CAPACITY AND ERROR PROBABILITY ANALYSIS OF NON-COHERENT MIMO SYSTEMS IN THE LOW SNR REGIME

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ABSTRACT

We investigate the non-coherent single-user MIMO channel in the low signal-to-noise (SNR) regime from two viewpoints: capacity and probability of error analysis. The novelty in both viewpoints is that an arbitrary correlation structure is allowed for the Gaussian observation noise. First, we look at the capacity of the spatially correlated Rayleigh fading channel. We investigate the impact of channel and noise correlation on the mutual information for the on-off and Gaussian signaling schemes. Our results establish that, in the low SNR regime, mutual information is maximized when the transmit antennas are fully correlated (the same holds for the receive array). Then, we consider the deterministic channel setup and perform a pairwise error probability (PEP) analysis for the GLRT receiver. This leads to a codebook design criterion on which we base the construction of new space-time constellations. Their performance is assessed by computer simulations and, as a byproduct, we show that our codebooks are also of interest for Bayesian receivers which decode constellations with non-uniform priors.

***Index Terms*—** MIMO systems, non-coherent communications, channel capacity, generalized likelihood ratio test (GLRT) receiver, colored noise

1. INTRODUCTION

In slowly fading scenarios, channel stability enables the receiver to be trained in order to acquire the channel state information (CSI) necessary for *coherent detection* of the transmitted codeword. The scope of this paper will be fast fading scenarios, where the channel coefficients fluctuate too rapidly to allow reliable channel estimation. Hence, CSI is no more accessible, and the receiver must then operate in a *non-coherent* mode. Also, we focus on the low signal-to-noise ratio (SNR) regime as in the third-generation mobile data systems almost 40% of geographical locations experience receiver SNR levels below 0 dB while only less than 10% display levels above 10 dB. This stems from the fact that the power limitations in the mobile device make the high SNR requirement difficult to observe. See [1, 2] for a more thorough discussion of this topic.

Previous work. Low SNR MIMO systems when CSI is available at the receiver have been treated in [1]. The interplay of rate, bandwidth, and power is analyzed in the region of energy per bit close to its minimum value. The scenario where no CSI is available at the receiver has been considered in [3]. It has been shown that the optimal signaling at low SNR achieves the same capacity as the known channel case for single transmit antenna systems. Verdu, in [4], has shown that knowledge of the first and second derivatives of capacity at low SNR gives us insight on bandwidth and energy efficiency for signal transmission. More precisely, these quantities tell us how spectral efficiently grows with energy-per-bit. In [5], a formula for the second-order expansion of the input-output mutual information at low SNR is obtained, whereas in [6] the capacity and the reliability function as the peak constraint tends to zero are considered for a discrete-time memoryless channel with peak constrained inputs. Similar results to [5, 6] have been obtained in [7] under weaker assumptions on the input signals. In the same work, Rao and Hassibi have demonstrated that the on-off signaling presented in [3] generalizes to the multi-antenna setting and attains the known channel capacity. The tradeoff between communication rate and average probability of decoding error using a framework of error-exponent theory has been investigated in [8]. It is argued that the advantage of having multiple antennas is best realized when the fading is fully correlated, i.e., a performance gain of MN and a peakiness gain of M^2NT can be achieved where M, N and T represent the number of transmit, receive antennas, and the length of the coherence interval, respectively. Recently, in [9], contrary to most approaches for the low SNR regime, the channel matrix is assumed deterministic, i.e., no stochastic model is attached to it. A low SNR analysis of the pairwise error probability (PEP) for the single transmit antenna is introduced. It has been shown that the problem of finding good codes corresponds to the very well known packing problem in the complex projective space. Some good packings were provided and it was demonstrated that these constellations perform substantially better than state-of-art known solutions which assume equal prior probabilities, and are also of interest for the constellations with unequal priors.

Contributions and paper organization. We study the non-coherent MIMO channel in the low SNR regime from the capacity and PEP viewpoints. The novel aspect is that we allow the Gaussian observation noise to have an arbitrary correlation structure, albeit known to the transmitter and the receiver. In section 2, the spatially correlated non-coherent MIMO block Rayleigh fading channel is analyzed. This extends the approach in [7] as we take into account both channel and noise correlation. The impact of channel correlation on the mutual information is obtained for the on-off and Gaussian signaling. The main conclusion is that mutual information is maximized when both the transmit and receive antennas are fully correlated. We also argue that the on-off signaling is optimal for this multi-antenna setting. In section 3, the channel matrix is assumed deterministic and a PEP analysis in the low SNR regime for the GLRT receiver is introduced. We obtain a codebook design criterion

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which is used to construct new space-time constellations for some particular wireless scenarios. Computer simulations show that these new codebooks are also of interest for Bayesian receivers which decode constellations with non-uniform priors. Section 4 contains the main conclusions of our paper.

2. RANDOM FADING CHANNEL: THE LOW SNR MUTUAL INFORMATION ANALYSIS

Data model and assumptions. The communication system comprises M transmit and N receive antennas and we assume a block fading channel model, which is widely used in the MIMO literature [7, 8], with coherence interval T. In complex base band notation we have the system model

$$\boldsymbol{X} = \boldsymbol{S}\boldsymbol{H} + \boldsymbol{E}, \tag{1}$$

where $\boldsymbol{S}$ is the $T \times M$ matrix of transmitted symbols, $\boldsymbol{X}$ is the $T \times N$ matrix of received symbols, $\boldsymbol{H}$ is the $M \times N$ channel matrix, and $\boldsymbol{E}$ is the $T \times N$ matrix of zero-mean additive observation noise. We work under the following assumptions: (**A1**) The popular *separable spatial correlation* model [1] is used, i.e., $\boldsymbol{H} = \sqrt{\frac{\rho}{M}}\boldsymbol{K}_t^{\frac{1}{2}}\boldsymbol{H}_w\left(\boldsymbol{K}_r^T\right)^{\frac{1}{2}}$ where $\boldsymbol{H}_w$ is a $M \times N$ matrix comprised of zero-mean and unit variance circularly symmetric complex Gaussian entries. The correlation coefficients between the M (N) transmit (receive) antennas are assembled into an $M \times M$ ($N \times N$) correlation matrix $\boldsymbol{K}_t$ ($\boldsymbol{K}_r^T$). The matrix $\boldsymbol{H}_w$ is not known at the receiver neither at the transmitter, but its distribution is, in addition to $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$. Also, we assume that $\text{tr}(\boldsymbol{K}_t) = M$ and $\text{tr}(\boldsymbol{K}_r) = N$; (**A2**) We impose the power constraint $\mathsf{E}[\text{tr}(\boldsymbol{S}^H\boldsymbol{S})] \leq TM$ for each transmitted symbol where $\mathsf{E}[\cdot]$ represents the expectation operator; (**A3**) The noise covariance matrix $\boldsymbol{\Upsilon} = \mathsf{E}[\text{vec}(\boldsymbol{E})\ \text{vec}(\boldsymbol{E})^H]$ is known at the transmitter and at the receiver (vec($\boldsymbol{E}$) stacks all columns of the matrix $\boldsymbol{E}$ on the top of each other, from left to right). Also, without loss of generality (w.l.o.g.), we assume $\text{tr}(\boldsymbol{\Upsilon}) = NT$. Note that in (**A3**), we let the data model depart from the customary assumption of spatio-temporal white Gaussian observation noise, which is clearly an approximation. In realistic scenarios the $\boldsymbol{E}$ term often exhibits a very rich correlation structure, e.g, see [1] and pp. 10,159,171 in [10]. The generalization to arbitrary noise covariance matrices $\boldsymbol{\Upsilon}$ encompasses many scenarios of interest as special cases: spatially colored or not jointly with temporally colored or not observation noise, multiuser environment, etc.

2.1. Mutual information: on-off signaling

The on-off signaling is defined as: for any $\epsilon > 1$, $\boldsymbol{S} = \boldsymbol{S}_{on}\rho^{-\frac{\epsilon}{2}}$ with probability (w.p.) ρ^{ϵ}; $\boldsymbol{S} = \boldsymbol{0}$ w.p. $1 - \rho^{\epsilon}$. In [7], it has been demonstrated that the on-off signaling presented in [3] generalizes to the multi-antenna setting and attains the known channel capacity. Here, we show that this is also the case for the correlated Rayleigh fading channel model with arbitrary noise covariance matrix. We maximize the mutual information with respect to (w.r.t.) the input signal $\boldsymbol{S}_{on}$, $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$. Thus, we view both $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$ as system parameters which we can control, e.g., by changing the antenna separation. At sufficiently low SNR, the mutual information between $\boldsymbol{X}$ and $\boldsymbol{S}$ up to first order in ρ is given by

$$I(\boldsymbol{X};\boldsymbol{S}) = \frac{\rho}{M}\,\text{tr}\left(\boldsymbol{\Upsilon}^{-1}\left(\boldsymbol{K}_r \otimes \boldsymbol{S}_{on}\boldsymbol{K}_t\boldsymbol{S}_{on}^H\right)\right) + o(\rho), \tag{2}$$

where $\otimes$ denotes Kronecker product. The proof of (2) is omitted due to paper length constraints. Now, we address the maximization of the mutual information w.r.t. $\boldsymbol{S}_{on}$, $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$, i.e.,

$$\max_{\substack{\|\boldsymbol{S}_{on}\|^2 \leq TM \\ \boldsymbol{K}_t \in \mathcal{P}_M \\ \boldsymbol{K}_r \in \mathcal{P}_N}} \text{tr}\left(\boldsymbol{\Upsilon}^{-1}\left(\boldsymbol{K}_r \otimes \boldsymbol{S}_{on}\boldsymbol{K}_t\boldsymbol{S}_{on}^H\right)\right) \tag{3}$$

where $\mathcal{P}_n = \{\boldsymbol{X} : n \times n \text{ such that } \boldsymbol{X} \succeq \boldsymbol{0} \text{ and tr}\,(\boldsymbol{X}) = n\}$. It can be shown (proof omitted) that the maximum in (3) is attained by

$$\widehat{\boldsymbol{S}}_{on} = \sqrt{TM}\begin{bmatrix}\hat{\boldsymbol{s}} & \boldsymbol{0}_{T\times(M-1)}\end{bmatrix},\ \widehat{\boldsymbol{K}}_r = N\hat{\boldsymbol{u}}\hat{\boldsymbol{u}}^H,\ \widehat{\boldsymbol{K}}_t(i,i) = M\delta_{i1} \tag{4}$$

where

$$(\hat{\boldsymbol{u}}, \hat{\boldsymbol{s}}) = \underset{\substack{\boldsymbol{u} \in \mathbb{C}^N, ||\boldsymbol{u}|| = 1 \\ \boldsymbol{s} \in \mathbb{C}^T, ||\boldsymbol{s}|| = 1}}{\arg\max} \left(\boldsymbol{u} \otimes \boldsymbol{s}\right)^H \boldsymbol{\Upsilon}^{-1}\left(\boldsymbol{u} \otimes \boldsymbol{s}\right) \tag{5}$$

with $\delta_{ij} = 1$ for $i = j$ and zero otherwise. Note that $\widehat{\boldsymbol{K}}_t$ is a diagonal matrix. The optimization problem in (5) always admit a solution (maximization of a continuous function over a compact set) but, in general, a closed form solution is not available. This would be the case if $\boldsymbol{\Upsilon}$ had a Kronecker structure, say $\boldsymbol{\Upsilon} = \boldsymbol{\Upsilon}_1 \otimes \boldsymbol{\Upsilon}_2$. In that situation, the optimal $\hat{\boldsymbol{u}}$ (resp. $\hat{\boldsymbol{s}}$) can be taken as any unit-norm eigenvector associated with the minimal eigenvalue of $\boldsymbol{\Upsilon}_1$ (resp. $\boldsymbol{\Upsilon}_2$). For the choice in (4), the maximal mutual information (per channel use) is equal to

$$\frac{1}{T}I(\boldsymbol{X};\boldsymbol{S}) = \rho\,N\,M\hat{\lambda} + o(\rho). \tag{6}$$

where $\hat{\lambda} = \left(\hat{\boldsymbol{u}} \otimes \hat{\boldsymbol{s}}\right)^H \boldsymbol{\Upsilon}^{-1}\left(\hat{\boldsymbol{u}} \otimes \hat{\boldsymbol{s}}\right)$.

Remarks. From (4) it is clear that both the transmit and receive antennas should be made as correlated as possible, as both the optimal $\boldsymbol{K}_t$ and $\boldsymbol{K}_r$ have rank one. Note that in (6) the mutual information is proportional to M. This is in sharp contrast with the case of uncorrelated Rayleigh fading channel model for which it has been shown that the maximal mutual information is independent of the number of transmit antennas [7]. Also, since $\text{tr}\left(\boldsymbol{\Upsilon}^{-1}\right) = \sum_{i=1}^{NT} 1/\lambda_i \geq \sum_{i=1}^{NT}(2 - \lambda_i) = NT$, where λ_i's are the eigenvalues of $\boldsymbol{\Upsilon}$, we can w.l.o.g. assume that, e.g., $\boldsymbol{\Upsilon}^{-1}(1,1) \geq 1$. Then, by choosing $\boldsymbol{u}_1 = \begin{bmatrix}1 & \boldsymbol{0}_{1\times(N-1)}\end{bmatrix}^T$ and $\boldsymbol{s}_1 = \begin{bmatrix}1 & \boldsymbol{0}_{1\times(T-1)}\end{bmatrix}^T$ we have $\hat{\lambda} \geq \left(\boldsymbol{u}_1 \otimes \boldsymbol{s}_1\right)^H \boldsymbol{\Upsilon}^{-1}\left(\boldsymbol{u}_1 \otimes \boldsymbol{s}_1\right) \boldsymbol{\Upsilon}^{-1}(1,1) \geq 1$. This result confirms the general principle that correlated noise is beneficial from the capacity point of view. Finally, a short exercise would show that the first order term in (6) corresponds to that of the capacity when the channel is known to the receiver.

2.2. Mutual information: Gaussian modulation

From a practical point of view, it is unreasonable to allow input signals with large peakiness as the previous on-off signaling. Hence, we compute the low SNR mutual information for the more realistic case of Gaussian modulation. Let $\boldsymbol{s} = \text{vec}(\boldsymbol{S})$ be a zero-mean random variable with covariance matrix $\boldsymbol{P}$ that follows a circularly symmetric, complex Gaussian distribution, i.e., $\boldsymbol{s}\sim\mathcal{CN}(\boldsymbol{0}, \boldsymbol{P})$. Clearly, $\text{tr}(\boldsymbol{P}) \leq TM$. Then, at sufficiently low SNR, the mutual information between $\boldsymbol{X}$ and $\boldsymbol{S}$ up to second order in ρ is given by

$$I(\boldsymbol{X};\boldsymbol{S}) = \frac{\rho^2}{2M^2}\,\text{tr}\left(\mathsf{E}[\boldsymbol{Z}^2] - (\mathsf{E}[\boldsymbol{Z}])^2\right) + o(\rho^2) \tag{7}$$

where $\boldsymbol{Z} = \boldsymbol{\Upsilon}^{-\frac{1}{2}}\left(\boldsymbol{K}_r \otimes \boldsymbol{S}\boldsymbol{K}_t\boldsymbol{S}^H\right)\boldsymbol{\Upsilon}^{-\frac{1}{2}}$. The proof of (7) is omitted due to paper length constraints. Note that we have $I(\boldsymbol{X};\boldsymbol{S}) =$

$f(\boldsymbol{K}_t, \boldsymbol{K}_r, \boldsymbol{P})$ for some function f, which we do not make explicit here. We now address the optimization problem

$$\max_{\substack{\boldsymbol{P} \succeq \boldsymbol{0}, \operatorname{tr}(\boldsymbol{P}) \leq TM \\ \boldsymbol{K}_t \in \mathcal{P}_M \\ \boldsymbol{K}_r \in \mathcal{P}_N}} f(\boldsymbol{K}_t, \boldsymbol{K}_r, \boldsymbol{P}). \tag{8}$$

It can be shown that the maximum of (8) is attained by the following signaling scheme: the optimal correlation matrices $\widehat{\boldsymbol{K}}_r$ and $\widehat{\boldsymbol{K}}_t$ are defined as in (4) and an optimal covariance matrix $\widehat{\boldsymbol{P}}$ is given by $\widehat{\boldsymbol{P}} = TM\boldsymbol{F}_1 \otimes \hat{\boldsymbol{s}}\hat{\boldsymbol{s}}^H$, where the vectors $\hat{\boldsymbol{u}}$ and $\hat{\boldsymbol{s}}$ are, as before, solutions to the optimization problem (5). The $M \times M$ matrix $\boldsymbol{F}_1$ has all the entries equal to zero except the entry (1,1) which is one. In this case, the mutual information (per channel use) is given by

$$\frac{1}{T} I(\boldsymbol{X}; \boldsymbol{S}) = \frac{\rho^2}{2} N^2\, T\, M^2\, \hat{\lambda}^2 + o(\rho^2). \tag{9}$$

where $\hat{\lambda} = (\hat{\boldsymbol{u}} \otimes \hat{\boldsymbol{s}})^H\, \boldsymbol{\Upsilon}^{-1}\, (\hat{\boldsymbol{u}} \otimes \hat{\boldsymbol{s}})$.

Remarks. In [7] it has been proved that for the uncorrelated Rayleigh fading channel only one transmit antenna should be employed. Here, we see from (9) that having more transmit (M) and receive (N) antennas can actually enhance the channel performance in terms of capacity significantly in the correlated setup. We see that the mutual information is proportional to N^2, whereas in [7] the increase is only linear in the number of the receive antennas. The conclusions herein presented are in concordance with [8, 12] and with the results of the previous subsection where it has been shown that channel correlation can actually improve the channel performance.

3. DETERMINISTIC FADING CHANNEL: THE LOW SNR PEP ANALYSIS

Data model and assumptions. We retain the data model (1), but the presumptions under which we work are the following: (**P1**) The channel matrix $\boldsymbol{H}$ is not known at the receiver neither at the transmitter, and no stochastic model is assumed for it; (**P2**) The codeword $\boldsymbol{S}$ is chosen from a finite codebook $\mathcal{S} = \{\boldsymbol{S}_1, \boldsymbol{S}_2, \ldots, \boldsymbol{S}_K\}$ known to the receiver, where K is the size of the codebook. We impose the power constraint $\operatorname{tr}(\boldsymbol{S}_k^H \boldsymbol{S}_k) = 1$ for each codeword. We further assume that each codeword is of full rank; (**P3**) The presumption 3 is equivalent to the assumption (**A3**) in Section 2.

Receiver. Under the above conditions, the conditional probability density function of the received vector $\boldsymbol{x} = \operatorname{vec}(\boldsymbol{X})$, given the transmitted matrix $\boldsymbol{S}$ and the unknown realization of the channel $\boldsymbol{g} = \operatorname{vec}(\boldsymbol{H})$, is given by $p(\boldsymbol{x}|\boldsymbol{S}, \boldsymbol{g}) = k \exp\{-||\boldsymbol{x} - (\boldsymbol{I}_N \otimes \boldsymbol{S})\boldsymbol{g}||^2_{\boldsymbol{\Upsilon}^{-1}}\}$, where $k = 1/\left(\pi^{TN} \det\boldsymbol{\Upsilon}\right)$ and the notation $||\boldsymbol{z}||^2_{\boldsymbol{A}} = \boldsymbol{z}^H \boldsymbol{A} \boldsymbol{z}$ is used. Since no stochastic model is attached to the channel propagation matrix, the receiver faces a multiple hypothesis testing problem with the channel $\boldsymbol{H}$ as a deterministic nuisance parameter. We assume a generalized likelihood ratio test (GLRT) receiver which decides the index k of the codeword as $\widehat{k} = \operatorname{argmax}\{p(\boldsymbol{x}|\boldsymbol{S}_k, \widehat{\boldsymbol{g}}_k) : k = 1, 2, \ldots, K\}$ where $\widehat{\boldsymbol{g}}_k = (\boldsymbol{\mathcal{S}}_k^H \boldsymbol{\mathcal{S}}_k)^{-1} \boldsymbol{\mathcal{S}}_k^H \boldsymbol{\Upsilon}^{-\frac{1}{2}} \boldsymbol{x}$ with $\boldsymbol{\mathcal{S}}_k = \boldsymbol{\Upsilon}^{-\frac{1}{2}} (\boldsymbol{I}_N \otimes \boldsymbol{S}_k)$.

Low SNR analysis. Recently, in [9], in contrast to other approaches for the low SNR regime, the channel matrix is assumed deterministic and a low SNR analysis of the PEP is introduced, for the special case $M = 1$ and spatio-temporal white Gaussian noise. Here, we generalize the approach in [9] for any number of transmit antennas and arbitrary noise covariance matrix. Let $P_{\boldsymbol{S}_i \to \boldsymbol{S}_j}$ be the probability of the GLRT receiver deciding $\boldsymbol{S}_j$ when $\boldsymbol{S}_i$ is sent. It can be shown (details omitted) that for $T \geq 2M$

$$P_{\boldsymbol{S}_i \to \boldsymbol{S}_j} \approx \mathsf{Prob}\left(Y > \boldsymbol{g}^H \boldsymbol{L}_{ij} \boldsymbol{g}\right), \tag{10}$$

with $\boldsymbol{L}_{ij} = \boldsymbol{\mathcal{S}}_i^H \boldsymbol{\Pi}_j^{\perp} \boldsymbol{\mathcal{S}}_i$, $\boldsymbol{\Pi}_j^{\perp} = \boldsymbol{I}_{TN} - \boldsymbol{\mathcal{S}}_j \left(\boldsymbol{\mathcal{S}}_j^H \boldsymbol{\mathcal{S}}_j\right)^{-1} \boldsymbol{\mathcal{S}}_j^H$, and $Y = \sum_{m=1}^{MN} \sin\alpha_m \left(|a_m|^2 - |b_m|^2\right)$ where a_m, b_m are independent and identically distributed (iid) circular complex Gaussian random variables with zero mean and unit variance, i.e., $a_m,\, b_m \overset{iid}{\sim} \mathcal{CN}(0, 1)$ for $m = 1, \ldots, MN$. The angles α_m are the *principal angles* between the subspaces spanned by $\boldsymbol{\mathcal{S}}_i$ and $\boldsymbol{\mathcal{S}}_j$. From (10), an upper bound on the PEP is readily found

$$P_{\boldsymbol{S}_i \to \boldsymbol{S}_j} \leq \mathsf{Prob}\left(Z > ||\boldsymbol{g}||^2\, \lambda_{\min}\left(\boldsymbol{L}_{ij}\right)\right), \tag{11}$$

where $Z = \sum_{m=1}^{MN} |a_m|^2$ and the symbol $\lambda_{\min}$ represents the minimal eigenvalue. The bound in (11) is admittedly loose, but allows us to come up with a workable codebook design criterion. The simulation results bellow will assess its effectiveness. The codebook design criterion in (11) is equivalent to the one for the high SNR regime that has been treated in [11]

Codebook construction. Using the codebook construction criterion methodology in [11], we have constructed codes for two special categories of noise covariance matrices $\boldsymbol{\Upsilon}$. In all simulations we assumed an uncorrelated Rayleigh fading model for the channel matrix, i.e., $h_{ij} \overset{iid}{\sim} \mathcal{CN}\left(0, \sigma^2\right)$.

Category 1: spatially white, temporally colored observation noise. The first category corresponds to spatially white-temporally colored observation noise, i.e., $\boldsymbol{\Upsilon} = \boldsymbol{I}_N \otimes \Sigma(\boldsymbol{\rho})$ where the vector $\boldsymbol{\rho} : T \times 1$ is the first column of the Toeplitz matrix $\Sigma(\boldsymbol{\rho})$. Figure 1 plots the result of the experiment for T=6, SNR= $\mathsf{E}\left[||\boldsymbol{X}_k \boldsymbol{H}^H||^2\right] / \mathsf{E}\left[||\boldsymbol{E}||^2\right] = \sigma^2/T$=-6dB and $\boldsymbol{\rho}$=[1; 0.85; 0.6; 0.35; 0.1; 0]. The solid, dash-dotted and dashed line represent the performances of our eight point constellations that match the noise statistics, when the GLRT receiver is implemented for $M = 1$, $M = 2$ and $M = 3$, respectively. The plus-signed dotted line represents the performance of our eight point constellation that is constructed for the spatio-temporal white noise case ($\boldsymbol{\Upsilon} = \boldsymbol{I}_{TN}$), when GLRT receiver is implemented and $M = 1$. For symbol error rate (SER) of 10^{-3}, and $M = 1$, we see that we can save up to two transmit antennas when we compare our eight point constellation matched to the noise statistics with the mismatched constellation constructed for $\boldsymbol{\Upsilon} = \boldsymbol{I}_{TN}$. The conclusion is that, for a GLRT receiver, one should construct codebook constellations with just one transmit antenna, but which are adapted to the noise statistics. Although our primal goal in this subsection is to address the deterministic channel case, figure 1 further shows that our codebook designs for $M = 1$ are also of interest for *maximum a posteriori* (MAP) receivers that assume knowledge of the channel statistics, see [9, 13] for more implementation details about MAP detectors and constellations with non-uniform priors. The gain we witness is concordance with the information theoretic results, presented here and in [7, 8], over the low SNR non-coherent Rayleigh fading channel under an average power constraint, which suggest that the capacity achieving distribution becomes peaky.

Category 2: $\boldsymbol{E} = \boldsymbol{s}\,\boldsymbol{\alpha}^T + \boldsymbol{E}_{\mathsf{temp}}$. We considered the case where the noise matrix is of the form $\boldsymbol{E} = \boldsymbol{s}\,\boldsymbol{\alpha}^T + \boldsymbol{E}_{\mathsf{temp}}$. This models an interfering source $\boldsymbol{s}$ (with known covariance matrix $\boldsymbol{\Upsilon}_s$) where the complex vector $\boldsymbol{\alpha}$ is the known channel attenuation between each receive antenna and the interfering source. The matrix $\boldsymbol{E}_{\mathsf{temp}}$ has a noise covariance matrix belonging to the first category. Figure 2 plots the result of the experiment for T=8, $N = 2$, K=32,

$\boldsymbol{s}$=[1;0.7;0.4;0.15;zeros(4,1)], $\boldsymbol{\rho}$ = [1;0.8;0.5;0.15;zeros(4,1)] and $\boldsymbol{\alpha}$ = [-1.146 + 1.189i;1.191- 0.038i]. For SER = 10^{-2} we experience a gain of 3dB when we compare the one transmit antenna constellation, constructed taking into account the noise statistics, with the one transmit constellation constructed for $\boldsymbol{\Upsilon} = \boldsymbol{I}_{TN}$. The conclusion we draw here, as before, is that for sufficiently low SNR one should construct codebook constellations with just one transmit antenna that match the noise statistics. Also, as expected, the $M = 2$ codebook construction, adapted to noise statistics, outperforms the one antenna constellation as SNR increases.

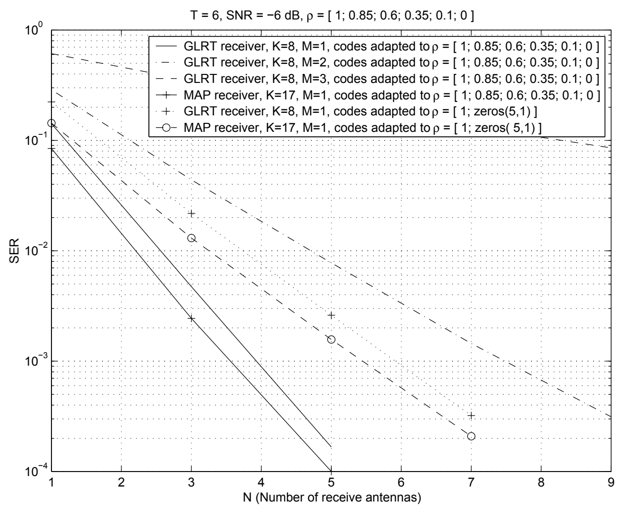

Fig. 1. Category 1 - spatially white - temporally colored: T=6, SNR=-6dB, $\boldsymbol{\rho}$=[1; 0.85; 0.6; 0.35; 0.1; 0].

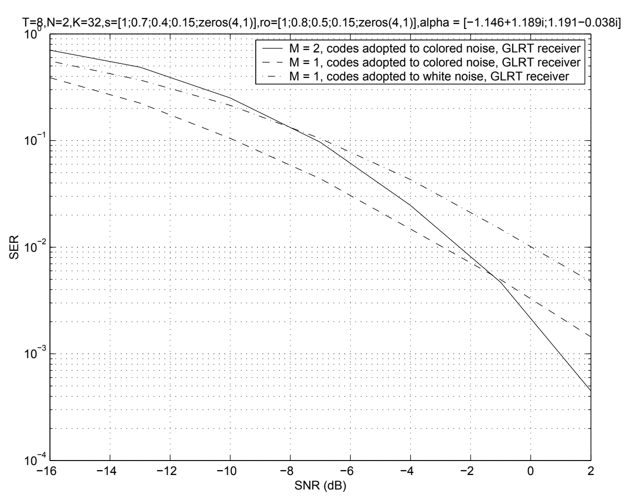

Fig. 2. Category 2 - T=8, $N = 2$, K=32, $\boldsymbol{s}$=[1;0.7;0.4;0.15;zeros(4,1)], $\boldsymbol{\rho}$ = [1;0.8;0.5;0.15;zeros(4,1)], $\boldsymbol{\alpha}$ = [-1.146 + 1.189i;1.191- 0.038i].

4. CONCLUSIONS

We have studied the MIMO channel in the low SNR regime from two perspectives: capacity and PEP analysis. The novel aspect is that we allow the Gaussian observation noise to have an arbitrary correlation structure. From the capacity analysis perspective for correlated Rayleigh fading channel, we have shown that, by maximizing the mutual information for the on-off and Gaussian signalings over the system's parameters (antenna correlation), the transmit (receive) antennas should be made as correlated as possible. Further, we have presented the PEP analysis for the low SNR deterministic channel setup and have shown how the noise statistics could be taken into account when constructing codebook constellations. We argued that one should construct codebooks for just one transmit antenna that match the noise statistics.

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