AVERAGE SEP LOSS ANALYSIS OF TRANSMIT BEAMFORMING FOR FINITE RATE FEEDBACK MISO SYSTEMS WITH QAM CONSTELLATION

Yogananda Isukapalli, Jun Zheng and Bhaskar D. Rao

Department of Electrical and Computer Engineering

University of California, San Diego, La Jolla, CA 92093, U.S.A

ABSTRACT

In this paper, utilizing high resolution quantization theory, we analyze the loss in average symbol error probability (SEP) for finite rate feedback MISO systems with rectangular M-QAM constellation. Assuming perfect channel estimation, no-feedback delay and error-less feedback, for spatially i.i.d and correlated channels we derive analytical expressions for loss in average SEP due to finite-rate channel quantization. We then consider the high-SNR regime and show that the loss associated with correlated case is related to the loss associated with the i.i.d case by a scaling constant given by the determinant of the correlation matrix. We also present simulation results in support of the analytical expressions.

Index Terms: MISO systems, transmit beamforming, channel state information, feedback, M-QAM, channel quantization, spatial correlation

1. INTRODUCTION

In a multiple-input and single-output (MISO) system, if the channel state information (CSI) is available at the transmitter, one can achieve both the diversity and array gains with transmit beamforming, whereas only diversity gain can be realized with space-time coding. In this paper we focus our attention on MISO systems where CSI is conveyed from the receiver to the transmitter through a finite-rate feedback link [1]-[4]. Optimum codebook design for ergodic capacity loss, a system performance metric, is proposed in [1]. In [3], the problem is studied from a source coding perspective by formulating the finite-rate quantized MISO system as a general vector quantization problem. By utilizing the high-resolution distortion analysis of the generalized vector quantizer, tight lower bounds of the ergodic capacity loss of a quantized MISO system over i.i.d and correlated fading channels with both optimal and mismatched channel quantizers were obtained [3].

Average SEP, another important system performance metric, for limited set of constellations has been studied with i.i.d fading channels based on approximating the statistical distribution of the key random variable that characterizes the system performance. Specifically both [1] and [2] characterized the absolute amplitude square of the inner product between the channel direction and its quantized version as a truncated beta distribution and used it to study effect of quantization on average SEP. Similar to the capacity analysis, SEP analysis for correlated channels using such statistical methods have not met with much success. In this paper we make use of the source coding based framework developed in [3] to study the average SEP loss in correlated Rayleigh fading channels with rectangular M-QAM constellation. The application of the theory in [3] to this problem is quite involved because of the complicated dependency of the objective function on the random variables involved and the results derived here serve to validate the general nature of the theory [3]. In addition, the results provide interesting insight into the more general and useful scenario of correlated channels with a more practical and relevant measure. The rest of this paper is organized as follows. In Section 2, we introduce our system model. In section 3 we present the high resolution analysis. The average SEP loss expressions are derived in Section 4. Numerical and simulation results are presented in Section 5. We conclude this paper in Section 6.

E-mail:yoga@ucsd.edu, juzheng@ucsd.edu, brao@ece.ucsd.edu. This research was supported in part by CoRe grant No.02-10109 sponsored by Ericsson and in part by the U. S. Army Research Office under the Multi-University Research Initiative (MURI) grant # W911NF-04-1-0224.

2. SYSTEM MODEL

We consider a MISO system with t antennas at the base station (BS) and one antenna at the mobile station (MS). The channel between the BS and the MS is modeled as a frequency-flat, slowly varying Rayleigh fading channel that is assumed to be constant over a block of symbols. Dropping time index, for the sake of simplicity, $\mathbf{h} \in \mathbb{C}^{t\times 1}$ is the correlated [1] MISO channel response with distribution given by $\mathbf{h} \sim \mathcal{NC}\big(0, \mathbf{\Sigma}_{\mathrm{h}}\big)$. Let us denote by $\mathbf{w} \in \mathbb{C}^{t\times 1}$, the unit norm beamforming vector at the BS. Then, the received signal at the MS is given by $y = \mathbf{w}^{\mathsf{H}}\mathbf{h}s_m + n$, where H is the Hermitian operator and n is a zero-mean circularly symmetric complex Gaussian random variable with $E[|n|^2] = 1$. The transmitted two dimensional modulation symbol is denoted by s_m, which belongs to the rectangular M-QAM constellation with $E[|s_m|^2] = \rho$. The CSI is assumed to be perfectly known at the receiver but only partially available at the transmitter through a finite-rate feedback link of B bits per channel update. Specifically, a codebook $\mathcal{C} = \big\{\widehat{\mathbf{v}}_1, \cdots, \widehat{\mathbf{v}}_N\big\}$, composed of transmit beamforming vectors, is assumed to be known to both the receiver and the transmitter, here $N = 2^B$. Based on the channel realization $\mathbf{h}$, the receiver selects the best code point $\widehat{\mathbf{v}}$ from the codebook and sends the corresponding index back to the transmitter. Assuming no errors and no delay in the feedback link, at the transmitter, the unit-norm vector $\widehat{\mathbf{v}}$ is employed as the beamforming vector, i.e. $\mathbf{w} = \widehat{\mathbf{v}}$. the received signal can now be written as $y = \langle\mathbf{h}, \widehat{\mathbf{v}}\rangle \cdot s_m + n = \sqrt{\alpha}\cdot\langle\mathbf{v}, \widehat{\mathbf{v}}\rangle \cdot s_m + n$, where $\mathbf{v} = \mathbf{h}/\|\mathbf{h}\|$, $\alpha = \|\mathbf{h}\|^2$ and $\langle\mathbf{x}, \mathbf{y}\rangle = \mathbf{x}^{\mathsf{H}}\mathbf{y}$.

3. HIGH RESOLUTION THEORY

In this section we briefly summarize the asymptotic distortion analysis of the generalized vector quantizer results in [3] that are relevant for the analysis of average SEP loss of M-QAM constellation. It is assumed that the source variable $\mathbf{h}$ is a two-vector tuple, $(\mathbf{v}, \alpha)$, where vector $\mathbf{v} \in \mathbb{Q}$ represents the actual quantization variable of dimension k_{q} and $\alpha \in \mathbb{Z}$ is the additional side information of dimension k_{z}. The *side information* α is available at the encoder (receiver) but not at the decoder (transmitter). The encoding or the quantization process is denoted as $\widehat{\mathbf{v}} = \mathcal{Q}(\mathbf{v}, \alpha)$. The distortion introduced by a finite-rate quantizer is defined as $D = E_{\mathbf{h}}\Big[D_{\mathrm{Q}}\big(\mathbf{v}, \widehat{\mathbf{v}}\,;\,\alpha\big)\Big]$, where $D_{\mathrm{Q}}\big(\mathbf{v}, \widehat{\mathbf{v}}\,;\,\alpha\big)$ is a general *non-mean-squared distortion* function between $\mathbf{v}$ and $\widehat{\mathbf{v}}$ that is parameterized by α. It is further assumed that function D_{Q} has a continuous second order derivative $\mathbf{W}_\alpha(\mathbf{v})$, the sensitivity matrix, with the $(i, j)^{\text{th}}$ element given by

$$w_{i,j} = \frac{1}{2}\cdot\frac{\partial^2}{\partial\, v_i \partial\, v_j} D_{\mathrm{Q}}\big(\,\mathbf{v}\,, \widehat{\mathbf{v}}\,;\,\alpha\big)\Bigg|_{\mathbf{v}=\widehat{\mathbf{v}}}\,. \quad (1)$$

[1] We normalize the channel covariance matrix such that the mean of the eigen values equals to one (equal to the i.i.d. channel case $\mathbf{\Sigma}_{\mathrm{h}} = I_t$).

3.1. Asymptotic Distortion Bounds

Under high resolution assumption (large N), the asymptotic distortion of the generalized finite-rate quantization system can be lower bounded by the following form

$$D_{\text{Low}} = 2^{-\frac{2B}{k_q}} \left(\int_{\mathbb{Q}} \left(I^{\text{w}}_{\text{opt}}(\mathbf{v}) \cdot p(\mathbf{v}) \right)^{\frac{k_q}{2+k_q}} \, d\,\mathbf{v} \right)^{\frac{2+k_q}{k_q}}, \quad (2)$$

where $I^{\text{w}}_{\text{opt}}(\mathbf{v})$ is the average optimal inertial profile defined as

$$I^{\text{w}}_{\text{opt}}(\mathbf{v}) = \int_{\mathbb{Z}} I_{\text{opt}}(\mathbf{v}\,;\,\alpha) \cdot p(\alpha|\,\mathbf{v}) \; d\,\alpha. \quad (3)$$

The normalized inertial profile of an optimal quantizer is defined as the minimum inertia of all admissible Voronoi regions. The inertial profile of any Voronoi shape, including the optimal inertial profile, $I_{\text{opt}}(\mathbf{v}\,;\,\alpha)$, can be tightly lower bounded by that of an M-shaped hyper-ellipsoid

$$I_{\text{opt}}(\mathbf{v}\,;\,\alpha) \gtrapprox \frac{k_q}{k_q+2} \cdot \left(\frac{|\mathbf{W}_\alpha(\mathbf{v})|}{\kappa^2_{k_q}} \right)^{\frac{1}{k_q}}, \quad (4)$$

where $|\cdot|$ represents determinant and κ_n is the volume of an n-dimensional unit sphere. The above results are derived assuming the quantization parameter v is unconstrained. But this is not generally the case. For instance the beamforming vector has a norm constraint $\|\mathbf{v}\| = 1$, and a phase constraint $\measuredangle\langle \mathbf{v}, \widehat{\mathbf{v}} \rangle = 0$. We denote the constrained space as $\mathbf{g}(\mathbf{v}) = 0$. For the constrained source, the asymptotic distortion bounds presented above are still valid with the following modification. First, the degrees of freedom in $\mathbf{v}$ reduce from k_q to $k'_q = k_q - k_c$, with k_c equal to the number of constraints. Here $k_c = 2$, which leads to $k'_q = 2t-2$. Next, the sensitivity matrix is replaced by its constrained version $\mathbf{W}_{c,\,\alpha}(\mathbf{v})$ given by

$$\mathbf{W}_{c,\,\alpha}(\mathbf{v}) = \mathbf{V}_2^{\mathsf{T}} \cdot \mathbf{W}_\alpha(\mathbf{v}) \cdot \mathbf{V}_2 \;, \quad (5)$$

where $\mathbf{V}_2 \in \mathbb{R}^{k_q \times k'_q}$ is an orthonormal matrix with its columns constituting an orthonormal basis for the null space $\mathcal{N}\left(\frac{\partial}{\partial\,\mathbf{v}}\,\mathbf{g}(\mathbf{v})\right)$. Subsequently $I_{\text{opt}}(\mathbf{v}\,;\,\alpha)$ now becomes a lower bound on the constrained optimal inertial profile $I_{\text{c,opt}}(\mathbf{v}\,;\,\alpha)$. Lastly, the multi-dimensional integrations used in evaluating the average distortions are over the constrained space $\mathbf{g}(\mathbf{v}) = 0$.

4. AVERAGE SEP LOSS ANALYSIS

In this section we derive $D_{\text{Q}}(\mathbf{v}, \widehat{\mathbf{v}}\,;\,\alpha)$, the *non-mean-squared distortion* function for the average SEP loss of rectangular M-QAM constellation, design the optimum codebook matched to the distortion function and derive the expressions for the loss in average SEP under spatially i.i.d and spatially correlated channel conditions. In the last subsection, we consider the high-SNR regime for insight into the effect of quantization on a correlated channel .

4.1. Distortion Function - Average SER of M-QAM

In this subsection, we derive the appropriate *non-mean-squared distortion* function, $D_{\text{Q}}(\mathbf{v}, \widehat{\mathbf{v}}\,;\,\alpha)$, for a rectangular M-QAM constellation. The transmitting symbol $s_m = s_x + js_y, m = 0, 1, \ldots, M-1$, $x = 0, 1, \ldots, M_1 - 1$, $y = 0, 1, \ldots, M_2 - 1$, where the M-QAM constellation is of size $M = M_1 M_2$. Here $s_x = a_x d$, and $s_y = a_y d$, where $a_x = -(M_1 - 1) + 2x$ (i.e., $a_x d$ is the in-phase M_1-PAM constellation symbol) and $a_y = -(M_2 - 1) + 2y$ (i.e., $a_y d$ is the quadrature-phase M_2-PAM constellation symbol). Average SEP without channel quantization for the in-phase M_1-PAM is given by [5]

$$P_{_{PM_1}} = 2\left(1 - \frac{1}{M_1}\right) \cdot E\left[Q\left(\sqrt{\lambda \cdot \alpha}\right)\right], \quad (6)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-u^2/2)du$ and $\lambda = \rho \cdot \phi$, where $\phi = \frac{6}{M_1^2 + M_2^2 - 2}$. The average SEP for Quadrature M_2-PAM is given by (6), with M_1 replaced by M_2. The average SEP of M-QAM with perfect feedback is given by $P_{_{P-QAM}} = P_{_{PM_1}} + P_{_{PM_2}} - P_{_{PM_1}} \cdot P_{_{PM_2}}$. The average SEP with finite rate channel quantization for M_1-PAM is given by,

$$P_{_{QM_1}} = 2\left(1 - \frac{1}{M_1}\right) \cdot E\left[Q\left(\sqrt{\lambda \cdot \alpha \cdot |\langle \mathbf{v}, \widehat{\mathbf{v}} \rangle|^2}\right)\right]. \quad (7)$$

The average SEP for M_2-PAM with channel quantization is given by (7), with M_1 replaced by M_2. The average SEP of M-QAM with finite rate quantization is given by $P_{_{Q-QAM}} = P_{_{QM_1}} + P_{_{QM_2}} - P_{_{QM_1}} \cdot P_{_{QM_2}}$. The finite-rate quantization effect is the loss in average SEP, which is given by $P_{_{Loss}} = P_{_{P-QAM}} - P_{_{Q-QAM}}$. The instantaneous SEP loss due to finite-rate CSI quantization is taken to be the system distortion function $D_{\text{Q}}(\mathbf{v},\, \widehat{\mathbf{v}}\,;\,\alpha)$ given by (8) (shown on the next page). Under high resolution assumptions, the quantized beamforming vector $\widehat{\mathbf{v}}$ is close to $\mathbf{v}$, and the inner product $|\langle \mathbf{v}, \widehat{\mathbf{v}} \rangle|$ is close to one. In this case, the distortion function $D_{\text{Q}}(\mathbf{v}, \widehat{\mathbf{v}}\,;\,\alpha)$ can be approximated by taking the first order Taylor series expansion w.r.t. the random variable $|\langle \mathbf{v}, \widehat{\mathbf{v}} \rangle|^2$. After some simplification the distortion function can be written as (9) (shown on next page). In this paper, we only consider the case with same distance d in both in-phase and quadrature-phase. The analysis can be easily extended to the case where the distances are not the same.

4.2. Optimum Codebook Design for Rectangular M-QAM

The codebook has to be designed to minimize the SEP loss. The cost function for SEP loss given in (9) is different compared to the ergodic capacity loss employed in previous work. However, the general framework can be used with appropriate modification. The criteria in this case is to maximize the following mean squared weighted inner product (MSwIP)

$$\max_{\mathcal{Q}(.)} E|\langle \tilde{\alpha}\mathbf{v},\, \mathcal{Q}(\mathbf{h}) \rangle|^2, \quad \mathcal{Q}(\mathbf{h}) = \hat{\mathbf{v}}, \quad (15)$$

where $\tilde{\alpha}^2 = \exp\left(-\frac{\lambda\,\alpha}{2}\right) \cdot \sqrt{\frac{\lambda\,\alpha}{8\pi}} \cdot \left[A + 2C \cdot Q\left(\sqrt{\lambda\alpha}\right)\right]$. With this new design criterion, the two conditions of Lloyd algorithm, the nearest neighbor-hood condition and centroid condition, are iterated until convergence. More details on the algorithm design can be found in [1]. It should be noted that similar to the case of capacity loss, because of the form of the SEP loss function, the codebook designed for spatially i.i.d capacity loss function is also optimum for the i.i.d case of SEP distortion analysis. A drawback with the new codebook is that the codebook has to be designed for each operating SNR, constellation and the correlation matrix. In the next subsections we quantify the loss due to quantization under i.i.d and correlated scenarios, under the assumption that the appropriate optimum codebook is used.

4.3. Distortion Analysis for spatially i.i.d Channels

We make use of the asymptotic distortion bounds presented in section 3.1 and show the steps required in arriving at the loss in average SEP for the M-QAM constellation. The relevant distortion function, (9), was derived in the previous section. Due to space limitations, we only outline the steps and present the final results.

The lower bound on asymptotic distortion given by (2), requires the computation of constrained sensitivity matrix (5), lower bound on constrained normalized inertial profile of an optimal quantizer (4) and the weighted constrained inertial profile (3). After some simplification the constrained sensitivity matrix for the distortion function of SEP loss can be shown to be given by (10). For spatially i.i.d and correlated channels, the optimal inertial profile is obtained by substituting (10), the constrained sensitivity matrix, into the hyper-ellipsoidal approximation given by (4). The optimal constrained

$$D_{\mathrm{Q}}(\mathbf{v},\widehat{\mathbf{v}}\,;\,\alpha)\triangleq\left[Q\left(\sqrt{\lambda\cdot\alpha}\right)-Q\left(\sqrt{\lambda\cdot\alpha\cdot|\langle\mathbf{v},\widehat{\mathbf{v}}\rangle|^2}\right)\right]\cdot\left[A+C\cdot\left(Q\left(\sqrt{\lambda\cdot\alpha}\right)+Q\left(\sqrt{\lambda\cdot\alpha\cdot|\langle\mathbf{v},\widehat{\mathbf{v}}\rangle|^2}\right)\right)\right], \tag{8}$$

$$A=2\left(2-\frac{1}{M_1}-\frac{1}{M_2}\right),\quad C=-4\left(1-\frac{1}{M_1}-\frac{1}{M_2}+\frac{1}{M_1M_2}\right).$$

$$D_{\mathrm{Q}}(\mathbf{v},\,\widehat{\mathbf{v}}\,;\,\alpha)\approx\exp\left(-\frac{\lambda\,\alpha}{2}\right)\cdot\sqrt{\frac{\lambda\,\alpha}{8\pi}}\cdot\left[A+2C\cdot Q\left(\sqrt{\lambda\alpha}\right)\right]\cdot\left(1-|\langle\mathbf{v},\widehat{\mathbf{v}}\rangle|^2\right) \tag{9}$$

$$\mathbf{W}_{\mathrm{c},\,\alpha}(\mathbf{v})=\exp\left(-\frac{\lambda\,\alpha}{2}\right)\cdot\sqrt{\frac{\lambda\,\alpha}{8\pi}}\cdot\left[A+2C\cdot Q\left(\sqrt{\lambda\alpha}\right)\right]\cdot I_{2t-2} \tag{10}$$

$$I_{\mathrm{c,opt}}\,(\mathbf{v}\,;\,\alpha)=(t-1)\cdot\exp\left(\frac{\lambda\,\alpha}{2}\right)\cdot\gamma_t^{-\frac{1}{t-1}}\cdot\sqrt{\frac{\lambda\,\alpha}{t^2\cdot 8\pi}}\cdot\left[A+2C\cdot Q\left(\sqrt{\lambda\alpha}\right)\right] \tag{11}$$

$$I_{\mathrm{opt}}^{\mathrm{w}}(\mathbf{v})=D_{\mathcal{A}}\cdot A\cdot\Gamma\left(t+\frac{1}{2}\right)+D_{\mathcal{A}}\cdot 2C\cdot\int_0^\infty Q\left(\sqrt{\mu y}\right)\cdot\exp\left(-y\right)\cdot y^{t-\frac{1}{2}}\,dy, \tag{12}$$

$$\int_0^\infty Q\left(\sqrt{\mu y}\right)\cdot\exp\left(-y\right)\cdot y^{t-\frac{1}{2}}\,dy=\frac{\Gamma\left(t+\frac{1}{2}\right)}{\pi}\int\limits_{\theta=0}^{\pi/2}\left(\frac{\sin^2\theta}{\kappa+\sin^2\theta}\right)^{t+\frac{1}{2}}d\theta, \tag{13}$$

$$\int\limits_{\theta=0}^{\pi/2}\left(\frac{\sin^2\theta}{\kappa+\sin^2\theta}\right)^{t+\frac{1}{2}}d\theta=\frac{\sqrt{\kappa\pi}\,\cdot\Gamma\left(t+1\right)}{2\left(1+\kappa\right)^{t+1}\Gamma\left(t+\frac{3}{2}\right)}\,{}_2F_1\left(1,t+1;t+\frac{3}{2};\frac{1}{1+\kappa}\right). \tag{14}$$

inertial profile is given by (11). For spatially i.i.d channel, $\mathbf{h}\sim\mathcal{NC}(\mathbf{0},\,I_t)$, the random variable α has a pdf

$$p_\alpha(x)=p_{\alpha|\mathbf{v}}\,(x)=\frac{\exp(-x)\cdot x^{t-1}}{(t-1)!}\,,\qquad x\geq 0\ . \tag{16}$$

Using (16) and (11) in (3), $I_{\mathrm{c,opt}}\big(\mathbf{v}\,;\,\alpha\big)$, the weighted constrained inertial profile coefficient can be obtained. After some simplification an intermediate step in the derivation is given by (12), where

$$D_{\mathcal{A}}=\frac{\sqrt{\lambda}(t-1)\cdot\gamma_t^{-\frac{1}{t-1}}}{\sqrt{8\pi}t!\left(\frac{\lambda}{2}+1\right)^{\left(t+\frac{1}{2}\right)}},\qquad\mu=\frac{2\lambda}{\lambda+2},$$

and $\Gamma(n)=\int\limits_0^\infty e^{-u}u^{n-1}du$ is the standard Gamma function. We use $Q(x)=\frac{1}{\pi}\int\limits_{\theta=0}^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta,\ x\geq 0$, an alternative definition of Q function [5] to simplify the second term with integral in (12) and arrive at (13), where $\kappa=\mu/2$. We make use of [5, Eqn. (5.17)] to arrive at a closed form expression shown in (14) for the finite integral in (13), where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the hypergeometric function. By substituting the weighted constrained inertial profile coefficient (12) and $p(\mathbf{v})=1/\gamma_t$, where γ_t is the volume of t-dimensional sphere, into the distortion integral (2), the average SEP loss of an i.i.d. MISO system can be shown to be given by (17) (shown on the next page).

4.4. Distortion Analysis for Spatially Correlated Channels

All the steps until the derivation of constrained normalized inertial profile (11) are same for both spatially i.i.d and correlated channels. For correlated MISO fading channels $\mathbf{h}\sim\mathcal{NC}\big(\mathbf{0},\,\mathbf{\Sigma}_{\mathrm{h}}\big)$ with channel correlation matrix $\mathbf{\Sigma}_{\mathrm{h}}$ having distinct eigen-values [2], i.e. $\lambda_{\mathrm{h},1}>\cdots>\lambda_{\mathrm{h},\,t}>0$. The marginal probability density functions of $\mathbf{v}$ and conditional distribution of $\alpha|\mathbf{v}$ can be shown to have the following form [6]

$$p_{\,\mathbf{v}}\,(\mathbf{x})\;=\;\gamma_t^{-1}\cdot|\mathbf{\Sigma}_{\mathrm{h}}|^{-1}\cdot\left(\mathbf{x}^{\mathsf{H}}\mathbf{\Sigma}_{\mathrm{h}}^{-1}\,\mathbf{x}\right)^{-t}, \tag{20}$$

$$p_{\alpha|\mathbf{v}}\,(x)\;=\;\frac{x^{t-1}\cdot\left(\mathbf{v}^{\mathsf{H}}\mathbf{\Sigma}_{\mathrm{h}}^{-1}\,\mathbf{v}\right)^t\cdot\exp\left(-x\cdot\mathbf{v}^{\mathsf{H}}\mathbf{\Sigma}_{\mathrm{h}}^{-1}\,\mathbf{v}\right)}{(t-1)!} \tag{21}$$

[2]In this paper, we assume that the channel covariance matrix $\mathbf{\Sigma}_{\mathrm{h}}$ has distinct positive eigen-values. The result can be extended to any covariance matrix that is positive definite. If the channel covariance matrix is singular, the quantization should be carried out in a space with reduced dimension.

By substituting the conditional pdf $p_{\alpha|\mathbf{v}}\,(x)$ given by (21) and the constrained normalized inertial profile (11) into equation (3), the average inertial profile can be obtained. Using (20) and the averaged inertial profile in (2), the average SEP loss of an spatially correlated MISO system is given by (18) (shown on the next page). To simplify the derivation, we use the alternative representation of Q function for this case also.

4.5. Distortion Analysis in High-SNR Regime

The analytical expressions for SEP loss of M-ary rectangular QAM constellation for transmit beamforming of a MISO system are given by (17) and by (18) for spatially i.i.d and correlated cases. The equations are lengthy and complex providing limited insight into the system behavior. In high-SNR regime it is easy to see that $\kappa\approx 1$. For spatially i.i.d. MISO fading channels, the average distortion, $D_{\text{Q-H-SNR-iid}}$, under high-SNR assumption can be simplified into (19) shown in the next page. From (19) it is clear that the diversity order is 't' and increasing the number of feedback bits has an exponential impact on the system distortion function, notice that this fact is true even without the high-SNR assumption. The rest of the terms in (19) depend on the number of transmitting antennas and the size of the rectangular QAM constellation. For spatially correlated channel, the functions $\beta_1\big(t,\lambda,\mathbf{\Sigma}_{\mathrm{h}}\big)$ and $\beta_2\big(t,\lambda,\mathbf{\Sigma}_{\mathrm{h}}\big)$ are difficult to evaluate. However, we can evaluate them in closed form under high SNR assumption

$$\beta_{1-H-SNR}\big(t,\lambda,\mathbf{\Sigma}_{\mathrm{h}}\big)\;=\;\lambda^{-\left(t+\frac{1}{2}\right)}\cdot 2^{\left(t+\frac{1}{2}\right)}\cdot\gamma_t^{\frac{t}{t-1}}, \tag{22}$$

$$\beta_{2-H-SNR}\big(t,\lambda,\mathbf{\Sigma}_{\mathrm{h}}\big)\;=\;2\cdot\lambda^{-(t+1)}\cdot\gamma_t^{\frac{t}{t-1}}. \tag{23}$$

Making use of the above equations, after some manipulations we arrive at an interesting simple relation between the loss associated with spatially correlated and i.i.d cases

$$D_{\text{Q-H-SNR-iid}}=|\mathbf{\Sigma}_{\mathrm{h}}|\cdot D_{\text{Q-H-SNR-cor}}. \tag{24}$$

The above equation tells us that in the correlated case the loss is a simple scaling of the loss associated with i.i.d case, the scaling factor being the determinant of the correlation matrix. Note that this analysis is quite general in the sense that we can have an arbitrary correlation structure across the antennas. The quantization parameter B, and number of antennas, t, both appear in the exponent for the correlated scenario under general and high-SNR regimes. In the

$$D_{\text{Q-iid}} = \left[\frac{\sqrt{\lambda}\cdot(t-1)\cdot A\cdot 2^{t-1}\cdot\Gamma\left(t+\frac{1}{2}\right)}{\sqrt{\pi}\cdot t!\cdot(\lambda+2)^{\left(t+\frac{1}{2}\right)}} + \frac{\lambda\cdot(t-1)\cdot C\cdot\Gamma\left(t+\frac{1}{2}\right)}{4\pi\cdot(1+\lambda)^{t+1}\cdot\Gamma\left(t+\frac{3}{2}\right)}\cdot {}_2F_1\left(1,t+1;t+\frac{3}{2};\frac{1}{1+\kappa}\right)\right]\cdot 2^{-\frac{B}{t-1}}. \tag{17}$$

$$D_{\text{Q-cor}} = \left[\beta_1\big(t,\lambda,\mathbf{\Sigma}_{\text{h}}\big)\cdot T_{\mathcal{D}} + \beta_2\big(t,\lambda,\mathbf{\Sigma}_{\text{h}}\big)\cdot\sqrt{\frac{\lambda}{2}}\cdot T_{\mathcal{E}}\right]\gamma_t^{-1}\cdot|\mathbf{\Sigma}_{\text{h}}|^{-1}\cdot 2^{-\frac{B}{t-1}}, \tag{18}$$

$$\beta_1\big(t,\lambda,\mathbf{\Sigma}_{\text{h}}\big) = \left(\int_{\mathbf{v}:\,\mathbf{g}(\mathbf{v})=0}\left(\frac{\lambda}{2}+\mathbf{v}^{\mathsf{H}}\mathbf{\Sigma}_{\text{h}}^{-1}\mathbf{v}\right)^{\frac{(1-t)(t+\frac{1}{2})}{t}} d\,\mathbf{v}\right)^{\frac{t}{t-1}},$$

$$\beta_2\big(t,\lambda,\mathbf{\Sigma}_{\text{h}}\big) = \left(\int_{\mathbf{v}:\,\mathbf{g}(\mathbf{v})=0}\left(\lambda+\mathbf{v}^{\mathsf{H}}\mathbf{\Sigma}_{\text{h}}^{-1}\mathbf{v}\right)^{\frac{(1-t)(t+1)}{t}}\cdot {}_2F_1\left(1,t+1;t+\frac{3}{2};\frac{1}{1+\nu}\right)^{\frac{t-1}{t}} d\,\mathbf{v}\right)^{\frac{t}{t-1}},$$

$$T_{\mathcal{D}} = \frac{\sqrt{\lambda}(t-1)\cdot\gamma_t^{-\frac{1}{t-1}}\cdot A\cdot\Gamma\left(t+\frac{1}{2}\right)}{\sqrt{8\pi}\cdot t!},\quad T_{\mathcal{E}} = \frac{\sqrt{\lambda}(t-1)\cdot\gamma_t^{-\frac{1}{t-1}}\cdot C\cdot\Gamma(t+1)\cdot\Gamma\left(t+\frac{1}{2}\right)}{\Gamma\left(t+\frac{3}{2}\right)\sqrt{8\pi}\,t!},\quad \nu = \frac{\lambda}{\left(2\,\mathbf{v}^{\mathsf{H}}\mathbf{\Sigma}_{\text{h}}^{-1}\mathbf{v}+\lambda\right)}.$$

$$D_{\text{Q-H-SNR-iid}} = \left[\frac{2^{t-1}\cdot(t-1)\cdot A\cdot\Gamma\left(t+\frac{1}{2}\right)}{\sqrt{\pi}\cdot t!\cdot\phi^t} + \frac{(t-1)\cdot C\cdot\Gamma\left(t+\frac{1}{2}\right)}{4\pi\cdot\Gamma\left(t+\frac{3}{2}\right)\cdot\phi^t}\cdot {}_2F_1\left(1,t+1;t+\frac{3}{2};\frac{1}{2}\right)\right]\cdot 2^{-\frac{B}{t-1}}\cdot\rho^{-t}. \tag{19}$$

correlated scenario, the additional loss in average SEP due to quantization is independent of the constellation size. The diversity order is also not effected as a result of quantization. Further, it can be shown that the system performance in terms of SEP is more sensitive to the finite-rate channel quantization in the high-SNR regime.

5. NUMERICAL AND SIMULATION RESULTS

A sample simulation is shown in Fig. 1. It plots the average SEP loss due to the finite rate quantization of the CSI versus feedback rate B, for a 3×1 MISO system over spatially i.i.d. and correlated Rayleigh fading channels with different rectangular M-QAM constellations at system SNRs $\rho = 10$dB, and 24dB, respectively. Codebooks are designed by using optimal MSwIP criterion, suitable for average SEP loss analysis, as explained in section 4.2. The spatially correlated channel is simulated by the correlation model in [7]: A linear antenna array with antenna spacing of half wavelength, angle of arrival $\phi = 0^\circ$ and uniform angular-spread in $[-30^\circ, 30^\circ]$.

Fig. 1 shows the analytical and simulation plots for both spatially i.i.d and correlated channels. The analytical expression for i.i.d is closed form, and for correlated channel the expression is closed form under high SNR assumption. The simulation and analytical results match well as the number of feedback bits increase. The distortion function we have is a first order approximation and this approximation becomes accurate as the number of feedback bits increase. Also note that the analytical expression for distortion is not optimum but a lower bound on the optimum, which becomes more tight as the number of feedback bits increases.

6. CONCLUSION

In this paper, we studied the average SEP loss analysis of finite rate feedback MISO system with rectangular M-QAM utilizing a source coding perspective . We derived the distortion function as a 1st order approximation of the instant SEP loss and designed a new optimum codebook. This codebook depends on SNR point, constellation, and the correlation matrix. For Rayleigh fading channels, assuming perfect channel estimation, no-delay and high resolution, for spatially i.i.d and correlated channels we provided analytical expressions for loss in average SEP due to finite-rate channel quantization. We then considered the high-SNR regime and showed that the loss associated with correlated case is the loss associated with the i.i.d case scaled by the determinant of the correlation matrix. The simulation results are in agreement with the analytical expressions. The present analysis

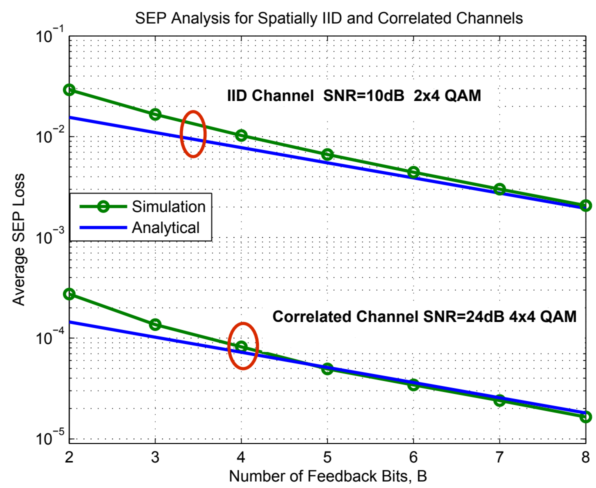

Fig. 1. Average SEP loss of M-QAM rectangular constellation.

framework can be extended to other two dimensional linear modulation schemes and also to study the effects of mismatched codebook.

7. REFERENCES

[1] J. C. Roh and B. D. Rao, "Transmit beamforming in multiple-antenna systems with finite rate feedback: A vq-vased approach," *IEEE Trans. Info. Theory,* vol. 52, no. 3, March 2006, pp. 1101-1112.

[2] S. Zhou, Z. Wang, and G. Giannakis, "Quantifying the power loss when transmit beamforming relies on finite-rate feedback," *IEEE Trans. Wireless Commun.,* vol. 4, no. 4, July 2005, pp. 1948-1957.

[3] J. Zheng, E. Duni, and B. D. Rao, " Analysis of multiple antenna systems with finite-rate feedback using high resolution quantization theory," *IEEE Trans. Signal. Proc,* to appear, available: dsp.ucsd.edu/∼jzheng.

[4] D. J. Love and R. W. Heath, Jr., "Limited Feedback Diversity Techniques for Correlated Channels," *IEEE Trans. on Veh. Tech.,* vol. 55, no. 2, March 2006, pp. 718-722.

[5] M. K. Simon and M.-S. Alouini, *Digital Communications Over Fading Channels: A Unified Approach to Performance Analysis,* Wiley Series, July 2000.

[6] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*, Wiley, New York, 1982.

[7] J. Salz and J. H. Winters, "Effect of fading correlation on adaptive arrays in digital mobile radio", *IEEE Trans. Vehic. Technology,*, vol. 43, no. 4, Nov. 1994, pp. 1049-1057.