LONG-TERM TRANSMIT BEAMFORMING FOR WIRELESS MULTICASTING

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ABSTRACT

This paper presents a low-complexity iterative algorithm for long-term transmit beamforming in multicast channels. Since it relies only on antenna correlations at the base station, the algorithm requires only infrequent feedback from the users. Multiantenna receivers are not required, but they are transparently accommodated. Simulation results within the context of the UMTS long-term evolution system indicate that, with half-wavelength antenna spacings and a typical power azimuth spectrum, the performance is close to optimal. The specific gains (in average SINR) then depend on the number of transmit antennas and the number of active users.

***Index Terms*—** Array signal processing, adaptive arrays, beam steering, multicast channels, mobile communication

1. INTRODUCTION

In a multicast channel, a transmitter communicates a common signal (video, audio, data, etc) to multiple receivers. In the wireless arena in particular, multicasting is shaping up as a central feature of nascent cellular systems such as UMTS LTE (long-term evolution) [1] and 1xEV-DO Revision C. Since another ubiquitous feature of emerging wireless systems is the availability of antenna arrays at the base stations, it is of obvious interest to find ways to utilize those arrays for the multicast transmissions [2].

This paper focuses on transmit beamforming (i.e., unit-rank transmit signal covariance), which accommodates but does not require multiantenna receivers and has low complexity requirements.

In contrast with individual unicast channels, where instantaneous CSI (channel state information) fast-feedback mechanisms exist, multicast channels are mostly one-directional. Usually, only long-term CSI reports can be conveyed back to the base station. Accordingly, our focus is on long-term transmit beamforming that operates on the basis of correlation information only. (Excellent treatments of the case where instantaneous CSI is assumed available at the base can be found in [3, 4].)

2. PROBLEM FORMULATION

Consider a base station, equipped with $n_{\rm T}$ transmit antennas, and K users, each featuring $n_{\rm R} \geq 1$ receive antennas. Denote by $\{\mathbf{H}_\ell\}_{\ell=1}^K$ the $n_{\rm R} \times n_{\rm T}$ channel matrices between the base station and each of the users. With a beamformed multicast transmission, the signal vector received at the ℓth user is

$$\mathbf{y}_\ell = \mathbf{H}_\ell \mathbf{w} x + \mathbf{n}_\ell \tag{1}$$

where x is the scalar transmit signal, with power $E[|x|^2] = P$, and $\mathbf{n}_\ell$ is the noise-plus-interference vector at the ℓth user, with covariance $\mathbf{\Sigma}_\ell = E[\mathbf{n}_\ell \mathbf{n}_\ell^\dagger]$. In turn, $\mathbf{w}$ is a common beamforming vector satisfying $\|\mathbf{w}\| = 1$.

The base station has only knowledge of the $n_{\rm T} \times n_{\rm T}$ correlation matrices

$$\mathbf{\Theta}_\ell = E\left[\mathbf{H}_\ell^\dagger \mathbf{\Sigma}_\ell^{-1} \mathbf{H}_\ell\right] \qquad \ell = 1, \ldots, K \tag{2}$$

each of which is estimated by the corresponding receiver and reported back to the base. Unlike $\{\mathbf{H}_\ell\}_{\ell=1}^K$, which are subject to fast fading, $\{\mathbf{\Theta}_\ell\}_{\ell=1}^K$ vary on a slow time scale and thus need to be reported back only infrequently.[1]

Our goal is to optimize $\mathbf{w}$ on the basis of only $\{\mathbf{\Theta}_\ell\}_{\ell=1}^K$. The optimization cannot therefore be driven by the instantaneous SINR (signal-to-interference-and-noise ratio) values, but only by their long-term averages. For the ℓth user, such average SINR is [5]

$$\begin{aligned} \bar{\gamma}_\ell &= E\left[\left\|\mathbf{\Sigma}_\ell^{-1/2} \mathbf{H}_\ell \mathbf{w} x\right\|^2\right] & (3)\\ &= P\, E\left[\mathbf{w}^\dagger \mathbf{H}_\ell^\dagger \mathbf{\Sigma}_\ell^{-1} \mathbf{H}_\ell \mathbf{w}\right] & (4)\\ &= P\, \mathbf{w}^\dagger \mathbf{\Theta}_\ell \mathbf{w}. & (5) \end{aligned}$$

Since, in order for a data rate to be sustainable in a multicast channel, it has to be achievable by all active users, the

[1] In OFDM, the scheme of choice for most emerging wideband wireless systems, each $\mathbf{\Theta}_\ell$ is further common to all tones and hence single long-term CSI reports by every user suffice. In contrast, every $\mathbf{H}_\ell$ may vary from tone to tone. Thus, a single long-term beamformer suffices while many distinct instantaneous beamformers could be required within the signal bandwidth.

optimum long-term beamformer is

$$\mathbf{w}^\star = \arg \max_{\mathbf{w}:\|\mathbf{w}\|=1} \min_{\ell=1,\ldots,K} P\,\mathbf{w}^\dagger \boldsymbol{\Theta}_\ell \mathbf{w}. \tag{6}$$

where, alas, the function $\min_\ell P\,\mathbf{w}^\dagger \boldsymbol{\Theta}_\ell \mathbf{w}$ is non-convex. Note that $\mathbf{w}^\star$ is not simply the vector that maximizes the average SINR for the user that, absent beamforming, would have the lowest such average. This would ignore how such beamforming vector impacts all other users. Not only is that solution suboptimal, but it may lead to a loss rather than a gain.

3. SEMIDEFINITE RELAXATION

Defining $\boldsymbol{\Phi} = P\mathbf{w}\mathbf{w}^\dagger$ as the unit-rank transmit covariance, (6) can be reformulated as

$$\boldsymbol{\Phi}^\star = \arg \max_{\substack{\boldsymbol{\Phi}:\mathrm{Tr}\{\boldsymbol{\Phi}\}=P \\ \boldsymbol{\Phi}\succeq 0, \mathrm{rank}\{\boldsymbol{\Phi}\}=1}} \min_{\ell=1,\ldots,K} \mathrm{Tr}\{\boldsymbol{\Phi}\boldsymbol{\Theta}_\ell\} \tag{7}$$

where the maximization is taken over all unit-rank positive-semidefinite matrices with trace equal to P. In its form in (7), the problem at hand is not only again seen to be non-convex (in this case as a result of the the unit-rank constraint), but it can further be shown to be NP-hard [4].[2]

An upper bound on $\min_\ell \mathrm{Tr}\{\boldsymbol{\Phi}\boldsymbol{\Theta}_\ell\}$ can be obtained by dropping the unit-rank constraint. Thus relaxed, (7) becomes a convex semidefinite problem that can be efficiently solved using modern interior point methods [6, Chapter 11]. Although, in general, the relaxed problem yields a covariance $\boldsymbol{\Phi}$ that is not unit rank, it provides a useful performance upper bound and it can serve as starting point for certain search methods developed in the optimization literature. (This is precisely the approach followed in [4] for the case of instantaneous beamforming.) Rather than going down that path, though, in the next section we present an iterative algorithm motivated purely by engineering intuition. By its very nature, this algorithm is well suited to dynamic implementations in the time-varying conditions of mobile wireless systems.

4. GRADIENT-PROJECTION LONG-TERM BEAMFORMING ALGORITHM

The idea behind the proposed algorithm is to, at every iteration, slowly steer $\mathbf{w}$ towards the beamforming vector that would maximize the average SINR of the worst user (in an average SINR sense) at that point. As this is done, the average SINR of that worst user rises until a different user becomes the worst, at which point $\mathbf{w}$ is steered towards the beamforming vector for this new worst user, and so on.

From (5), the gradient of the ℓth user's average SINR with respect to $\mathbf{w}$ can be found to be

$$\nabla_{\mathbf{w}}\, \bar{\gamma}_\ell = \boldsymbol{\Theta}_\ell \mathbf{w} \tag{8}$$

[2]Although the proof of NP-hardness given in [4] is for a beamformer driven by instantaneous SINR values, the argumentation carries over to (7).

Hence, denoting by $\mathbf{w}^{(t)}$ the value at iteration t, our basic update equation is

$$\begin{aligned} \mathbf{w}^{(t+1)} &= \mathbf{w}^{(t)} + \mu\, \boldsymbol{\Theta}_m \mathbf{w}^{(t)} && (9) \\ &= (\mathbf{I} + \mu\, \boldsymbol{\Theta}_m)\, \mathbf{w}^{(t)} && (10) \end{aligned}$$

where μ is a step-size parameter and $m = \arg\min_\ell \boldsymbol{\Theta}_\ell$ is the index of the worst user at iteration t. After each update, the new $\mathbf{w}^{(t+1)}$ is projected back onto the admissible set via

$$\mathbf{w}^{(t+1)} = \frac{\mathbf{w}^{(t+1)}}{\|\mathbf{w}^{(t+1)}\|} \tag{11}$$

The algorithm starts by initializing $\mathbf{w}^{(0)}$ and, once in steady state, it can track the changes in average SINR's caused by user motion as long as μ is properly chosen. A complete flowchart of the algorithm is shown in Fig. 1.

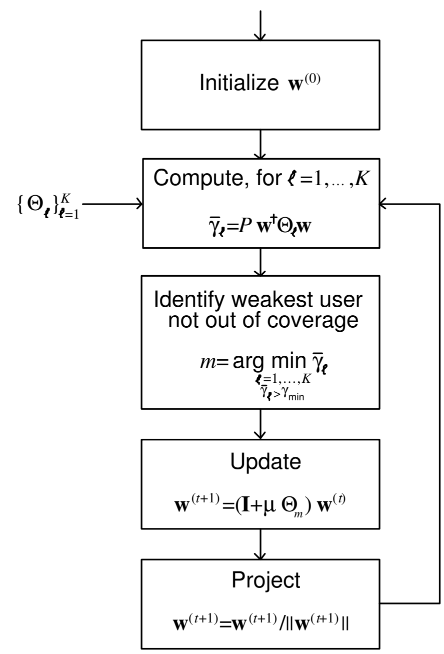

Fig. 1. Algorithm flowchart.

Note that the selection of the worst user (index m) considers only users with $\bar{\gamma}_\ell > \gamma_{\min}$. This refinement recognizes the fact that users with a very low average SINR would represent an excessive burden to the system and are hence deemed out of coverage. The algorithm thus seeks to maximize the minimum average SINR among those users not out of coverage.

The two parameters in the algorithm are (*i*) the step size μ, determined primarily by the degree of user mobility, and (*ii*) the threshold $\gamma_{\min}$, which should be adjusted on the basis of the target coverage level and of the number of users, K.

Although the algorithm is not guaranteed to converge to the optimal beamforming solution because of the non-convexity of the problem, the results in the next section illustrate its excellent performance.

Table 1. UMTS LTE Simulation Parameters

Layout	Hexagonal grid
Frequency reuse	Universal
Base station separation	1.732 Km
Sectors per cell	3
Sector antenna pattern	See Fig. 2
P	43 dBm
Path-loss v. distance (r in Km)	$128.1 + 37.6 \log_{10} r$ dB
Shadowing	Log-normal, 8-dB std
Shadowing correlation between base stations	50%
Receiver noise figure	9 dB
Bandwidth	5 MHz

5. APPLICATION TO UMTS LTE

5.1. Simulation Methodology

To exemplify the performance of the proposed algorithm, we simulate its behavior within the context of the UMTS LTE system. The relevant parameters of the simulation methodology specified in [1] for UMTS LTE evaluation are summarized in Table 1. Average SINR statistics are computed over a large number of drops, on each of which users are randomly placed and connected to the base from which they receive the strongest signal. Once K users are connected to every sector, the algorithm is initialized at $\mathbf{w}^{(0)} = [1\ 0\ \ldots\ 0]^T$ and run until convergence. There is no user motion and thus the step-size μ is consequential only in terms of the granularity noise in the converged solution.

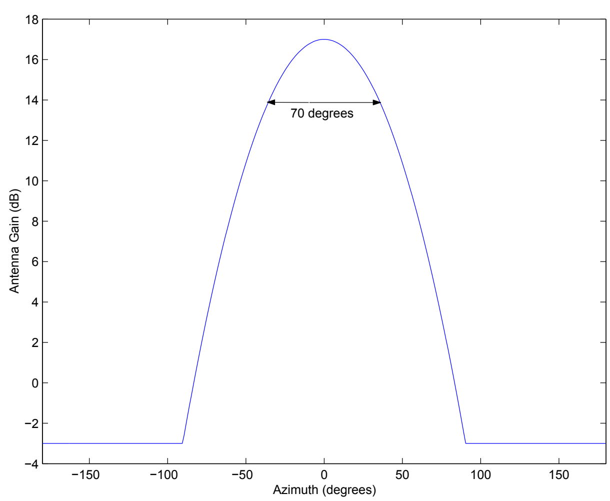

Fig. 2. Sector antenna pattern (17-dB gain, 70° at 14 dB).

5.2. Antenna Correlations

The feature of the wireless channel that most critically impacts a long-term beamformer is the correlation between the transmit antennas at the base station [3]. Denoting by $(\cdot)_{p,q}$ the (p,q)th entry of a matrix, the correlation coefficient between transmit antennas j and k from the standpoint of the ith receive antenna at the ℓth user is

$$\frac{E\left[(\mathbf{H}_\ell)_{i,j}(\mathbf{H}_\ell)^*_{i,k}\right]}{\sqrt{E\left[|(\mathbf{H}_\ell)_{i,j}|^2\right] E\left[|(\mathbf{H}_\ell)_{i,k}|^2\right]}} = \int_{-\pi}^{\pi} A_\ell(\theta) e^{\mathrm{j}2\pi d_{j,k}\sin(\theta)} d\theta \tag{12}$$

where $A_\ell(\theta)$ is the PAS (power azimuth spectrum) and $d_{j,k}$ is the spacing between antennas j and k in wavelengths. Since it relies thereupon, long-term beamforming requires strong antenna correlations, which in turn requires small antenna spacings and narrow angular spreads at the base station. In accordance with [7], for the simulations that follow we consider half-wavelength antenna spacings and a Laplacian PAS

$$A_\ell(\theta) = \frac{e^{-\sqrt{2}|\theta-\bar{\theta}_\ell|/\sigma} G(\theta)}{\int_{-\pi}^{\pi} e^{-\sqrt{2}|\theta-\bar{\theta}_\ell|/\sigma} G(\theta) d\theta} \tag{13}$$

where $G(\theta)$ is the antenna pattern in Fig. 2 while $\bar{\theta}_\ell$ is the azimuth location of the ℓth user relative to the array normal and σ is the angular spread, set to 8°. (This is a realistic value for elevated base stations in urban and sub-urban environments, and a conservative one for rural environments [7].)

From (12), (13), and the user locations and shadowings, the entries of $\{\mathbf{\Theta}_\ell\}_{\ell=1}^K$ are generated for every simulated drop. In the actual system, the ℓth user would estimate $\mathbf{\Theta}_\ell$ by low-pass filtering $\|\mathbf{\Sigma}_\ell^{-1/2}\mathbf{y}_\ell\|^2$ upon transmission of pilot signals $\mathbf{x}$ with covariance $E[\mathbf{x}\mathbf{x}^\dagger] = \frac{P}{n_\mathrm{T}}\mathbf{I}$ in lieu of $\mathbf{w}x$ in (1). In the simulator, perfect estimation of $\{\mathbf{\Theta}_\ell\}_{\ell=1}^K$ is considered.

For $n_\mathrm{R} > 1$, receive antenna correlations affect $\{\bar{\gamma}\}_{\ell=1}^k$ but are immaterial to the algorithm.

5.3. Results

In Fig. 3, the cumulative distributions of average SINR over a large number of drops are depicted in solid lines, parameterized by K, for $n_\mathrm{T} = 4$ and $n_\mathrm{R} = 1$. These distributions give the complement of the coverage level, i.e., they give the fraction of system locations in which the average SINR in less than a certain value. The target coverage for the algorithm was set to 90% and thus $\gamma_{\min}$ has been optimized, for every value of K, to yield the highest average SINR at that level. Also shown in Fig. 3, in dashed, is the distribution of average SINR in the absence of beamforming (i.e., with $n_\mathrm{T} = 1$) for arbitrary K. Notice that, for $K = 1$, the beamforming distribution is a replica of the non-beamforming one only shifted by 5.5 dB, which is the single-user beamforming gain with $n_\mathrm{T} = 4$ in an 8°-degree Laplacian PAS.

Focusing on the 90% coverage level, Fig. 4 presents the gains in average SINR with $n_\mathrm{T} = 2$, $n_\mathrm{T} = 4$, and $n_\mathrm{T} =$

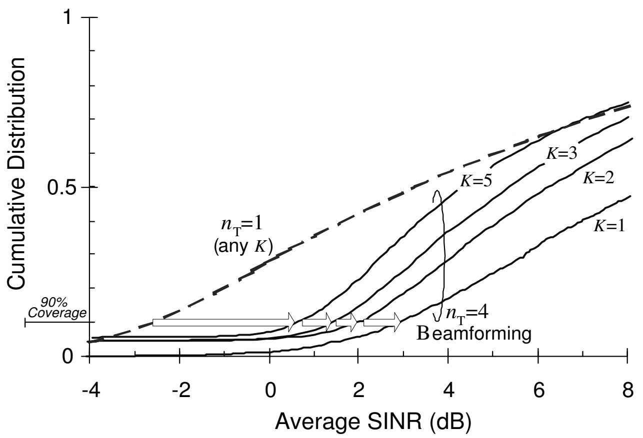

Fig. 3. Cumulative distributions of average SINR for the K active users. In dashed, no beamforming ($n_{\mathrm{T}} = 1$). In solid, beamforming with $n_{\mathrm{T}} = 4$ for $K = 1, 2, 3$ and 5.

8, as function of K. Also shown, for each configuration, is the upper bound obtained by solving the relaxed semidefinite problem in (7). We observe that:

(*i*) The algorithm performs close to the (in general unachievable) relaxation upper bound. This excellent performance with an arbitrary initialization suggests that, with the correlations of interest and with the exclusion of the 5% worst locations, the problem is close to being convex.

(*ii*) The gain in average SINR diminishes with K, as is to be expected, but relatively slowly. (With an isotropic user population, the gain is sure to vanish for $K \to \infty$.)

(*iii*) With growing n_{T}, the gain saturates as the beam narrows and becomes comparable to the angular spread.

For n_{T} in the range 4–8, which are values in line with those in emerging systems, and for the 8° Laplacian PAS typical of the corresponding deployment scenarios, average SINR gains in the range 3–7.5 dB are feasible for $K < 10$. (Note that K is not the total number of users connected to a sector, but only the number of users per sector that are actively tuned onto a specific multicast program.) These gains, in turn, would allow for sizeable increases in the multicast data rates.

An alternative embodiment of the algorithm, driven by the ergodic mutual information $E[\mathcal{I}]$ rather than the average SINR, can be implemented by simply replacing the gradient in (8) by [8]

$$\nabla_{\mathbf{w}} E[\mathcal{I}_\ell] = E\left[e_\ell \, \mathbf{H}_\ell^\dagger \boldsymbol{\Sigma}_\ell^{-1/2} \mathbf{H}_\ell \right] \mathbf{w} \tag{14}$$

where e_ℓ is the MMSE in the estimation of the transmit signal x from the observation of $\mathbf{y}_\ell$. In this case, the feedback required from each user would be the expected quantity in (14).

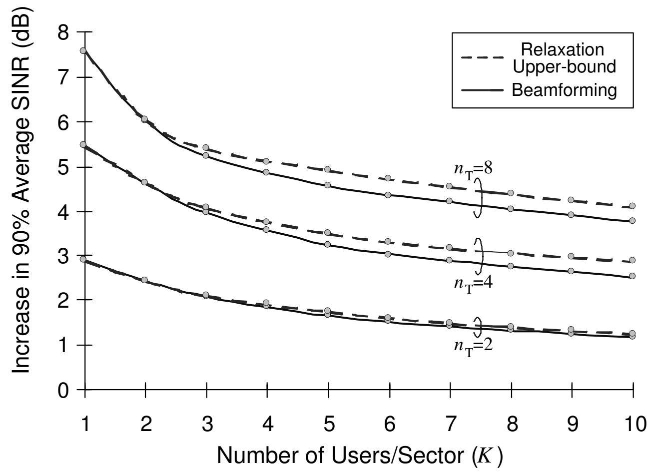

Fig. 4. Increase in average SINR achieved on 90% of the locations with long-term beamforming ($n_{\mathrm{T}} = 2$, 4 and 8) with respect to non-beamforming ($n_{\mathrm{T}} = 1$). Also shown are the respective upper bounds obtained by solving (7).

6. REFERENCES

[1] 3rd Generation Partnership Project (3GPP), *TR 25.814 V1.2.3, Physical Layer Aspects for Evolved UTRA*, 2006.

[2] N. Jindal and Z.-Q. Luo, "Capacity limits of multiple antenna multicast," *IEEE Int'l Symp. on Inform. Theory (ISIT'06)*, July 2006.

[3] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, "Efficient use of side information in multiple-antenna data transmission over fading channels," *IEEE J. Sel. Areas in Comm.*, vol. 16, no. 8, pp. 1423–1436, Oct. 1998.

[4] N. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. on Signal Processing*, vol. 54, no. 6, pp. 2239–2250, June 2006.

[5] A. Lozano, A. M. Tulino, and S. Verdú, "Multiple-antenna capacity in the low-power regime," *IEEE Trans. on Inform. Theory*, vol. 49, pp. 2527–2544, Oct. 2003.

[6] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge Univ. Press, 2004.

[7] 3rd Generation Partnership Project (3GPP), *TR 25.996 V6.1.0, Spatial Channel Model for Multiple-Input Multiple-Output (MIMO) Simulations*, Sept. 2003.

[8] D. P. Palomar and S. Verdu, "Gradient of mutual information in linear vector Gaussian channels," *IEEE Trans. Inform. Theory*, vol. 52, no. 1, pp. 141–154, Jan. 2006.