TRANSMIT BEAMFORMING WEIGHTS FOR CHANNEL ESTIMATION ERROR AND FEEDBACK LATENCY IN OFDM

Timothy A. Thomas
Motorola Labs - Communication Systems Research Laboratory
1301 E. Algonquin Road, Schaumburg, IL 60196
T.Thomas@motorola.com

ABSTRACT

This paper presents a new method for computing transmit beamforming weights in OFDM when there is channel state information (CSI) uncertainty at the base. The channel uncertainty may be caused by channel estimation error, quantization effects, and/or channel variations from when the CSI is measured and when the beamforming weights are applied. The weight calculation automatically trades off using long-term statistics (e.g., the average spatial covariance matrix) and short-term beamforming (i.e., maximal ratio transmission). Simulation results show that the new weights have very promising performance with the benefit of not requiring a means to decide whether the short-term or long-term beamforming should be used.

Index terms – MIMO systems, Array signal processing, OFDM

1. INTRODUCTION

Large gains can be obtained by using transmit beamforming on the downlink of a broadband communication system such as OFDM if channel state information (CSI) is available at the base station. Obtaining CSI at the base station can be accomplished in a time division duplex (TDD) communication system through uplink sounding [7] and in frequency division duplex (FDD) through feedback methods such as direct channel feedback (DCFB) [2] or codebook feedback [8]. However the CSI obtained through either method can suffer from estimation error and also errors from feedback latency (i.e., when the channel changes from when the CSI is obtained at the base and when the beamforming weights are applied).

Previous methods for computing transmit weights such as maximal ratio transmission (MRT) [1] and Eigen beamforming (EBF) [4] are not optimized for a given channel estimation error and feedback latency. In [9] methods are described that choose between MRT and EBF given the channel conditions (e.g., angular spread), the channel estimation error, and the Doppler frequency. However, improved beamforming weights may be found if a model of the effects of feedback latency and channel estimation error are used in the design of the weights themselves.

In [10], transmit weights are designed for flat-fading channels with CSI error (e.g., from quantization error or channel estimation error). These weights are a good first step for a design, but for transmit weights in OFDM the weights should be designed by exploiting the channel correlation (in frequency) that is present (exploiting the frequency correlation helps mitigate the effects of CSI uncertainty due to channel estimation error). In addition, these weights did not consider feedback latency and hence further gains can be made by trading off short-term beamforming gain and long-term beamforming gain (e.g., using an estimate of the spatial covariance matrix). This paper describes a new beamforming weight design that will account for the feedback latency as well as the frequency correlation present in OFDM systems.

2. BEAMFORMING WEIGHTS DESIGNED WITH CSI UNCERTAINTY

This section gives the detailed derivation of transmit weights designed for a given feedback latency and channel estimation error. Assume that the base station has M_T transmit antennas and the mobile has M_R receive antennas. Also assume that the downlink transmission occurs at time b and that the downlink channel estimates used to compute the transmit weights are found at time 0. The received $M_R\times 1$ signal at the mobile on subcarrier k and symbol time b is given as:

$$\mathbf{Y}(k,b)=\mathbf{H}(k,b)\mathbf{v}(k,b)x(k,b)+\mathbf{N}(k,b) \tag{1}$$

where $\mathbf{H}(k,b)$ is the $M_R\times M_T$ channel matrix, $\mathbf{v}(k,b)$ is the $M_T\times 1$ transmit weight vector, $x(k,b)$ is the transmitted symbol, and $\mathbf{N}(k,b)$ is additive noise with covariance matrix, $\sigma_n^2\mathbf{I}_{M_R}$ ($\mathbf{I}_m$ is an $m\times m$ identity matrix). It will be assumed that there are a total of K subcarriers (i.e., $0\le k\le K$-1). Note that although there is only a single data stream in (1), as described in Section 3 the following transmit weight design is easily extendable for multiple data streams to one terminal.

The idea behind the design of the transmit weights is to maximize the expected power of the received signal in (1) given downlink channel estimates at time 0 (e.g., the channel estimates can be found with uplink sounding [7], codebook type feedback [8], or direct channel feedback [2]). Thus the transmit weights are designed as follows (i.e., trying to maximize the expected received power):

$$\mathbf{v}(k,b)=\arg\max\{E[\mathbf{v}^H(k,b)\mathbf{H}^H(k,b)\mathbf{H}(k,b)\mathbf{v}(k,b)\,|\,\hat{\mathbf{H}}_1,\ldots,\hat{\mathbf{H}}_{M_R}]\} \tag{2}$$

where $\hat{\mathbf{H}}_m$ is a $KM_T\times 1$ vector containing the downlink channel estimates for mobile antenna m and is given as:

$$\hat{\mathbf{H}}_m=\begin{bmatrix}\hat{\mathbf{H}}_m^T(0,0)\\ \hat{\mathbf{H}}_m^T(1,0)\\ \vdots\\ \hat{\mathbf{H}}_m^T(K-1,0)\end{bmatrix} \tag{3}$$

where $\hat{\mathbf{H}}_m(k,0)$ is the $1\times M_T$ downlink channel estimate for receive antenna m at subcarrier k at time 0. It will be assumed that the downlink channel estimate can be modeled as:

$$\hat{\mathbf{H}}_m(k,0)=\mathbf{H}_m(k,0)+\mathbf{E}(k) \tag{4}$$

where $\mathbf{H}_m(k,0)$ is the true downlink channel at time 0 (i.e., row m of $\mathbf{H}(k,0)$) and $\mathbf{E}(k)$ is a $1\times M_T$ vector of additive Gaussian noise with covariance matrix $\sigma_e^2 \mathbf{I}_{M_T}$ where σ_e^2 is the expected channel estimation error of the downlink channel estimates. Thus $\hat{\mathbf{H}}_m$ can also be expressed as:

$$\hat{\mathbf{H}}_m = \begin{bmatrix} \mathbf{H}_m^T(0,0) \\ \mathbf{H}_m^T(1,0) \\ \vdots \\ \mathbf{H}_m^T(K-1,0) \end{bmatrix} + \mathbf{E} \tag{5}$$

where $\mathbf{E}$ is a $KM_T \times 1$ vector of additive Gaussian noise with covariance matrix $\sigma_e^2 \mathbf{I}_{KM_T}$. It should be noted that the channel estimation error is typically a function of frequency since it tends to be higher at the band edges. Thus the derivation of the transmit weights can be further improved by tracking the channel estimation error as a function of frequency. However, to derive the low computational complexity version of the transmit weights it is advantageous to assume that the channel estimation error is constant across frequency.

The solution is to choose $\mathbf{v}(k,b)$ as the eigenvector associated with the largest eigenvalue of the following matrix:

$$E[\mathbf{H}^T(k,b)\mathbf{H}^*(k,b) \mid \hat{\mathbf{H}}_1, \ldots, \hat{\mathbf{H}}_{M_R}] \tag{6}$$

To make the problem easier to solve this equation will be broken up so the expectation is found for each receive antenna. So the solution becomes to choose $\mathbf{v}(k,b)$ as the eigenvector associated with the largest eigenvalue of:

$$\sum_{m=1}^{M_R} E[\mathbf{H}_m^T(k,b)\mathbf{H}_m^*(k,b) \mid \hat{\mathbf{H}}_m] \tag{7}$$

Thus what is needed is the expected spatial correlation matrix at each receive antenna given the channel estimates (across all subcarriers) for the receive antennas. This spatial correlation matrix can be found by finding the pdf of $\mathbf{H}_m(k,b)$ given $\hat{\mathbf{H}}_m$ which is denoted as $f(\mathbf{H}_m(k,b) \mid \hat{\mathbf{H}}_m)$. This pdf can be found using the conditional density formulas from [5] (pages 68-69) as:

$$f(\mathbf{H}_m(k,b) \mid \hat{\mathbf{H}}_m) = \frac{f(\hat{\mathbf{H}}_m \mid \mathbf{H}_m(k,b)) f(\mathbf{H}_m(k,b))}{f(\hat{\mathbf{H}}_m)} \tag{8}$$

Because the resulting pdf is a multivariate complex Gaussian distribution, the correlation can be read directly from the pdf and can be shown to be:

$$E[\mathbf{H}_m^T(k,b)\mathbf{H}_m^*(k,b) \mid \hat{\mathbf{H}}_m] = \tilde{\mathbf{Q}}(k,b) + \tilde{\mathbf{Q}}(k,b)\hat{\mathbf{H}}_m^T(k,b)\hat{\mathbf{H}}_m^*(k,b)\tilde{\mathbf{Q}}(k,b) \tag{9}$$

where $M_T \times M_T$ $\tilde{\mathbf{Q}}(k,b)$ and $M_T \times 1$ $\hat{\mathbf{H}}_m^T(k,b)$ are given as:

$$\tilde{\mathbf{Q}}(k,b) = \left(J^2(b)\mathbf{R}_k^H \mathbf{Z}^{-1}(k,b)\mathbf{R}_k + \mathbf{Q}^{-1}\right)^{-1} \tag{10}$$

$$\hat{\mathbf{H}}_m^T(k,b) = J(b)\mathbf{R}_k^H \mathbf{Z}^{-1}(k,b)\hat{\mathbf{H}}_m \tag{11}$$

where $J(b)$ is the expected time correlation (assuming a Jake's spectrum) and can be expressed as (Δ_t equals the time between OFDM symbols and f_d is the maximum Doppler frequency):

$$J(b) = J_0(2\pi f_d b \Delta_t) \tag{12}$$

where $KM_T \times M_T$ $\mathbf{R}_k$ is a matrix containing the frequency correlations and is given as:

$$\mathbf{R}_k = \begin{bmatrix} r(k)\mathbf{I}_{M_T} \\ r(k-1)\mathbf{I}_{M_T} \\ \vdots \\ r(k-K+1)\mathbf{I}_{M_T} \end{bmatrix} \tag{13}$$

where the frequency correlation is (Δ_f is the subcarrier spacing (in Hz) and $\tau_{\max}$ is the maximum expected delay (in seconds)):

$$r(f) = e^{-j\pi\tau_{\max} f \Delta_f} \frac{\sin(\pi\tau_{\max} f \Delta_f)}{\pi\tau_{\max} f \Delta_f} \tag{14}$$

where $KM_T \times KM_T$ $\mathbf{Z}(k,b)$ is given as:

$$\mathbf{Z}(k,b) = \sigma_e^2 \mathbf{I}_{KM_T} + \begin{bmatrix} \mathbf{Q} & r(1)\mathbf{Q} & r(2)\mathbf{Q} & \cdots & r(K-1)\mathbf{Q} \\ r^*(1)\mathbf{Q} & \mathbf{Q} & r(1)\mathbf{Q} & \cdots & r(K-2)\mathbf{Q} \\ r^*(2)\mathbf{Q} & & \mathbf{Q} & & \vdots \\ \vdots & & & \ddots & \\ r^*(K-1)\mathbf{Q} & r^*(K-2)\mathbf{Q} & \cdots & & \mathbf{Q} \end{bmatrix} - \tag{15}$$

$$J^2(b) \begin{bmatrix} |r(k)|^2\mathbf{Q} & r(k)r^*(k-1)\mathbf{Q} & r(k)r^*(k-2)\mathbf{Q} & \cdots & r(k)r^*(k-K+1)\mathbf{Q} \\ r^*(k)r(k-1)\mathbf{Q} & |r(k-1)|^2\mathbf{Q} & r(k-1)r^*(k-2)\mathbf{Q} & \cdots & r(k-1)r^*(k-K+1)\mathbf{Q} \\ r^*(k)r(k-2)\mathbf{Q} & r^*(k-1)r(k-2)\mathbf{Q} & |r(k-2)|^2\mathbf{Q} & & \vdots \\ \vdots & \vdots & & \ddots & \\ r^*(k)r(k-K+1)\mathbf{Q} & r^*(k-1)r(k-K+1)\mathbf{Q} & \cdots & & |r(k-K+1)|^2\mathbf{Q} \end{bmatrix}$$

and finally where M_TxM_T $\mathbf{Q}$ is the average spatial correlation matrix and may be estimated as follows:

$$\mathbf{Q} = \frac{1}{M_R K} \sum_{m=1}^{M_R} \sum_{k=0}^{K-1} \hat{\mathbf{H}}_m^T(k,0)\hat{\mathbf{H}}_m^*(k,0) \tag{16}$$

3. ALGORITHM DETAILS

It should be clear from the equations in the previous section that computation of the transmit weights would be extremely complex since a different $KM_T \times KM_T$ inverse needs to be taken on each subcarrier. However, an implementable weight computation algorithm can be found by using the observation that the correlation across subcarriers quickly decreases in frequency and that each subcarrier sees the same frequency correlation if looking at just a small window of frequencies around the subcarrier of interest (the resulting weight calculation is not too dissimilar from FIR filtering ideas). First, the same coefficients are applied on each (overlapping) group of L subcarriers and hence only one inverse need to be found. Second, L is chosen to be much smaller than K and hence not only is one inverse needed, but the inverse is now of size $LM_T \times LM_T$. The following equations reflect this lower complexity version. It will be assumed that L is odd and first define $\bar{L}$ as $(L-1)/2$. In place of $\mathbf{Z}(k,b)$, use:

$$\tilde{\mathbf{Z}}(b) = \sigma_e^2 \mathbf{I}_{LM_T} + \begin{bmatrix} \mathbf{Q} & r(1)\mathbf{Q} & r(2)\mathbf{Q} & \cdots & r(L-1)\mathbf{Q} \\ r^*(1)\mathbf{Q} & \mathbf{Q} & r(1)\mathbf{Q} & \cdots & r(L-2)\mathbf{Q} \\ r^*(2)\mathbf{Q} & & \mathbf{Q} & & \vdots \\ \vdots & & & \ddots & \\ r^*(L-1)\mathbf{Q} & r^*(L-2)\mathbf{Q} & \cdots & & \mathbf{Q} \end{bmatrix} - \tag{17}$$

$$J^2(b) \begin{bmatrix} |r(\bar{L})|^2\mathbf{Q} & r(\bar{L})r^*(\bar{L}-1)\mathbf{Q} & r(\bar{L})r^*(\bar{L}-2)\mathbf{Q} & \cdots & r(\bar{L})r^*(-\bar{L})\mathbf{Q} \\ r^*(\bar{L})r(\bar{L}-1)\mathbf{Q} & |r(\bar{L}-1)|^2\mathbf{Q} & r(\bar{L}-1)r^*(\bar{L}-2)\mathbf{Q} & \cdots & r(\bar{L}-1)r^*(-\bar{L})\mathbf{Q} \\ r^*(\bar{L})r(\bar{L}-2)\mathbf{Q} & r^*(\bar{L}-1)r(\bar{L}-2)\mathbf{Q} & |r(\bar{L}-2)|^2\mathbf{Q} & & \vdots \\ \vdots & \vdots & & \ddots & \\ r^*(\bar{L})r(-\bar{L})\mathbf{Q} & r^*(\bar{L}-1)r(-\bar{L})\mathbf{Q} & \cdots & & |r(-\bar{L})|^2\mathbf{Q} \end{bmatrix}$$

For the channel estimate on subcarrier k and time b, $\hat{\mathbf{H}}_m(k,b)$,use:

$$\tilde{\mathbf{H}}_m^T(k,b) = J(b)\tilde{\mathbf{R}}^H \tilde{\mathbf{Z}}^{-1}(b) \begin{bmatrix} \hat{\mathbf{H}}_m^T(k-\bar{L},0) \\ \hat{\mathbf{H}}_m^T(k-\bar{L}+1,0) \\ \vdots \\ \hat{\mathbf{H}}_m^T(k+\bar{L},0) \end{bmatrix} \tag{18}$$

where $LM_T \times M_T$ $\tilde{\mathbf{R}}$ is given as:

$$\tilde{\mathbf{R}} = \begin{bmatrix} r(\bar{L})\mathbf{I}_{M_T} \\ r(\bar{L}-1)\mathbf{I}_{M_T} \\ \vdots \\ r(-\bar{L})\mathbf{I}_{M_T} \end{bmatrix} \tag{19}$$

Note that in (18) if channel estimate values, $\hat{\mathbf{H}}_m(k,0)$, go outside of the allowable subcarriers (i.e., k<0 or k>K-1) then set these channel estimate values to zero.

Also, replace $\tilde{\mathbf{Q}}(k,b)$ with the following:

$$\mathbf{Q}'(b) = \left(J^2(b)\tilde{\mathbf{R}}^H \tilde{\mathbf{Z}}^{-1}(b)\tilde{\mathbf{R}} + \mathbf{Q}^{-1}\right)^{-1} \tag{20}$$

Finally, the expected spatial correlation matrix on subcarrier k and time b given the observations is approximated by:

$$E[\mathbf{H}_m^T(k,b)\mathbf{H}_m^*(k,b) \mid \hat{\mathbf{H}}_m] \approx \mathbf{Q}'(b) + \mathbf{Q}'(b)\tilde{\mathbf{H}}_m^T(k,b)\tilde{\mathbf{H}}_m^*(k,b)\mathbf{Q}'(b) \tag{21}$$

The transmit weights on subcarrier k and OFDM symbol time b are thus given by the Eigenvector associated with the largest Eigenvalue of the correlation matrix in (21) (noting that $\tilde{\mathbf{R}}$ can be precomputed based on the expected maximum time of arrival). If multi-stream transmission is desired, the data streams can be sent out of the strongest Eigenmodes of the correlation matrix in (21).

In summary, the steps of computing the weights are:

1. Have $r(k)$ (and hence $\tilde{\mathbf{R}}$ in (19)) precomputed assuming a maximum delay spread (e.g., 8.0 µsec).
2. Estimate the channel estimates (e.g., using uplink sounding) to find $\hat{\mathbf{H}}_m(k,0)$ for $0 \le k \le K$-1.
3. Estimate the channel estimation error, σ_e^2, estimate the expected Doppler frequency, f_d, and compute $J(b)$ from (12).
4. Estimate the spatial correlation matrix, $\mathbf{Q}$, from (16). Since $\mathbf{Q}$ is a correlation matrix it may be best to normalize $\mathbf{Q}$ by the largest diagonal element and hence the resulting $\mathbf{Q}$ will have 1 as its largest diagonal element (the simulation results did this normalization step).
5. Compute $\tilde{\mathbf{Z}}^{-1}(b)$ using (17).
6. Compute $\mathbf{Q}'(b)$ using (20).
7. For k=0,…, K-1, compute $\tilde{\mathbf{H}}_m^T(k,b)$ using (18).
8. For m=1,…,M_R, compute $E[\mathbf{H}_m^T(k,b)\mathbf{H}_m^*(k,b) \mid \hat{\mathbf{H}}_m]$ using (21).
9. Determine the transmit weight on subcarrier k and time b, $\mathbf{v}(k,b)$, as the eigenvector associated with the largest eigenvalue of $\sum_{m=1}^{M_R} E[\mathbf{H}_m^T(k,b)\mathbf{H}_m^*(k,b) \mid \hat{\mathbf{H}}_m]$.

Note that different weights are still found for each OFDM symbol in time. To further simplify calculations, a single set of weights can be fixed for multiple OFDM symbols.

4. SIMULATION RESULTS

The simulations are for a TDD communication system with a COST-259 style spatial channel model [3] consisting of a single scattering zone having 100 discrete multipath rays, 2.0 µsec RMS delay spread and a 15° multipath angular spread with respect to the base antenna array. The base has a uniform linear array of four antennas with a ½ wavelength spacing between the antenna elements and the mobiles have one receive antenna. The OFDM system uses a 512-point FFT with a 15 kHz subcarrier spacing at a 2 GHz carrier frequency. The number of subcarriers with data is K=300 which span 4.5 MHz. The cyclic prefix length is 37 (4.8 µsec) and the total OFDM symbol duration is 71.42 µsec. The data allocation is a single OFDM symbol on the downlink and ideal channel information is assumed at the mobile (i.e., no downlink channel estimation is performed). For the simulations, the data stream is encoded using the 3GPP turbo code with max-log-map decoding and 12 iterations (the frame size is 288 bits for rate ½ QPSK and 1338 bits for rate ¾ 64-QAM). For all algorithms the channel is obtained at the base using uplink sounding [7] using decimation separability with a decimation factor of four (the decimation value is the separation in number of subcarriers between pilot symbols). The channel estimator used for uplink sounding was a DFT estimator designed for 11 µsec maximum delay spread (i.e., the DFT estimator simply takes an IFFT of the pilot symbols and only keeps time taps up to 11 µsec). The delay between when the uplink sounding waveform is used and when the beamforming weights are applied is five OFDM symbols (0.357 msec). In all of the following algorithms it is assumed that the base station knows the speed of the mobile (i.e., maximum Doppler frequency) and also the uplink SNR (used to estimate the channel estimation error).

The following algorithms are compared in the simulation results: 1) ULS-MRT is per-subcarrier MRT with uplink sounding used to get the CSI. 2) ULS-EBF is EBF with uplink sounding used to get the CSI. 3) ULS-CE/DOP is the weights given in Section 3 (using uplink sounding to get the CSI) with L=17, $\tau_{\max}$=8 µsec, and the Doppler frequency matched to the given mobile speed. 4) 1Tx is a single transmit antenna (used to show the gain of each technique.).

Figure 1 shows the downlink SNR required to get a 0.01 FER for various velocities for rate ½ QPSK when the uplink SNR (used for channel sounding) equals the downlink SNR. At all speeds the new transmit weights (ULS-CE/DOP) performed better than MRT and EBF. Figure 3 and Figure 4 show similar results for rate ½ QPSK and rate ¾ 64-QAM respectively when the uplink SNR being 10 dB below the downlink SNR (this is done to model the power mismatch between the mobile and the base). At the lower SNRs (i.e., QPSK results) EBF is best because of the gain it gets over noise and the new weights are close to the EBF performance (the use of the truncation to L=17 subcarriers may explain the slight performance loss over EBF). At the higher SNRS (i.e., 64-QAM results) the new weights tracked the MRT performance at the lower speeds and at higher speeds the new weights did better than EBF and MRT. Thus in general the new weights performed well and also have the benefit of not requiring a means to switch between EBF and MRT.

5. CONCLUSION

This paper described the design of transmit weights for feedback latency and channel estimation error in the channel state information at the base station. The weights are designed to automatically trade off long-term statistical beamforming (i.e., Eigen beamforming, EBF) with short-term beamforming (i.e., maximal ratio transmission, MRT). Besides having a performance improvement, the new weights provide an alternative to needing a mechanism to switch between EBF and MRT. Simulation results show that the new weights provide good performance for different levels of channel estimation error and feedback latency.

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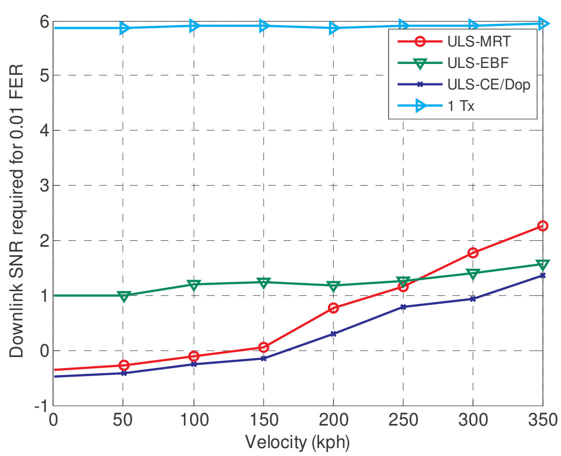

Figure 1. Downlink SNR required for a 0.01 FER with rate ½ QPSK and uplink SNR equal to the downlink SNR.

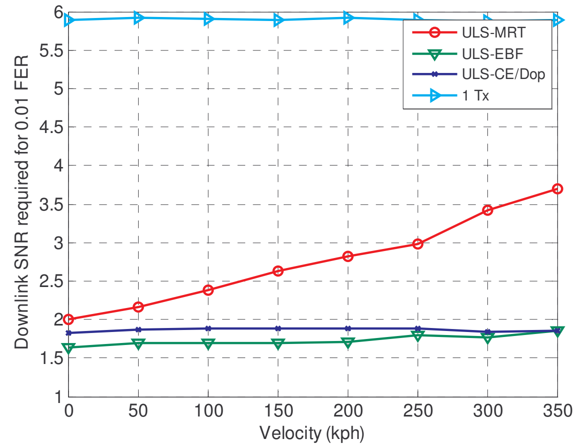

Figure 2. Downlink SNR required for a 0.01 FER with rate ½ QPSK and uplink SNR 10 dB below the downlink SNR.

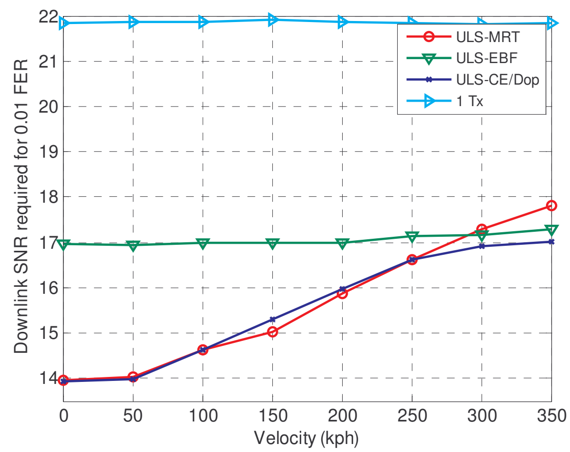

Figure 3. Downlink SNR required for a 0.01 FER with rate ¾ 64-QAM and uplink SNR 10 dB below the downlink SNR.