BLIND IDENTIFICATION OF MIMO-OFDM CHANNELS DRIVEN BY COLORED SOURCE WITH UNKNOWN STATISTICS

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ABSTRACT

We present a closed-form solution for blind multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) channel estimation driven by colored sources with unknown statistics. The uniqueness of the solution is proved by exploiting the inherent structural relationship of the source and received signal autocorrelation matrices. Computer experiments results are presented to illustrate the performance of the proposed algorithm.

***Index Terms*—** Blind channel estimation, MIMO-OFDM, colored source

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most promising physical layer technologies for high data rate wireless communications due to its robustness to frequency selective fading, high spectral effciency, and low computational complexity. OFDM can be used in conjunction with a Multiple-Input Multiple-Output (MIMO) transceiver to increase the diversity gain and/or the system capacity by exploiting spatial domain. [1, 2]

Blind channel estimation for MIMO-OFDM system has been a very active area of research in recent years. A subspace channel estimation algorithm by exploiting the cyclostationarity of MIMO-OFDM channel outputs is presented in [1]. A blind subspace algorithm with the assistance of redundant linear precoding is proposed in [3]. In [4, 5], the virtual subcarriers are exploited to estimate the MIMO-OFDM channel. However, both the precoding redundancy and the virtual subcarriers impair the bandwidth efficiency.

In this paper, we proposed a novel closed-form solution for blind estimation of MIMO-OFDM channels driven by colored source with unknown sources. We further exploited the inherent properties of the autocorrelation matrices of the received data blocks. A few lemmas are introduced to prove the uniqueness of the solution to the estimation criterion function.

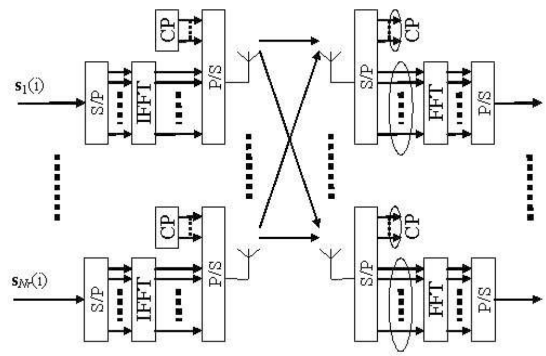

Fig. 1. MIMO-OFDM System Block Diagram

2. SYSTEM MODEL

Consider the MIMO-OFDM system shown in Fig. 1, which is equipped with N_T and N_R transmit and receive antennas respectively. Define the i^{th} block of symbols to be transmitted before OFDM modulation as

$$\mathbf{s}(i) \triangleq \left[\mathbf{s}^T(0,i), \cdots, \mathbf{s}^T(M-1,i)\right]^T \in \mathcal{C}^{MN_T\times 1} \tag{1}$$

where $\mathbf{s}(m,i) \in \mathcal{C}^{N_T\times 1}$ denote the symbols modulating the m^{th} subcarrier, and M is the number of subcarriers. In fact, the source symbols may be colored with known or unknown statistics. For example, colored sources arise in the preceded OFDM [6], where the precoding matrices are known a prior or not. In particular, we consider the latter case. In OFDM, each source symbol block is sent to an IFFT unit, and appended with the cyclic prefix of length M_g . The corresponding output can be expressed as

$$\mathbf{u}(i) = \mathbf{F}_{cp}\mathbf{s}(i) \in \mathcal{C}^{PN_T\times 1} \tag{2}$$

where $P = M + M_g$, and

$$\mathbf{F}_{cp} \triangleq \left(\begin{bmatrix} \mathbf{0}_{M_g\times(M-M_g)} \quad \mathbf{I}_{M_g} \\ \mathbf{I}_M \end{bmatrix} \otimes \mathbf{I}_{N_T}\right)\left(\mathbf{F}_M^H \otimes \mathbf{I}_{N_T}\right) \tag{3}$$

where $\mathbf{F}_M$ is the $M \times M$ FFT matrix, $\mathbf{I}_D$ denotes the $D \times D$ identity matrix, $\mathbf{0}_D$ denotes the $D \times D$ zero matrix, and $\otimes$

denotes the Kronecker product [9]. Consequently, the modulated symbols are transmitted through FIR channels. Let $h^{(r,t)}[l]$, $(t = 1, \cdots, N_T,\ r = 1, \cdots, N_R,\ l = 1, \cdots, L)$ denote the time domain channel impulse response between the $(t,r)^{th}$ transiver pair, where L is the maximum channel order. Then the i^{th} block of received signal can be expressed as

$$\mathbf{x}(i) = \mathbf{H}_0\mathbf{u}(i) + \mathbf{H}_1\mathbf{u}(i-1) + \mathbf{v}(i) \tag{4}$$

where $\mathbf{v}(i)$ is the i^{th} block of AWGN noise, $\mathbf{H}_0$ and $\mathbf{H}_1$ are $(M+M_g)N_R \times (M+M_g)N_T$ block triangular Toeplitz matrices constructed by $[\mathbf{h}^T[0], \cdots, \mathbf{h}^T[L]]^T$ and $[\mathbf{h}^T[0], \cdots, \mathbf{h}^T[L]]^T$ respectively [2], while $\mathbf{h}[l]$ $(l = 0, \cdots, L)$ is defined by

$$\mathbf{h}[l] \triangleq \begin{bmatrix} h^{(1,1)}[l] & \cdots & h^{(1,N_T)}[l] \\ \vdots & \ddots & \vdots \\ h^{(N_R,1)}[l] & \cdots & h^{(N_R,N_T)}[l] \end{bmatrix} \in \mathcal{C}^{N_R \times N_T} \tag{5}$$

We adopt the following basic assumptions: 1) the channel $\mathbf{h}(z)$ is irreducible. 2) The maximum channel order is known a priori. 3) Source signal is zero mean, wide sense stationary colored with unknown statistics. 4) Additive noise are spatially and temporally white, and they are statistically independent of the source. Our objective is to estimate the channel impulse response by utilizing the second-order statistics of the observed output data.

3. PROPOSED CHANNEL ESTIMATION METHOD

Define the source autocorrelation matrix with block delay lag k as

$$\mathbf{R_s}[k] \triangleq E\{\mathbf{s}(i)\mathbf{s}^H(i-k)\} \tag{6}$$

where we assume that $\mathbf{R_X}[0]$ is of full rank. Thus, from (2), (4), and the assumption that the noise is independent from the the source signal, then the autocorrelation matrix of the received signal blocks with block delay lag k can be expressed as

$$\begin{aligned} \mathbf{R_x}[k] &\triangleq E\{\mathbf{x}(i)\mathbf{x}^H(i-k)\} \\ &= \mathbf{H}_0\mathbf{F}_{cp}\mathbf{R_s}[k]\mathbf{F}_{cp}^H\mathbf{H}_0^H + \mathbf{H}_1\mathbf{F}_{cp}\mathbf{R_s}[k]\mathbf{F}_{cp}^H\mathbf{H}_1^H \\ &\quad +\mathbf{H}_0\mathbf{F}_{cp}\mathbf{R_s}[k+1]\mathbf{F}_{cp}^H\mathbf{H}_1^H \\ &\quad +\mathbf{H}_1\mathbf{F}_{cp}\mathbf{R_s}[k-1]\mathbf{F}_{cp}^H\mathbf{H}_0^H + \mathbf{R_v}[k] \end{aligned} \tag{7}$$

where $\mathbf{R_v}[k]$ is noise autocorrelation matrix, which can be expressed as follows:

$$\mathbf{R_v}[k] = \begin{cases} \sigma_v^2\mathbf{I}_{PN_R} & k = 0 \\ \mathbf{0}_{PN_R} & k = \pm1, \pm2, \cdots \end{cases} \tag{8}$$

where σ_v^2 is the noise level. Next, we will exploit the structures of the autocorrelation matrices of the received signals to estimate the channel. Define:

$$\mathbb{R}_\mathbf{x}[k] = \sum_{j=-k}^{k}\mathbf{R_x}[j] \qquad \mathbb{R}_\mathbf{s}[k] \triangleq \sum_{j=-k}^{k}\mathbf{R_s}[j] \tag{9}$$

Substitute to (7), we have

$$\begin{aligned} \mathbb{R}_\mathbf{x}[k] &= \sum_{j=-k}^{k}\Big[\mathbf{H}_0\mathbf{F}_{cp}\mathbf{R_s}[j]\mathbf{F}_{cp}^H\mathbf{H}_0^H + \mathbf{H}_1\mathbf{F}_{cp}\mathbf{R_s}[j]\mathbf{F}_{cp}^H\mathbf{H}_1^H \\ &\quad +\mathbf{H}_0\mathbf{F}_{cp}\mathbf{R_s}[j+1]\mathbf{F}_{cp}^H\mathbf{H}_1^H \\ &\quad +\mathbf{H}_1\mathbf{F}_{cp}\mathbf{R_s}[j-1]\mathbf{F}_{cp}^H\mathbf{H}_0^H\Big] + \sigma_v^2\mathbf{I}_{PN_R} \\ &= (\mathbf{H}_0+\mathbf{H}_1)\mathbf{F}_{cp}\mathbb{R}_\mathbf{s}[k-1]\mathbf{F}_{cp}^H(\mathbf{H}_0+\mathbf{H}_1)^H \\ &\quad +\Phi_k + \sigma_v^2\mathbf{I}_{PN_R} \end{aligned} \tag{10}$$

where

$$\begin{aligned} \Phi_k &= \mathbf{H}_0\mathbf{F}_{cp}\mathbf{R_x}[k+1]\mathbf{F}_{cp}^H\mathbf{H}_1^H + \mathbf{H}_1\mathbf{F}_{cp}\mathbf{R_x}[-k-1]\mathbf{F}_{cp}^H\mathbf{H}_0^H \\ &\quad +\mathbf{H}_0\mathbf{F}_{cp}\mathbf{R_s}[k]\mathbf{F}_{cp}^H(\mathbf{H}_0+\mathbf{H}_1)^H \\ &\quad +(\mathbf{H}_0+\mathbf{H}_1)\mathbf{F}_{cp}\mathbf{R_x}[-k]\mathbf{F}_{cp}^H\mathbf{H}_0^H \end{aligned} \tag{11}$$

Without lose of generality, we can assume that the source signal block are correlated only within the lag $k \leqslant K$, where $K = 0, 1, 2, \cdots$, or in other words, $\mathbf{R_x}[k] = \mathbf{R_x}[-k] = \mathbf{0}$, for $k > K$. Under this assumption, and let $k = K+1$, then we have

$$\Phi_{K+1} = \mathbf{0}$$

and (10) can be rewritten as

$$\begin{aligned} \mathbb{R}_\mathbf{x}[K+1] &= (\mathbf{H}_0+\mathbf{H}_1)\mathbf{F}_{cp}\mathbb{R}_\mathbf{s}[K]\mathbf{F}_{cp}^H(\mathbf{H}_0+\mathbf{H}_1)^H \\ &\quad +\Phi_{K+1} + \sigma_v^2\mathbf{I}_{PN_R} \\ &= \tilde{\mathbf{H}}\mathbf{F}_{cp}\mathbb{R}_\mathbf{s}[K]\mathbf{F}_{cp}^H\tilde{\mathbf{H}}^H + \sigma_v^2\mathbf{I}_{PN_R} \end{aligned} \tag{12}$$

where we define $\tilde{\mathbf{H}} \triangleq \mathbf{H}_0 + \mathbf{H}_1 \in \mathcal{C}^{PN_R \times PN_T}$, which is a block circulant matrix, and can be written as:

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{h}[0] & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}[L] & \cdots & \mathbf{h}[1] \\ \mathbf{h}[1] & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \ddots & \mathbf{h}[L] \\ \mathbf{h}[L] & & \ddots & \ddots & \ddots & & \mathbf{0} \\ \mathbf{0} & \ddots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}[L] & \cdots & \mathbf{h}[1] & \mathbf{h}[0] \end{bmatrix} \tag{13}$$

Next, we will derive the proposed blind channel estimation method by exploiting the properties of $\mathbb{R}_\mathbf{x}[K+1]$. In order to do this, we first introduce three lemmas.

Lemma 1: Given the channel matrix $\mathbf{h}[z]$ is irreducible, and $\tilde{\mathbf{H}}$ defined by (13), then $\tilde{\mathbf{H}}\mathbf{F}cp$ is of full column rank.

Proof: According to the property of block circulant the matrix, $\tilde{\mathbf{H}}$ can be written as

$$\tilde{\mathbf{H}} = \left(\mathbf{F}_P^H \otimes \mathbf{I}_{N_R}\right)\mathcal{H}\left(\mathbf{F}_P^H \otimes \mathbf{I}_{N_T}\right) \tag{14}$$

where $\mathcal{H}$ is the block diagonal matrix, with the block diagonal submatrices being the frequency domain channel responses. Given the channel matrix $\mathbf{h}[z]$ is irreducible, then $\mathcal{H}$ must be full column rank [2]. Since both $\left(\mathbf{F}_P^H \otimes \mathbf{I}_{N_R}\right)$ and $\left(\mathbf{F}_P^H \otimes \mathbf{I}_{N_T}\right)$

are of full rank, then $\tilde{\mathbf{H}}$ must be of full column rank. The proof ends here. ■

Lemma 2: Given $\mathbf{a}[l]$ and $\hat{\mathbf{a}}[l]$ ($l = 0, \cdots, L$) are $m \times n$ full column rank matrices, $\tilde{\mathbf{A}}$ and $\hat{\tilde{\mathbf{A}}}$ are the $Tm \times Tn$ ($T \geqslant L$) block circulant matrices with the same structure of $\tilde{\mathbf{H}}$, but constructed from $\mathbf{a}[l]$ and $\hat{\mathbf{a}}[l]$ respectively. If span($\tilde{\mathbf{A}}$)=span($\hat{\tilde{\mathbf{A}}}$), then there exits a $m \times n$ invertible matrix $\mathbf{Q}$, such that

$$\hat{\mathbf{a}}[l] = \mathbf{a}[l]\mathbf{Q}, \qquad l = 0, \cdots, L \tag{15}$$

Proof: Denote $\mathbf{F} \triangleq \mathbf{F}_T^H \otimes \mathbf{I}_n$, which is obviously full rank. Consequently, we have

$$\mathrm{span}(\tilde{\mathbf{H}}) = \mathrm{span}(\hat{\tilde{\mathbf{H}}}) \Leftrightarrow \mathrm{span}(\tilde{\mathbf{H}}\mathbf{F}) = \mathrm{span}(\hat{\tilde{\mathbf{H}}}\mathbf{F}) \tag{16}$$

It is easy to verify that (16) is achieved if and only if there exists a $Tn \times Tn$ invertible matrix B such that

$$\hat{\tilde{\mathbf{A}}}\mathbf{F} = \tilde{\mathbf{A}}\mathbf{F}\mathbf{B} \Leftrightarrow \hat{\tilde{\mathbf{A}}} = \tilde{\mathbf{A}}\mathbf{F}\mathbf{B}\mathbf{F}^H \tag{17}$$

Since $\hat{\tilde{\mathbf{A}}}$ and $\tilde{\mathbf{A}}$ are both block circulant matrices, then $\mathbf{F}\mathbf{B}\mathbf{F}^H$ must be also a block circulant matrix, which leads to the result that $\mathbf{B}$ is block diagonal. We denote the $n \times n$ block diagonal entries of $\mathbf{B}$ being $\mathbf{b}_t$, where $t = 0, \cdots, T-1$, and substitute to (16), we have

$$\mathbf{b}_0 = \mathbf{b}_1 = \cdots = \mathbf{b}_{T-1} \tag{18}$$

Thus $\hat{\mathbf{a}}[l] = \mathbf{a}[l]\mathbf{Q}$, where $\mathbf{Q} = \mathbf{b}_0$. ■

Lemma 3: Given span($\tilde{\mathbf{H}}\mathbf{F}_{cp}$)=span($\hat{\tilde{\mathbf{H}}}\mathbf{F}_{cp}$), then their exists an invertible matrix $\mathbf{Q}$ such that

$$\hat{\mathbf{h}}[l] = \mathbf{h}[l]\mathbf{Q}, \qquad l = 0, \cdots, L \tag{19}$$

Proof: Since the first $M_g N_T$ rows of $\mathbf{F}_{cp}$ is the repeating of the last $M_g N_T$ rows of $\mathbf{F}_M^H \otimes \mathbf{I}_{N_T}$, then

$$\tilde{\mathbf{H}}\mathbf{F}_{cp} = \tilde{\mathbf{H}}'(\mathbf{F}_M^H \otimes \mathbf{I}_{N_T}), \qquad \hat{\tilde{\mathbf{H}}}\mathbf{F}_{cp} = \hat{\tilde{\mathbf{H}}}'(\mathbf{F}_M^H \otimes \mathbf{I}_{N_T}) \tag{20}$$

where $\tilde{\mathbf{H}}'$ is constructed by removing the first $M_g N_T$ columns of $\tilde{\mathbf{H}}$, and adding them to the last $M_g N_T$ columns of the resulted matrix. It can be verified that span($\tilde{\mathbf{H}}\mathbf{F}_{cp}$)=span($\hat{\tilde{\mathbf{H}}}\mathbf{F}_{cp}$) if and only if there exists an invertible matrix $\mathbf{B}$ such that

$$\tilde{\mathbf{H}}'(\mathbf{F}_M^H \otimes \mathbf{I}_{N_T}) = \hat{\tilde{\mathbf{H}}}'(\mathbf{F}_M^H \otimes \mathbf{I}_{N_T})\mathbf{B} \tag{21}$$

Note that the matrices $\tilde{\mathbf{H}}''$ and $\hat{\tilde{\mathbf{H}}}''$ containing the last MN_T rows of $\tilde{\mathbf{H}}'$ and $\hat{\tilde{\mathbf{H}}}'$ respectively are also block circulant, and from (21), $\tilde{\mathbf{H}}''(\mathbf{F}_M^H \otimes \mathbf{I}_{N_T}) = \hat{\tilde{\mathbf{H}}}''(\mathbf{F}_M^H \otimes \mathbf{I}_{N_T})\mathbf{B}$. Thus by invoking Lemma 2, it can be concluded that (19) is true. ■

Now, recall (10), it can be verified that the smallest eigenvalue of matrix $\mathbb{R}_{\mathbf{x}}[K+1]$ is σ_v^2, and there are $q = PN_R - MN_T$ co-orthogonal eigenvectors corresponding to the smallest eigenvalue. These eigenvectors are denoted by β_i ($i = 0, \cdots, q-1$). Based on a simple mathematical derivation used in the standard subspace method [8], we know that

$$\beta_i^H \tilde{\mathbf{H}} = \mathbf{0}, \qquad i = 0, \cdots, q-1 \tag{22}$$

or equivalently,

$$\tilde{\mathbf{H}}^H \beta_i = \mathbf{0}, \qquad i = 0, \cdots, q-1 \tag{23}$$

By dividing the vector β_i into blocks as

$$\beta_i = \left[\beta_i^T[M-1], \cdots, \beta_i^T[0]\right]^T \tag{24}$$

where $\beta_i^T[m]$, ($m = 0, \cdots, M-1$) are $N_R \times 1$ vectors. It is not difficult to verify that the circulant convolution of (22) is equivalent to the following equation:

$$\tilde{\mathbf{G}}_i \mathbf{h} = 0 \qquad i = 0, \cdots, q-1 \tag{25}$$

where $\tilde{\mathbf{G}}_i$ is the truncated block circulant matrix constructed by β_i, i.e.

$$\tilde{\mathbf{G}}_i = \begin{bmatrix} \beta_i^T[0] & \beta_i^T[1] & \cdots & \beta_i^T[L] \\ \beta_i^T[1] & \beta_i^T[2] & \cdots & \beta_i^T[L+1] \\ \vdots & \vdots & \ddots & \vdots \\ \beta_i^T[P-1] & \beta_i^T[P] & \cdots & \beta_i^T[L-2] \\ \beta_i^T[P] & \beta_i^T[0] & \cdots & \beta_i^T[L-1] \end{bmatrix} \tag{26}$$

and $\mathbf{h} = [\mathbf{h}^T[0], \cdots, \mathbf{h}^T[L]]^T$. Therefore, $\mathbf{h}$ can be obtained by the following criterion:

$$\hat{\mathbf{h}} = \arg \min_{\|\mathbf{h}\|=1} \sum_{i=0}^{q-1} \|\tilde{\mathbf{G}}_i \mathbf{h}\|^2 \tag{27}$$

which is equivalent to obtain the N_T eigenvectors of $\tilde{\mathbf{G}}^H \tilde{\mathbf{G}}$, corresponding to the smallest eigenvalue, where

$$\tilde{\mathbf{G}} \triangleq \left[\tilde{\mathbf{G}}_0^T, \cdots, \tilde{\mathbf{G}}_{q-1}^T\right]^T \tag{28}$$

Since we have the constraint $\|\mathbf{h}\|^2 = 1$, and invoking Lemma 3, $\mathbf{h}$ can be estimated up to an unitary ambiguity matrix $\mathbf{Q}$.

In this section, we have proposed a closed-form solution to blind MIMO-OFDM channel estimation problem by exploiting the inherent properties of the autocorrelation matrices of the received symbol blocks with different block lags. To achieve the the proposed estimation method, we have assumed that the source correlation matrix with block lag k is $\mathbf{0}$ when $k > K$. In most of the situations, it is reasonable to assume that the upper bound K is fairly small, because the stack memory could not be too large. For example, in a very popular precoded OFDM system [6], every two blocks of source symbols are grouped and precoded, i.e. $K = 1$. In [7], precoding is processed in one OFDM block, hence $K = 0$. In these cases, the computational cost of our proposed algorithm is not high, since the correlation matrices to be calculated is not too much. However, it is still possible that K is large. In such case, we can propose an approximation method to avoid the high computational cost. But due to the space limit, this can not be discussed in this paper.

4. SIMULATION RESULTS

We now present simulation results to illustrate the performance of our proposed algorithm. The simulated OFDM system is modeled containing 64 subcarriers, i.e. $M = 64$. Each OFDM frame consists of 68 symbols including the CP of length 4, i.e. $M_g = 4$. Every 2 blocks of source data are grouped and precoded, using the optimized precoder proposed in [6], which means $K = 1$. The system is equipped with 2 transmit antennas and 2 receive antennas, and the channel model used is a 3-tap FIR filter with tap coefficients independently chosen from a white Gaussian process. As a comparison, we also simulate the subspace based algorithm that exploits the cyclostationarity of the output data, which is proposed in [1].

To evaluate the channel estimation error, we employed the normalized-root-mean-square-error (NRMSE), which is defined as

$$\text{NRMSE} = \sqrt{\frac{1}{N_R N_T N_M} \sum_{j=1}^{N_R} \sum_{i=1}^{N_T} \sum_{t=1}^{N_M} \frac{\| \hat{\mathbb{H}}_{ji}^{(t)} - \mathbb{H}_{ji} \|^2}{\| \mathbb{H}_{ji} \|^2}} \quad (29)$$

where N_M is the number of Monte Carlo runs for each channel realization. $\hat{\mathbb{H}}_{ji}^{(t)}$ is the estimation of channel $\mathbb{H}_{ji}$ from the t^{th} run. We simulate 30 channel realizations, each for 100 Monte Carlo runs.

Fig.2 illustrate the NRMSE as a function of SNR, where 300 symbol blocks are collected to compute the correlation matrices. Fig.3 illustrate the NRMSE as a function of number of observation symobl blocks (NoS), under the condition that SNR being 20 dB. This figure indicates that our proposed method is more sensitive to the number of observation data blocks.

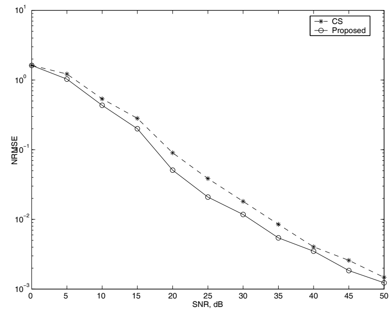

Fig. 2. NRMSE performance as a function of SNR

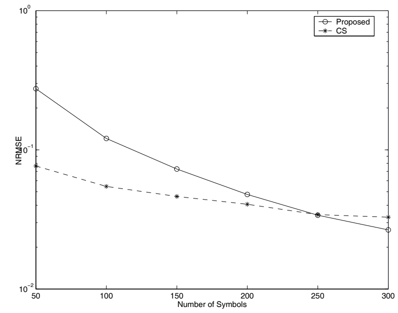

Fig. 3. NRMSE performance as a function of NoS

5. CONCLUSION

In this paper, we present a new second order statistics based method that admits a closed-form solution for blind MIMO-OFDM channel estimation driven by colored source with unknown statistics. The uniqueness of the solution is proved by exploiting the inherent properties of the correlation matrices of the received data block with different block lags. In face, our method still works even if the source signals are white. Simulation results show that our proposed algorithm achieves better performance than the subspace method proposed in [1] if the observation window is big enough.

6. REFERENCES

[1] W. Bai, C. He, L. Jiang, and H. Zhu, "Blind channel estimation in MIMO-OFDM systems," *Proc. GLOBECOM*, Taipei, Taiwan, Nov. 2002, vol.1, pp. 317-321.

[2] H. Ali, A. Doucet, and Y. Hua, "Blind SOS Subspace channel estimation and equalizatoin techniques exploiting spatial diversity in OFDM systems", *Digital Signal Processing*, Vol. 14, No. 2, Mar. 2004

[3] S. Zhou, B. Muquet, and G. B. Giannakis, "Subspace-based (semi-) blind channel estimation for block precoded space-time OFDM", *IEEE Trans. Signal Processing*, Vol 50, May 2002, pp. 1215-1228.

[4] W. Bai, and Z. Bu, "Subspace based channel estimation in MIMO-OFDM system," *Proc. VTC* , May 2004, vol. 2, pp. 598 - 602.

[5] C. Shin, and E. J. Powers, "Blind channel estimation for MIMO-OFDM systems using virtual carriers," *Proc. GLOBECOM*, 29 Nov.-3 Dec. 2004 , vol. 4, pp. 2465 - 2469.

[6] R. Lin, and A. P. Petropulu, "Linear block precoding for blind channel estimation in OFDM systems", in *Proc. ISSPA 2003*, vol 2, pp. 395 398, 1-4 Jul. 2003.

[7] X. Chen, A. R. Leyman, and J. Fang, "Blind Channel Estimation for Linearly Precoded MIMO-OFDM," *Proc. ICASSP 2006*, Vol. 4, pp.IV-381 IV-384, 14 19 May, 2006.

[8] K. Abed-Meraim, P. Loubaton, and E. Moulines, "A subspace algorithm for certain blind identification problem," *IEEE Trans. Inform. Theory*, vol. 32, pp. 499-511, Aro. 1997.

[9] J. N. Buxton, R. F. Churchouse, and A. B. Tayler, *Matrices: Methods and Applications*, Oxford University Press, New York, 1990