUCTION

In mobile communications, high speeds of terminals and scatterers cause Doppler effects that can seriously affect the reception performance. Thus, channel estimation is a major challenge for reliable wireless transmissions.

The channel is usually charaterized by its physical propagation parameters such as the path delays, path phases, path frequencies, path angles of arrival, etc. Therefore, in an environnment with no direct line-of-sight, the statistical model presented by Jakes [2] is widely accepted as a realistic channel model. Thus, the real and imaginary parts of the channel are decorrelated and have the same power spectral density (PSD) which is assumed to be bandlimited, U-shaped, exhibiting twin peaks at $\pm f_d$ where f_d is the maximum Doppler frequency. The corresponding discrete-time normalized autocorrelation function is then given by:

$$R_h^{theo}[k] = J_0\left(2\pi f_{d_n}\,|k|\right)\ \forall\ k\ \in Z \qquad (1)$$

where $J_0(.)$ denotes the zero-order Bessel function of the first kind and f_{d_n} is the normalised maximum Doppler frequency.

The most conventional strategy to estimate the channel is to transmit pilot bits. In that case, two families of methods have been proposed.

One approach consists in estimating the channel from the noisy observations of the channel, but does not exploit any statistical properties of the Rayleigh fading channel such as (1). Thus, Kalofonos et al. [1] propose to estimate the channel by means of the Least Mean Square (LMS) or Recursive Least Square (RLS) algorithms. However, both adaptive estimators are outperformed by model based methods, which constitutes the second family of pilot-aided estimators.

The purpose of this second approach is to choose an *a priori* model for the channel process and to use Kalman algorithm. AutoRegressive (AR) or AR Moving Average (ARMA) models are usually considered for the sake of simplicity [3]-[6]. Nevertheless, two issues must be investigated: the selection of the order and the parameter estimation. In the following, let us focus our attention on the AR model.

In [3], an off-line AR parameter estimation is based on the Yule-Walker equations, using the theoritical autocorrelation function (1) of the Rayleigh fading channel. However, this approach requires the preliminary estimation of the maximum Doppler frequency. In [3], Wu et al. suggest using a second-order AR model. Nevertheless, their solution is debatable because the resulting spectrum of a low-order AR process exhibits twin peaks at $\pm\frac{f_d}{\sqrt{2}}$ and results in a poor match to the desired bandlimited channel process. A better approximation of the spectral characteristics of a Rayleigh fading channel can be obtained by considering a much higher order AR model. However, the higher the order, the higher the computational cost. In addition, according to Baddour et al. [7], the channel autocorrelation matrix used in Yule-Walker equations may become ill-conditioned. To overcome this difficulty, the authors propose to add a small value ϵ (for instance $\epsilon = 10^{-8}$ when $f_{d_n} = 0.05$) to the diagonal of the channel autocorrelation matrix. In that case, the resulting channel corresponds to the Jakes process disturbed by a white Gaussian noise with variance ϵ.

Another method consists in estimating the AR parameters from the noisy observations with no information on f_d. It can be carried out by using an Error-in-variables (EIV) based method, which can be seen as the Noise Compensated Yule-Walker equations [4]. As an alternative, the AR parameters and the channel process can be jointly estimated by using mutually interactive filters. Thus, Davis et al. [6] propose to couple a RLS estimator which provides the AR parameters and a Kalman filter which provides the channel sample estimates.

However, as only an infinite-order AR process can lead to the bandlimited DSP [7], one has to select an order as high as possible.

In this paper, instead of selecting an *a priori* model and explicitly estimating the corresponding parameters, we propose to view the channel estimation as a realization issue. For this purpose, we use subspace methods for system identification initially developed in the field of control [8]. They provide the matrices in the state-space representation by using an estimation of the correlation function of the noisy observations. Once this so-called "realisation issue" is solved, a Kalman filter can be used to estimate the channel samples during the training step. Thus, the proposed approach does not require neither the preliminary Doppler frequency estimation nor an *a priori* explicit model for channel. To avoid any confusion, it should be noted that this method is not related to the subspace based approach for identification of multichannel FIR filter developped by Moulines et al. in [10].

The remainder of the paper is organized as follows: In section 2, the state space representation of the channel process is recalled. In section 3, the subspace identification methods is presented. In section 4 , the relevance of the proposed estimator is illustrated by numerical results.

2. STATE SPACE REPRESENTATION

Let us consider a radio communication system using pilot bits for channel estimation. After a preprocessing, one can obtain the following noisy observation $z(k)$ of the channel sample $h(k)$ at time k:

$$z(k) = h(k) + v(k) \tag{2}$$

where $v(k)$ is a complex white Gaussian noise of variance σ_v^2.

Since subspace methods for identification can be used providing the data are real, we propose to deal with the real part $z_R(k)$ of the noisy observations $z(k)$. Indeed, given its statistical properties, the complex fading process, its real part h_R and its imaginary part can be described by the same system matrices in the state-space representation.

For this purpose, let us introduce a state vector $\underline{x}(k) \in \mathbf{R^p}$ at time k, where p denotes the model order[1]. The state vector $\underline{x}(k)$ and $z_R(k)$ satisfy the following observation equation:

$$z_R(k) = \underbrace{H\underline{x}(k)}_{h_R(k)} + v_R(k) \tag{3}$$

where $v_R(k)$ is the real part of the complex noise $v(k)$.

It should be noted that the explicit relation between the state $\underline{x}(k)$ and $h_R(k)$ is not known since no *a priori* model of channel is considered.

Moreover, the state vector is assumed to be updated as follows:

$$\underline{x}(k+1) = \Phi\underline{x}(k) + \underline{w}(k) \tag{4}$$

where Φ and H are real matrices of appropriate dimensions and $\underline{w}(k) \in \mathbf{R^p}$ is a noise vector, assumed to be white, zero-mean and stationary with covariance matrix Q. We also assume that the state $\underline{x}(k)$ is decorrelated with $\underline{w}(k)$ and $v_R(k)$.

In the following, we take advantage of the subspace methods for identification [8] to estimate the quadruple $[H, \Phi, Q, \sigma_{v_R}^2]$. Then, a Kalman filter is used to retrieve the complex fading channel samples.

[1]The choice of the model order will be explained in section 3.

3. STOCHASTIC SUBSPACE METHODS FOR IDENTIFICATION

This section reviews subspace methods [8] that make it possible to identify the state-space representation of a stochastic process directly from the noisy observations.

3.1. Main principles

The core of the subspace methods for identification is to estimate, from noisy observations, the extended ($s > p$) observability matrix of a state-space representation (3)-(4), defined as follows:

$$\Gamma_s = \begin{bmatrix} H \\ H\Phi \\ \vdots \\ H\Phi^{s-1} \end{bmatrix} \tag{5}$$

Indeed, H and Φ can be retrieved from Γ_s, since H corresponds to the first row of Γ_s and Φ is given by:

$$\Phi = \Gamma_{s-1}^{\dagger}\overline{\Gamma}_s \tag{6}$$

where $\dagger$ denotes the pseudo-inverse operator and $\overline{\Gamma}_s$ corresponds to Γ_s without its first row.

For this purpose, the $2s \times (N-2s+1)$ Hankel matrix $Z_{0/2s-1}$ constructed from the N observations $z_R(k)$ is introduced:

$$Z_{0/2s-1} = \begin{bmatrix} Z_{0/s-1} \\ Z_{s/s} \\ Z_{s+1/2s-1} \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} z_R(0) & z_R(1) & \cdots & z_R(N-2s) \\ z_R(1) & z_R(2) & \cdots & z_R(N-2s+1) \\ \vdots & \vdots & \ddots & \vdots \\ z_R(s) & z_R(s+1) & \cdots & z_R(N-s) \\ z_R(s+1) & z_R(s+2) & \cdots & z_R(N-s+1) \\ \vdots & \vdots & \ddots & \vdots \\ z_R(2s-1) & z_R(2s) & \cdots & z_R(N-1) \end{bmatrix}$$

In the following, $N' = N - 2s + 1$.

Van Overschee et al. proved that the extended observability matrix Γ_s can be estimated from the two following orthogonal projections:

$$O_s = Z_{s/2s-1}/Z_{0/s-1} \tag{8}$$

$$O_{s-1} = Z_{s+1/2s-1}/Z_{0/s} \tag{9}$$

Indeed, when $N' \to \infty$, these orthogonal projections can be expressed as the following products:

$$O_s = \Gamma_s X_s \tag{10}$$

$$O_{s-1} = \Gamma_{s-1} X_{s+1} \tag{11}$$

where X_s and X_{s+1} respectively denote the state sequence generated by a bank of non-steady Kalman filters working in parallel on each of the columns of the block Hankel matrix $Z_{0/s-1}$ and $Z_{0/s}$. Moreover, one has:

$$\text{Span}(X_s^T) = \text{Span}(O_s^T) \tag{12}$$

$$\text{Span}(X_{s+1}^T) = \text{Span}(O_{s-1}^T) \tag{13}$$

$$\text{Span}(\Gamma_s) = \text{Span}(O_s) \tag{14}$$

$$\text{Span}(X_{s+1}) = \text{Span}(O_{s-1}) \tag{15}$$

To obtain Γ_s and X_s, the authors in [8] propose to use the Singular Value Decomposition (SVD) of the weighted orthogonal projection $W_1 O_s W_2$:

$$W_1 O_s W_2 = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} \tag{16}$$

where S_1 is a diagonal matrix that contains the p non-zero singular values of $W_1 O_s W_2$. $W_1 \in \mathbf{R}^{\mathbf{s}\times\mathbf{s}}$ and $W_2 \in \mathbf{R}^{\mathbf{N}'\times\mathbf{N}'}$ are two weighting matrices such that W_1 is full rank and $\text{rank}(Z_{0/s-1}) = \text{rank}(Z_{0/s-1} W_2)$.

The choice of W_1 and W_2 determine the state-space basis in which the representation will be identified. Different choices have been investigated, leading to various published methods. Then, Γ_s can be obtained as follows:

$$\Gamma_s = W_1^{-1} U_1 S_1^{1/2} \tag{17}$$

At that stage, H and Φ can be easily retrieved. In addition, Van Overschee et al. proposed a way to estimate the covariance matrix Q and the variance σ_v^2 from the least square residuals, $\rho_{w(k)}$ and $\rho_{v(k)}$, defined as follows:

$$\begin{pmatrix} \rho_{w(k)} \\ \rho_{v(k)} \end{pmatrix} = \begin{bmatrix} X_{s+1} \\ Z_{s/s} \end{bmatrix} - \begin{bmatrix} \Phi \\ H \end{bmatrix} \hat{X}_s. \tag{18}$$

and

$$\frac{1}{N'} \begin{pmatrix} \rho_{w(k)} \\ \rho_{v(k)} \end{pmatrix} \begin{pmatrix} \rho_{w(k)} \\ \rho_{v(k)} \end{pmatrix}^T = \begin{bmatrix} Q & S \\ S & \sigma_v^2 \end{bmatrix}. \tag{19}$$

In the next section, we will focus on the N4SID method [8] in which $W_1 = I_{s\times s}$ and $W_2 = I_{N'\times N'}$.

3.2. Application to channel estimation

We propose to use the N4SID method for channel estimation. Indeed, in the framework of channel estimation, the number of available observations corresponds to the number of pilot bits. Therefore, N is finite and has to be as low as possible. Van Overschee et al. studied the behavior of the subspace identification methods when N is finite. They suggest using the RQ decomposition $\frac{1}{\sqrt{N'}} Z_{0/2s-1}$ to compute the orthogonal projections O_s and O_{s-1}.

$$\frac{1}{\sqrt{N'}} \begin{bmatrix} Z_{0/s-1} \\ Z_{s/s} \\ Z_{s+1/2s-1} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{bmatrix} \tag{20}$$

Thus, O_s and O_{s-1} can be written as follows:

$$\hat{O}_s = \begin{bmatrix} R_{21} \\ R_{31} \end{bmatrix} Q_1^T \text{ and } \hat{O}_{s-1} = \begin{bmatrix} R_{31} & R_{32} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \end{bmatrix} \tag{21}$$

In addition, as Q_1 is an orthogonal matrix, the SVD of O_s can be calculated by means of the SVD of $\begin{bmatrix} R_{21} & R_{31} \end{bmatrix}^T$. One has:

$$\begin{bmatrix} R_{21} \\ R_{31} \end{bmatrix} = \begin{bmatrix} \hat{U}_1 & \hat{U}_2 \end{bmatrix} \begin{bmatrix} \hat{S}_1 & 0 \\ 0 & \hat{S}_2 \end{bmatrix} \begin{bmatrix} \hat{V}_1^T \\ \hat{V}_2^T \end{bmatrix}. \tag{22}$$

where $\hat{S}_1$ corresponds to the p dominant singular values. The estimate of Γ_s corresponds to:

$$\hat{\Gamma}_s = \hat{U}_1 \hat{S}_1^{1/2}. \tag{23}$$

As the N4SID method is consistent [9], Γ_{s-1} and X_{s+1} can be estimated from $\hat{\Gamma}_s$. Therefore, using (18) and (19) makes it possible to get the estimation of H, Φ, Q and $\sigma_{v_R}^2$.

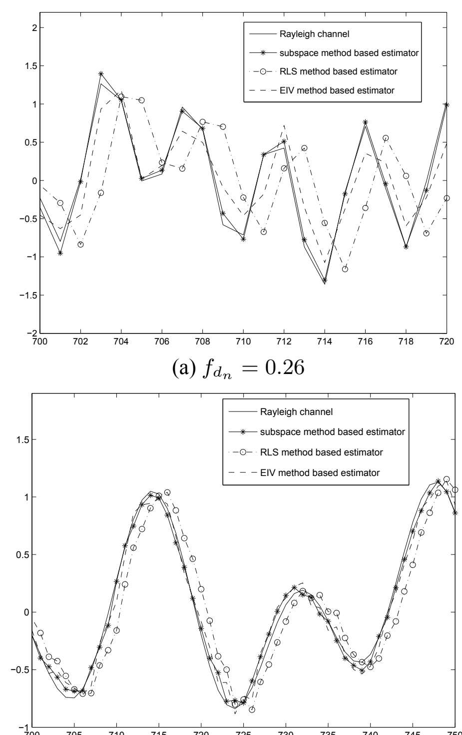
Once the quadruple $[H, \Phi, Q, \sigma_{v_R}^2]$ is estimated, a Kalman filter is used to retrieve the complex fading channel sample.

4. NUMERICAL RESULTS AND CONCLUSION

In this section, we study the relevance of the proposed method for channel estimation. For this purpose, we carry out a comparative study between the method presented here, the EIV estimator [4] and the RLS methods [1].

In our simulations, the fading process to be estimated is simulated using the Jakes' model [2] with 2048 simulators. p is assigned to 5. The normalized maximum Doppler frequency varies from 0.0625 to 0.260. The number of available channel observations N is equal to 80. The simulations illustrated in Fig.1 and Fig.2 shows that the proposed method provides better estimation than the EIV method based estimator and the RLS, whatever the maximum Doppler frequency may be.

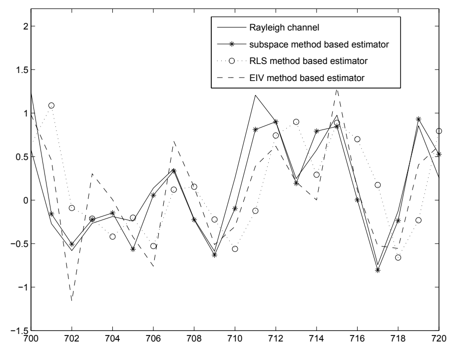
The method we propose is not iterative, which is a great advantage. Like various channel estimators, it requires at least 80 pilots, which is slightly higher than the number proposed in the cdma2000 recommandations [12].



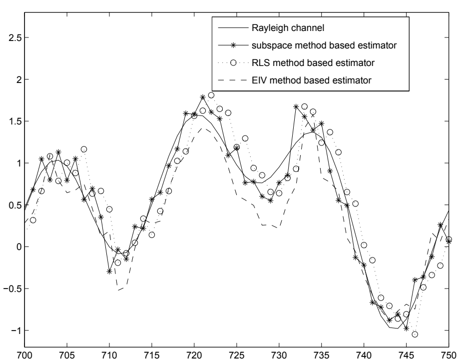
(a) $f_{d_n} = 0.26$

(b) $f_{d_n} = 0.0625$

Fig. 1. Real part of the estimated channel for SNR=20dB



(a) $f_{d_n} = 0.26$



(b) $f_{d_n} = 0.0625$

Fig. 2. Real part of the estimated channel for SNR=10dB

APPENDIX: is it relevant to retrieve a canonical parameterization ?

A linear stochastic process can be represented by an infinite number of linear state-space representations which are equivalent up to a non-singular transformation T. However, the tranfer function is unique.

In some previous studies such as [11], the authors suggest modeling the channel by a p^{th} order AR process. To estimate the AR parameters, they propose to use the subspace methods for identification to obtain the quadruple $[\mathbf{\Phi}, \mathbf{H}, \mathbf{Q}, \mathbf{R}]$ corresponding to the following state-space representation:

$$\begin{cases} \underline{x}(k+1) = \mathbf{\Phi}\underline{x}(k) + \underline{w}(k) \\ \underline{z}_R(k) = \mathbf{H}\underline{x}_T(k) + \underline{v}_R(k) \end{cases} \tag{24}$$

where

$$\underline{z}_R(k) = [z_R(k)\ z_R(k-1)\ \cdot\ z_R(k-p+1)]^T \tag{25}$$

and $\underline{v}_R(k)$ is a $p \times 1$ white noise vector, $\mathbf{H}$ and $\mathbf{\Phi}$ are $p \times p$ matrices.

Then, they propose to explicitly find the transformation $\mathbf{T}$ that makes it possible to obtain the canonical state-space representation of the system, in which the canonical transition matrix is a companion matrix $\mathbf{A}$ defined from the AR parameters $a_{i=1..p}$ and the canonical observation matrix is the identity matrix $I_{p\times p}$. For this purpose, they propose to use the extended observability matrix which should satisfy:

$$\mathbf{\Gamma}_s\mathbf{T} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H\Phi} \\ \mathbf{H\Phi}^2 \\ \vdots \\ \mathbf{H\Phi}^{s-1} \end{bmatrix} \mathbf{T} = \begin{bmatrix} I_{p\times p} \\ \mathbf{A} \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^{s-1} \end{bmatrix} \tag{26}$$

Given (26), $\mathbf{T}$ should correspond to the inverse of $\mathbf{H}$.

However, in real cases, this method is debatable for the following reasons:

- The finite number of pilot data induces an error in the estimation of $\mathbf{\Gamma}_s$. The resulting estimation of the observation matrix $\mathbf{H}$ may be singular.
- When $\hat{\mathbf{H}}$ is non-singular, we have noticed that the resulting estimated AR parameters could correspond to an unstable system. Indeed, $\hat{\mathbf{A}}$ is not exactly a companion matrix since the elements of the subdiagonal for instance are not exactly equal to 1. As the fading channel process is not a finite-order AR process [7], the state-space representation 24 is not equivalent to a state-space representation of an AR process.

Therefore, it is hazardous to search the transformation $\mathbf{T}$.

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