# A LOW COMPLEXITY ITERATIVE RECEIVER FOR CODED MIMO-OFDM SYSTEMS

Wenfeng Lin, Chen He

Dept of Electronic Eng., Shanghai Jiao Tong University, Shanghai, 200240, China {linwenfeng1978, chenhe}@sjtu.edu.cn

# ABSTRACT

A novel low-complexity iterative receiver for coded multiple-output (MIMO) multiple-input orthogonal frequency division multiplexing (OFDM) systems is proposed in this paper. The soft-in soft-out (SISO) detector is simply a parallel interference cancellation (PIC) maximum ratio combining (MRC) operation. Usually, the probability density function (PDF) of PIC-MRC detector output is approximated as Gaussian, whose variance is calculated with soft information fed back from the channel decoder. With this approximation, the log likelihood ratios (LLRs) of transmitted bits are under-estimated. Thus the LLRs are multiplied by a constant factor to achieve a performance gain. The constant factor is optimized according to the extrinsic information transfer (EXIT) chart of the channel decoder. Simulation results show that the proposed iterative receiver can significantly improve the system performance and converge to the matched filter bound (MFB) with low computational complexity at high signal-to-noise ratios (SNRs).

Index Terms-MIMO-OFDM, Iterative, PIC-MRC, EXIT

#### **1. INTRODUCTION**

In recent years, multiple-input multiple-output (MIMO) communication system has received considerable attention because of its potential to improve system performance and capacity [1]. To achieve this advantage, turbo BLAST (Bell Laboratories Lavered Space-Time) techniques are developed for flat fading MIMO systems [2]. A linear minimum mean square error (MMSE) detector is developed in the systems. In [3], turbo equalization is developed to combat frequency selective channels. But it introduces a significant increase in computational complexity for the large number of receive antennas and long delay spreads due to the matrix inversion. Since the orthogonal frequency division multiplexing (OFDM) can transfer frequency selective channels in time domain to flat fading channels in

frequency domain, it would be more attractive for wideband signals. In order to reduce the complexity of the MMSE detector, the soft-in soft-out (SISO) detector is simply a parallel interference cancellation (PIC)-maximum ratio combining (MRC) operation in this paper. Then the probability density function (PDF) of the PIC-MRC detector output is approximated as Gaussian to obtain the log likelihood ratios (LLRs) of the transmitted bits. With this approximation, the LLRs are often under-estimated. Therefore, they are multiplied by a constant factor to improve the system performance. Simulation results show that the proposed scheme significantly improves the system performance and converges to the matched filter bound (MFB) with low computational complexity.

The rest of the paper is organized as follows. In Section 2, we describe MIMO OFDM system model. In Section 3, the receiver structure for iterative detection and decoding is outlined. The proposed low-complexity PIC-MRC based detector is introduced in Section 4. Simulation results and performance analysis are provided in Section 5. Finally, conclusions are drawn in Section 6.

### **2. SYSTEM MODEL**

Consider a MIMO-OFDM system equipped with  $N_t$  transmit antennas and  $N_r$  receive antennas. At the transmitter, a sequence of binary information bits is encoded, bit-interleaved and modulated into symbols. Then the symbols are demultiplexed to  $N_t$  sub-streams. Symbol blocks of size *K* are formed and each block is OFDM modulated through inverse fast Fourier transform (IFFT) at each transmit antenna. Before being transmitted through a multipath fading channel, a cyclic prefix (CP) of a length equal to or larger than that of the channel delay spread is inserted to eliminate inter block interference (IBI). After CP removal and fast Fourier transform (FFT) demodulation, the received vector at the *k* th subcarrier is expressed as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k \tag{1}$$

where  $\mathbf{H}_k$  is an  $N_r \times N_t$  channel matrix whose (i, j) th element,  $h_k(i, j)$ , is the fading coefficient between the *j* th transmit antenna and the *i* th receive antenna.  $\mathbf{x}_k = [x_{k,1}, x_{k,2}, \mathbf{K}, x_{k,N_t}]^T$  is the  $N_t \times 1$  transmitted symbol

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vector;  $\mathbf{n}_k$  is the zero mean complex Gaussian noise with variance  $\sigma^2$  per complex dimension.



Fig.1 MIMO-OFDM Iterative receiver

In this section, we give a brief review on the receiver for iterative detection and decoding based on the MMSE criteria [2]-[4]. The iterative receiver is shown in Fig.1. For the MMSE detector, the constant factor  $\alpha$  is set to one. In the turbo processing, soft information is exchanged between the detector and the decoder to improve the system performance.

At each iteration, the SISO detector eliminates coantenna interference (CAI) among multiple transmit antennas. This operation uses a priori information fed back from the channel decoder to compute the mean value of each transmitted symbol. The mean value  $\overline{x}_{k,n}$  is given in [4], where *n* represents the transmit antenna index.

$$\overline{x}_{k,n} = E[x_{k,n}] = \sum_{\mathbf{d} \in \mathbb{S}} \phi(\mathbf{d}) P(x_{k,n} = \phi(\mathbf{d}))$$
(2)

 $P(x_{k,n} = \phi(\mathbf{d}))$  is function of *a priori* information  $L_a(d_{k,n}(i))$ , i = 1, 2, K,  $M_c(M_c)$  is the number of bits per constellation symbol). **d** is a  $M_c \times 1$  vector of data bits with its entity  $d_{k,n}(i)$ .  $\phi(\mathbf{d})$  is a constellation symbol which is obtained using the mapping function. S is the set of possible values of **d**. And we have

$$P(x_{k,n} = \phi(\mathbf{d})) = \prod_{i=1}^{M_c} \frac{1}{2} \left[ 1 + (2d_{k,n}(i) - 1) \tanh\left(\frac{1}{2}L_a(d_{k,n}(i))\right) \right]$$
(3)

The mean of transmitted symbol vector is denoted as

$$\overline{\mathbf{x}}_{k} = \left[ \overline{x}_{k,1} \ \overline{x}_{k,2} \ \mathbf{L} \ \overline{x}_{k,N_{t}} \right]^{T}$$

Then the output of parallel interference cancellation (PIC) is given by

$$\overline{\mathbf{y}}_{k,n} = \mathbf{y}_k - \mathbf{H}_k \left( \overline{\mathbf{x}}_k - \overline{\mathbf{x}}_{k,n} \mathbf{e}_n \right)$$
(4)

where  $\mathbf{e}_n$  is a  $N_t \times 1$  all zero column vector except its *n* th entry is equal to one. In order to suppress residual CAI, the PIC output vector  $\overline{\mathbf{y}}_{k,n}$  is fed to a MMSE detector

$$J(\mathbf{w}_{k,n}) = E \left| x_{k,n} - \mathbf{w}_{k,n}^{H} \overline{\mathbf{y}}_{k,n} \right|^{2}$$
(5)

with the solution [2][3]

$$\mathbf{w}_{k,n} = \left(\mathbf{H}_{k}\mathbf{V}_{k}\mathbf{H}_{k}^{H} + (1-\nu_{k,n})\mathbf{h}_{k,n}\mathbf{h}_{k,n}^{H} + \sigma^{2}\mathbf{I}_{N_{r}}\right)^{-1}\mathbf{h}_{k,n} (6)$$

where  $\mathbf{h}_{k,n} = \mathbf{H}_k \mathbf{e}_n$ ,  $\mathbf{V}_k = diag[v_{k,1} \mathbf{L} \ v_{k,N_t}]$  is the diagonal covariance matrix of the transmitted symbol vector and  $v_{k,n}$ 

is its (n, n) th element. Thus the output of PIC-MMSE detector can be written as

$$z_{k,n} = \mathbf{w}_{k,n}^{H} \left( \mathbf{y}_{k} - \mathbf{H}_{k} \overline{\mathbf{x}}_{k} + \overline{x}_{k,n} \mathbf{h}_{k,n} \right)$$
(7)

As described in [2]-[5], the output of the MMSE detector can be viewed as the output of an equivalent AWGN channel with input  $x_{k,n}$ 

$$z_{k,n} = \mu_{k,n} x_{k,n} + \eta_{k,n} \tag{8}$$

where  $\mu_{k,n}$  is the equivalent amplitude of the output signal, and  $\eta_{k,n}$ : N(0, $\sigma_{k,n}^2$ ) is the equivalent Gaussian noise. The parameters  $\mu_{k,n}$  and  $\sigma_{k,n}^2$  are calculated as follows

$$\mu_{k,n} = \mathbf{w}_{k,n}^H \mathbf{h}_{k,n} \tag{9}$$

$$\sigma_{k,n}^{2} = \mathbf{w}_{k,n}^{H} \left( \mathbf{H}_{k} \mathbf{V}_{k} \mathbf{H}_{k}^{H} - v_{k,n} \mathbf{h}_{k,n} \mathbf{h}_{k,n}^{H} + \sigma^{2} \mathbf{I}_{N_{r}} \right) \mathbf{w}_{k,n} (10)$$

Using the Bayes' theorem and independence of codeword, the extrinsic LLRs of the transmitted bits can be obtained as in [4].

### 4. PIC-MRC BASED SISO DETECTOR

In this section, the proposed PIC-MRC based detector is outlined. As shown in Section 3, the MMSE detector requires matrix inversion at each iteration, which has  $O((N_r)^3)$  computational complexity. It is still too complex in practice. In order to further reduce the computational complexity, the SISO detector is simply matched filter realizing maximum ration combining (MRC) operation. Thus the weight  $\mathbf{w}_{k,n}$  in (6) is simply to  $\mathbf{h}_{k,n}$ . Then the PIC-MRC output is expressed as

$$z_{k,n} = \mathbf{h}_{k,n}^{H} \left( \mathbf{y}_{k} - \mathbf{H}_{k} \overline{\mathbf{x}}_{k} + \overline{\mathbf{x}}_{k,n} \mathbf{h}_{k,n} \right)$$
(11)

The PDF of PIC-MRC detector output is still approximated as Gaussian. The parameters  $\mu_{k,n}$  and  $\sigma_{k,n}^2$  are computed as follows.

$$\boldsymbol{\mu}_{k,n} = \mathbf{h}_{k,n}^{H} \mathbf{h}_{k,n} \tag{12}$$

$$\sigma_{k,n}^{2} = \mathbf{h}_{k,n}^{H} \left( \mathbf{H}_{k} \mathbf{V}_{k} \mathbf{H}_{k}^{H} - v_{k,n} \mathbf{h}_{k,n} \mathbf{h}_{k,n}^{H} + \sigma^{2} \mathbf{I}_{N_{r}} \right) \mathbf{h}_{k,n} (13)$$

Therefore, the computational complexity is significantly reduced without requiring matrix inversion as in [2]. However, the MRC operation would result in large performance loss because of residual CAI, especially at the first iteration when no *a priori* information is available. In the above derivation, the PDF of PIC-MRC detector output is roughly approximated as Gaussian. No theoretical analysis has been done for this approximation to the best of our knowledge, though the theoretical analysis of approximating PDF of the MMSE detector output as Gaussian is provided in [5]. The LLRs based on this approximation may be under-estimated compared with its actual PDF. Thus the LLRs of PIC-MRC detector output are multiplied by a constant factor  $\alpha$  to improve the system performance in our scheme. The constant factor  $\alpha$  is optimized according to the extrinsic information transfer

(EXIT) chart of the channel decoder. The method can simplify the EXIT chart analysis since the channel types and system parameters are not indispensable in this case.

The EXIT chart is an effective technique to examine the convergence property of iterative detection and decoding [6]. It models receiver components as devices mapping a sequence of observations and the *a priori* information  $L_a$  to a new sequence of extrinsic information  $L_e$ . In our scheme, the channel decoder with the constant factor  $\alpha$  is treated as an equivalent SISO decoder. To obtain the transfer chart, the information bit stream is randomly generated and encoded with the channel encoder. The *a priori* information of encoded bits is generated according to  $L_a : N(\sigma_a^2/2, \sigma_a^2) . N(\sigma_a^2/2, \sigma_a^2)$  is a Gaussian distribution and its variance is twice its mean. The mutual information (MI) between the encoded bits and the *a priori* information is computed as

$$I_{a} = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{a}^{2}}} \exp\left(-\frac{l - \sigma_{a}^{2}/2}{2\sigma_{a}^{2}}\right) \log_{2}\left(1 + e^{-l}\right) dl \quad (14)$$

Using the *a priori* information multiplied by a constant factor  $\alpha$ , the channel decoder estimates LLRs of the encoded bits. Then MI between the extrinsic information  $L_e$  and the encoded bits *C* is approximately computed as [7]

$$I_{e}(L_{e};C) = 1 - \frac{1}{N} \sum_{n=1}^{N} \log_{2} \left[ 1 + \exp(-c(n)L_{e}(n)) \right]$$
(15)

where N is the number of encoded bits. The constant factor  $\alpha$  is set to the minimum value that can almost result in the maximum MI  $I_e(L_e;C)$ , which is shown in the next section. For the MMSE detector, the  $\alpha$  is set to one because the PDF of its output is well approximated as Gaussian [5]. No optimization of  $\alpha$  is needed for it.

# **5. SIMULATION RESULTS**

In this section, we provide simulation results that show the bit error rate (BER) performance of the proposed iterative receiver. Eight transmit antennas and eight receive antennas are employed. The carrier frequency is 2.0GHz and the bandwidth is 5MHz. The system has 512 subcarriers with 64 cyclic prefix. Information bits are encoded using a rate-1/2 recursive systematic convolution (RSC) code with constraint length 3 and generation function (7, 5). Encoded bits are randomly interleaved and QPSK modulated. The channel model is the international telecommunicational union (ITU) "Vehicular A" (VA) model with mobile velocity of 30km/h. Perfect channel state information is available at the receiver. We first show the EXIT charts of the SISO decoder under different  $\alpha$  in Fig.2, where  $\alpha = 1.0$  corresponds to the conventional SISO decoder. If  $\alpha$  is set too small, more iterations would be needed for which iterative receiver to converge, will be computationally inefficient. If  $\alpha \rightarrow \infty$ , it would reduce the

SISO detector to a hard-decision detector, resulting in performance loss. It is observed that  $\alpha = 2.0$  is an approximately optimal solution.



Fig.2 EXIT charts of the receiver under different  $\alpha$ , SNR=5dB At the low input MI, it maximizes the output MI  $I_e(L_e;C)$ , which can reduce the SNR threshold requiring for the turbo cliff region. Further increase of  $\alpha$  would not improve the output MI at low input MI region. Under different  $\alpha$ , the output MI  $I_e(L_e;C)$  is nearly the same at the high input MI region. The EXIT charts of the PIC-MRC detector and the MMSE detector at SNR=5dB are also shown in Fig.2. For the MMSE detector, the MI fed to the channel decoder is already high enough at the first iteration. At that region, the LLRs multiplication by a constant factor  $\alpha > 1.0$  would not improve the MI of channel decoder output. On the other hand, if LLRs of MMSE detector output is multiplied by a constant, the MI between the transmitted bits and the LLRs degrades since the MMSE detector output is well approximated as Gaussian [5]. Thus no optimization of  $\alpha$ is needed for the MMSE detector. However, for the PIC-MRC detector, the MI fed to the channel decoder is still low at the first iteration. Thus LLRs output of the PIC-MRC detector multiplied by a constant factor  $\alpha$  can improve the system performance.

The BER performance of the conventional PIC-MRC detector ( $\alpha = 1.0$ , dashed line) is compared with that of the proposed one ( $\alpha = 2.0$ , solid line) in Fig.3. Five iterations are implemented in both schemes. The matched filter bound (MFB) is also presented as a lower bound of BER of the iterative receiver. This is obtained when the interference from other transmit antennas is completely removed. For the conventional PIC-MRC detector, an error floor is observed. BER performance of the proposed scheme is much better than that of the conventional one after two iterations. It converges to the MFB at high SNRs. The BER performance of the proposed PIC-MRC detector ( $\alpha = 2.0$ , solid line) in Fig.4. During the first three iterations, the performance loss of our scheme is larger than the MMSE detector. This is

because the MRC operation ignores the existence of residual CAI. But the performance loss decreases with more iterations.



Fig.3 BER performance comparison between the conventional PIC-MRC detector ( $\alpha = 1.0$ , dashed line) and the proposed one ( $\alpha = 2.0$ , solid line); VA



Fig.4 BER performance comparison between the MMSE detector (dashed line) and the proposed PIC-MRC detector ( $\alpha = 2.0$ , solid line); VA

However, our scheme is much simpler than the MMSE detector [2] since matrix inversion is avoided at each iteration. In Fig.5, we consider the MIMO-OFDM systems with different number of transmit antennas, from two transmit antennas to eight transmit antennas. The number of receive antennas is the same with the number of transmit antennas. BER performance is shown at SNR=7dB with the same simulation parameters as Fig.3. The constant factor  $\alpha = 2.0$  is also used in this case. The BER performance of conventional PIC-MRC detector degrades with the increase of transmit antennas. This is due to the increase in CAI as the number of transmit antennas increases. However, the BER performance of our scheme still converges to the MFB after five iterations. The simulation result of the MSME detector after five iterations is also given in Fig.5, which approaches the MFB. This scheme is also effective for other channel types such as "Pedestrian A" (PA),

"Pedestrian B (PB)" and "Vehicular B" (VB). We don't give the simulation results due to page limit.



Fig.5 BER performance comparison between the conventional PIC-MRC detector ( $\alpha = 1.0$ , dashed line), the proposed one ( $\alpha = 2.0$ , solid line) and the MMSE detector for different antennas configurations, SNR=7dB; VA

#### 6. CONCLUSIONS

In this paper, we have proposed a novel low-complexity iterative receiver for coded MIMO-OFDM system. The SISO detector is simply PIC-MRC operation. Then the LLRs of PIC-MRC detector output are multiplied by a constant factor  $\alpha$  to achieve a performance gain. A little additional complexity at SISO detector is incurred in the proposed scheme. However it obtains significant performance gain and converges to the MFB with low computational complexity at high SNRs.

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