SUM CAPACITY (SUB)OPTIMALITY OF ORTHOGONAL TRANSMISSIONS IN VECTOR GAUSSIAN MULTIPLE ACCESS CHANNELS

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ABSTRACT

We study in this paper the sum capacity achievability of orthogonal transmissions in vector Gaussian multiple access channels (MAC). Specifically, we derive the sufficient and necessary conditions, in terms of channel matrices and transmitter power constraints, for orthogonal transmission to achieve the sum capacity of a vector Gaussian MAC. The obtained conditions provide a unified framework in explaining many of the results that are intuitively true. They also enable us to explore cases that have not been addressed to determine the (sub)optimality of orthogonal transmissions compared with the overlay transmission.

Index Terms— Sum capacity, vector Gaussian multiple access channel, frequency division multiple access

1. INTRODUCTION

The capacity region of a multiple access channel (MAC) was established in [1]. While it is shown that the capacity region is achievable using overlay transmission, it is also well known that, for a scalar Gaussian MAC, orthogonal transmissions, i.e., frequency division multiple access (FDMA) or time division multiple access (TDMA) under an average power constraint, can achieve *the sum capacity*. As such, if only the system throughput is of concern, orthogonal transmissions are sufficient, resulting in a much simplified transceiver structure, i.e., no successive interference cancellation is needed.

With vector Gaussian MAC, the above claim - that orthogonal transmissions achieve the sum capacity - is not necessarily true. Indeed, it is observed that in most cases orthogonal transmissions fall well short of achieving the sum capacity of a vector Gaussian MAC [2]. Focusing on FDMA, our goal is to establish the sufficient and necessary conditions for orthogonal transmissions to be optimal in achievable sum rate for a vector Gaussian MAC. The established conditions provide a unified framework behind many intuitive and well known results. In addition, it allows us to examine cases that have not been explored before in terms of the (sub)optimality of orthogonal transmissions for vector Gaussian MAC.

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The paper is organized as follows. In Section 2, we present the channel model and give the main results, namely the sufficient and necessary conditions for FDMA to achieve the sum capacity. In Section 3 we examine several cases using this unified framework to determine the (sub)optimality of orthogonal transmissions. We conclude in Section 4.

2. MAIN RESULTS

Consider a two-user vector Gaussian MAC

$$\mathbf{y} = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \mathbf{n}$$

where $\mathbf{H}_i$ is a $r \times t_i$ channel matrix, $\mathbf{x}_i$ and $\mathbf{y}$ are $t_i \times 1$ transmit and $r \times 1$ receive signal vectors respectively, $\mathbf{S}_i$ is the $t_i \times t_i$ covariance matrix of x_i with power constraint $tr(\mathbf{S}_i) \leq P_i$, $\mathbf{n}$ is a $r \times 1$ noise vector, with $E[\mathbf{n}\mathbf{n}^\dagger] = \mathbf{I}$. Both the transmitter and receiver have full channel state information. For simplicity, we use $MAC(\mathbf{H}_1, \mathbf{H}_2, P_1, P_2)$ to denote this vector Gaussian MAC, of which, the sum capacity is

$$C = \max_{tr(\mathbf{S}_1)\leq P_1, tr(\mathbf{S}_2)\leq P_2} \log \left|\mathbf{H}_1\mathbf{S}_1\mathbf{H}_1^\dagger + \mathbf{H}_2\mathbf{S}_2\mathbf{H}_2^\dagger + \mathbf{I}\right| \quad (1)$$

It was established in [3] that the sufficient and necessary condition to achieve the sum capacity is the mutually waterfilling scheme. On the other hand, the maximum achievable sum rate by using FDMA is

$$\begin{aligned} C_F \;=\; & \max_{tr(\mathbf{S}_1)\leq P_1, tr(\mathbf{S}_2)\leq P_2, 0\leq\alpha\leq 1} \\ & \left\{\alpha \log \left|\frac{1}{\alpha}\mathbf{H}_1\mathbf{S}_1\mathbf{H}_1^\dagger + \mathbf{I}\right| + \bar{\alpha} \log \left|\frac{1}{\bar{\alpha}}\mathbf{H}_2\mathbf{S}_2\mathbf{H}_2^\dagger + \mathbf{I}\right|\right\} \quad (2) \end{aligned}$$

where α is the fraction of bandwidth allocated to the first user and $\bar{\alpha} = 1 - \alpha$. Implicitly used in the above definition is the narrowband channel assumption, i.e., flat fading channels. Observe that for a given α, the maximum sum rate, denoted by $C_F(\alpha)$, is obtained by two independent single user water fillings in their respective channels. In addition, we have [4]

Proposition 1 *$C_F(\alpha)$ is a concave function of α.*

This proposition guarantees the convergence to the global maximum C_F using simple gradient methods [5]. Our goal is to find the sufficient and necessary conditions such that $C_F = C$. Our main result is summarized below.

Theorem 1 *For $MAC(\mathbf{H}_1, \mathbf{H}_2, P_1, P_2)$, FDMA can achieve its sum capacity if and only if there exist $0 < \alpha < 1$, $\mathbf{S}_{1opt}$, and $\mathbf{S}_{2opt}$ that jointly satisfy*

$$\frac{1}{\alpha}\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger = \frac{1}{\bar{\alpha}}\mathbf{H}_2\mathbf{S}_{2opt}\mathbf{H}_2^\dagger \tag{3}$$

$$\mathbf{S}_{1opt} = \arg\max_{tr(\mathbf{S}_1)\le P_1} \log\left|\frac{1}{\alpha}\mathbf{H}_1\mathbf{S}_1\mathbf{H}_1^\dagger + \mathbf{I}\right| \tag{4}$$

$$\mathbf{S}_{2opt} = \arg\max_{tr(\mathbf{S}_2)\le P_2} \log\left|\frac{1}{\bar{\alpha}}\mathbf{H}_2\mathbf{S}_2\mathbf{H}_2^\dagger + \mathbf{I}\right| \tag{5}$$

Proof: We first introduce the following lemma, established by considering a MAC with $\mathbf{H}_1 = \mathbf{H}_2 = \mathbf{H}$, $tr(\mathbf{S}_1) \le \beta P$, $tr(\mathbf{S}_2) \le \bar{\beta}P$, and invoking Theorem 1 of [3]:

Lemma 1 *If $\mathbf{S}_{opt} = arg\max_{tr(\mathbf{S}\le P)} \log\left|\frac{1}{\alpha}\mathbf{HSH}^\dagger + \mathbf{I}\right|$, where $\alpha > 0$ is a constant, then for any $\beta \in (0,1)$, $\frac{\beta}{\alpha}\mathbf{HS}_{opt}\mathbf{H}$ and $\frac{\bar{\beta}}{\alpha}\mathbf{HS}_{opt}\mathbf{H}$ satisfy the mutually waterfilling condition.*

Sufficiency From Eqs. (3)-(5), by choosing $\mathbf{S}_1 = \mathbf{S}_{1opt}$ and $\mathbf{S}_2 = \mathbf{S}_{2opt}$, the achievable sum rate is

$$\begin{aligned}&\log\left|\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger + \mathbf{H}_2\mathbf{S}_{2opt}\mathbf{H}_2^\dagger + \mathbf{I}\right| \\ &= \log\left|\frac{1}{\alpha}\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger + \mathbf{I}\right| = \max_{tr(\mathbf{S}_1)\le P_1} \log\left|\frac{1}{\alpha}\mathbf{H}_1\mathbf{S}_1\mathbf{H}_1^\dagger + \mathbf{I}\right|\end{aligned}$$

From Lemma 1, $\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger$, and $\frac{\bar{\alpha}}{\alpha}\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger$ (or $\mathbf{H}_2\mathbf{S}_{2opt}\mathbf{H}_2^\dagger$) satisfy the mutually waterfilling condition. From Theorem 1 of [3], they achieve the sum capacity.

Apply FDMA to the same channel with $\mathbf{S}_1 = \mathbf{S}_{1opt}$, $\mathbf{S}_2 = \mathbf{S}_{2opt}$ and the bandwidth allocation factor α, the sum rate is

$$\begin{aligned}C_F &= \alpha\log\left|\frac{\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger}{\alpha} + \mathbf{I}\right| + \bar{\alpha}\log\left|\frac{\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger}{\alpha} + \mathbf{I}\right| \\ &= \log\left|\frac{1}{\alpha}\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger + \mathbf{I}\right| = C\end{aligned}$$

i.e., it achieves the sum capacity.

Necessary condition Assume FDMA can achieve the sum capacity with $\alpha, \mathbf{S}_{1opt}, \mathbf{S}_{2opt}$. We only need to show that (3) must be satisfied. Since

$$\begin{aligned}C &= \alpha\log\left|\frac{\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger}{\alpha} + \mathbf{I}\right| + \bar{\alpha}\log\left|\frac{\mathbf{H}_2\mathbf{S}_{2opt}\mathbf{H}_2^\dagger}{\bar{\alpha}} + \mathbf{I}\right| \\ &\overset{(a)}{\le} \log\left|\mathbf{H}_1\mathbf{S}_{1opt}\mathbf{H}_1^\dagger + \mathbf{H}_2\mathbf{S}_{2opt}\mathbf{H}_2^\dagger + \mathbf{I}\right| \overset{(b)}{\le} C\end{aligned}$$

where (a) follows from the concavity of $\log|\cdot|$ with equality if and only if (3) is true, and (b) follows from the sum capacity definition. Since equality must hold, (3) must be true. Q.E.D.

Conditions (4) and (5) can be interpreted as that $\mathbf{S}_{iopt}$ waterfills $\mathbf{H}_i$ for the given α. To be able to dissect more complicated cases, we now present a set of conditions derived directly from Theorem 1. Before proceeding, we assume that the channel matrices admit respective singular value decompositions $\mathbf{H}_i = \mathbf{U}_i\mathbf{\Sigma}_i\mathbf{V}_i^\dagger$, $i = 1, 2$, and the singular values are denoted respectively by σ_{1j} and σ_{2l} with corresponding left singular vectors $\mathbf{u}_{1j}$ and $\mathbf{u}_{2l}$. Furthermore, define $r_i \triangleq rank(\mathbf{H}_i)$. Without loss of generality, the singular values are assumed in descending order. We have

Theorem 2 *For a $MAC(\mathbf{H}_1, \mathbf{H}_2, P_1, P_2)$, FDMA achieves the sum capacity if and only if there exists an integer $1 \le m \le \min\{r_1, r_2\}$ that satisfies the following conditions.*

Singular value conditions *For some constant k,*

$$\frac{\sigma_{11}^2}{\sigma_{21}^2} = \cdots = \frac{\sigma_{1m}^2}{\sigma_{2m}^2} = k \tag{6}$$

Singular vector conditions *For any $\sigma_{1n_1-1} \ne \sigma_{1n_1} = \sigma_{1n_1+1} = \cdots = \sigma_{1n_2} \ne \sigma_{1n_2+1}$ where $1 \le n_1 \le n_2 \le m$,*

$$\mathcal{S}\{\mathbf{u}_{1n_1}, \cdots, \mathbf{u}_{1n_2}\} = \mathcal{S}\{\mathbf{u}_{2n_1}, \cdots, \mathbf{u}_{2n_2}\} \tag{7}$$

where $\mathcal{S}\{\mathbf{u}_1, \cdots, \mathbf{u}_L\}$ denotes the subspace spanned by $\mathbf{u}_1$, $\cdots$, $\mathbf{u}_L$. In the event that all singular values are distinct, we have $\mathbf{u}_{1i} = \pm\mathbf{u}_{2i}$ for $1 \le i \le m$.

Power constraint conditions

$$v_1 P_2 = v_2 P_1 \tag{8}$$

where

$$\sum_{i=1}^{r_1}\left(v_1 - \frac{\alpha}{\sigma_{1i}^2}\right)^+ = \sum_{i=1}^{m}\left(v_1 - \frac{\alpha}{\sigma_{1i}^2}\right) = P_1 \tag{9}$$

$$\sum_{i=1}^{r_2}\left(v_2 - \frac{\bar{\alpha}}{\sigma_{2i}^2}\right)^+ = \sum_{i=1}^{m}\left(v_2 - \frac{\bar{\alpha}}{\sigma_{2i}^2}\right) = P_2 \tag{10}$$

$$\alpha = \frac{kP_1}{kP_1 + P_2} \tag{11}$$

where $(x)^+ \triangleq \max\{x, 0\}$.

Eqs. (6) and (7) establish that the two channel matrices must have proportional singular values and perfectly aligned singular vectors, while the last condition dictates that the corresponding power constraints must be such that the respective waterfilling uses the same number of eigenmodes for the two users in the FDMA transmission for the optimal α.

3. APPLICATIONS

The sufficient and necessary conditions in Theorem 1 or 2 appear to be overly restrictive, i.e., such conditions are rarely satisfied for the general vector Gaussian MAC. The results, however, provide a unified approach in determining the sum capacity optimality of orthogonal transmissions. More importantly, Theorem 2 also allow us to gain insight into how to quantify the suboptimality of orthogonal transmissions as demonstrated in this section.

3.1. FDMA is optimal

Example 1 $r = 1$, $t_1, t_2 \ge 1$.

Example 2 $\mathbf{H}_1 = \gamma\mathbf{H}_2\mathbf{A}$, *$\gamma$ is a constant and* $\mathbf{AA}^\dagger = \mathbf{I}$.

Example 3 $r \le \min\{t_1, t_2\}$, $\mathbf{H}_1$ *and* $\mathbf{H}_2$ *have identical singular values* $\sigma_{ij} = \sigma_i$, $i = 1, 2; j = 1, 2, \cdots, r$.

One can readily show that $C_F = C$ for Examples 1-3 using Theorem 2. Fig.1 is a special case of Example 2 with $C_F = C$ when $\alpha = 0.8$. The example in [3, page 148] is also achievable by FDMA with $\alpha = 0.5$. In the above examples the channel matrices make the power constraint automatically satisfied regardless of the values of P_1 and P_2. There are cases that, even if the channel matrices satisfy the singular value/vector constraints, one still need the right P_1 and P_2.

Example 4 *Assume that for some* $m \ge 1$, *Eq. (6) as well as the associated singular vector conditions are satisfied but* $\frac{\sigma^2_{1,m}}{\sigma^2_{1,m}} \neq \frac{\sigma^2_{1,m+1}}{\sigma^2_{1,m+1}}$, *if* $P_i > \frac{m}{\sigma^2_{1,m+1}} - \sum_{j=1}^{m} \frac{1}{\sigma^2_{i,j}}$ *for either* $i = 1$ *or* 2, *the power conditions are violated and FDMA is suboptimal due to the generous power constraint, which favors overlay transmission with successive interference cancellation.*

To elaborate, consider $\mathbf{H}_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}, \mathbf{H}_2 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$. Then from Eqs. (8)-(11), $C_F = C$ only when $P_1 + \frac{P_2}{4} < 3$.

3.2. FDMA is suboptimal

A simple example for FDMA to be subpotimal is $\frac{\sigma_{11}}{\sigma_{21}} \neq \frac{\sigma_{12}}{\sigma_{22}}$ and $\mathbf{u}_{11} \neq \pm\mathbf{u}_{21}$. Next, we decouple the singular value and vector conditions, and use Theorem 2 to evaluate their individual impact on the sum capacity achievability of FDMA.

3.2.1. Singular vector

We first develop a mechanism that allows us to quantify the relation of singular vectors and $\frac{C_F}{C}$. The singular value conditions are assumed to be satisfied, but the subspaces spanned by the corresponding singular vectors are now different. The difference can be measured by the *distance* of subspaces, defined as [6]

$$\text{dist}(\mathcal{U}_1, \mathcal{U}_2) \triangleq ||Q_1 - Q_2||_2 = \sigma_{max}\left(\mathbf{Q}_1 - \mathbf{Q}_2\right) \qquad (12)$$

where $\mathcal{U}_i, i = 1, 2$ are the subspaces, Q_i is the orthogonal projection matrix for $\mathcal{U}_i$, and the 2-norm of $Q_1 - Q_2$ is its largest singular value. When $\mathcal{U}_1$ and $\mathcal{U}_2$ have the same dimension, their largest principal angle ϕ is shown to be [6]

$$\phi \quad = \quad \sin^{-1}\left(\text{dist}\left(\mathcal{U}_1, \mathcal{U}_2\right)\right) \qquad (13)$$

If $\mathbf{H}_1$ and $\mathbf{H}_2$ are $n \times n$ real matrices, the unitary matrix $\mathbf{U}_2$ can be obtained by rotating $\mathbf{U}_1$ along an axis defined by the subspace $\mathcal{A}$ of dimension $n - 2$ by an angle θ, $\mathbf{U}_2 = \text{rot}\{\mathbf{U}_1, \mathcal{A}, \theta\}$. By choosing $\mathcal{A}$ and let θ vary in $[0, 2\pi]$, different $\mathbf{U}_2$ is generated. This mechanism allows us to quantify the relation of $\frac{C_F}{C}$ and ϕ. Here is an example.

Example 5 $\mathbf{U}_1 = \mathbf{V}_1 = \mathbf{V}_2 = \mathbf{I}$, $\mathbf{\Sigma}_1 = diag\left(1, 1, \frac{1}{9}, \frac{1}{10}\right)$, $\mathbf{\Sigma}_2 = diag\left(2, 2, \frac{1}{8}, \frac{1}{15}\right)$, $\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^T$, $\mathbf{U}_2 =$ *rot*$(\mathbf{I}, \mathcal{A}, \theta)$, $\theta \in [0, 2\pi]$, $P_1 = P_2 = 1$.

The singular value conditions are satisfied and power is allocated to only the first two eigenmodes. Signals are transmitted in the subspaces $\mathcal{U}_i$ spanned by $[\mathbf{u}_{i1}, \mathbf{u}_{i2}]$, where $\mathbf{u}_{i1}$ and $\mathbf{u}_{i2}$ are unitary vectors. The projection matrix for $\mathcal{U}_i$ is,

$$\mathbf{Q}_i = [\mathbf{u}_{i1}, \mathbf{u}_{i2}][\mathbf{u}_{i1}, \mathbf{u}_{i2}]^T \qquad (14)$$

From Eqs. (12)-(14), the angle of $\mathcal{U}_1$ and $\mathcal{U}_2$ is

$$\phi = \sin^{-1}\left(\sigma_{max}\left\{\mathbf{u}_{11}\mathbf{u}_{11}^T + \mathbf{u}_{12}\mathbf{u}_{12}^T - \mathbf{u}_{21}\mathbf{u}_{21}^T - \mathbf{u}_{22}\mathbf{u}_{22}^T\right\}\right)$$

The results are shown in Figs.2 and 3. While different $\mathcal{A}$ results in different curves of θ and ϕ, for all the cases, $\frac{C_F}{C}$ is monotonically decreasing with ϕ. When $\theta = 0, 2\pi$, $\phi = 0$, $\mathcal{U}_1$ and $\mathcal{U}_2$ coincide, the conditions of Theorem 2 are satisfied and $\frac{C_F}{C} = 1$. This is when the mutual interferences from the two users are the worst, and FDMA benefit the most via orthogonization. When $\theta = 0.66\pi, 1.33\pi, \phi = 0.5\pi, \mathcal{U}_1$ and $\mathcal{U}_2$ are orthogonal to each other and $\frac{C_F}{C}$ becomes the minimum. This agrees with intuition: the orthogonality of the subspaces allows both users to communicate at maximum rate without interfering each other.

3.2.2. Singular value

We still assume $\mathbf{H}_1$ and $\mathbf{H}_2$ are $n \times n$ real matrices. The singular vector conditions are satisfied, but the singular value conditions are not. Without loss of generality, we can assume $\mathbf{U}_1 = \mathbf{U}_2 = \mathbf{V}_1 = \mathbf{V}_2 = \mathbf{I}$.

Example 6 $\mathbf{H}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{H}_2 = \begin{bmatrix} 1 & 0 \\ 0 & \sigma \end{bmatrix}$, $-20dB \le \sigma \le 20dB$, $P_1 = P_2 = 1$.

Table 1. Overlay transmission.

σ (dB)	user 1	user 2	C (bit)
< 0	$(0, 1)$	$(1, 0)$	2
$= 0$	$(a, 1 - a)$	$(1 - a, a)$	2
> 0	$(1, 0)$	$(0, 1)$	$1 + \log(1 + \sigma^2)$

Table 2. FDMA transmission.

σ (dB)	user 1	user 2	C (bit)
≤ -4.8	$(\frac{1}{2}, \frac{1}{2})$	$(1, 0)$	1.79
> -4.8	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{\alpha_o}{2} + \frac{1-\alpha_o}{2\sigma^2}, 1 - \frac{\alpha_o}{2} - \frac{1-\alpha_o}{2\sigma^2})$	$\max C_F(\alpha)$

The results are shown in Fig. 4, the sum rate and optimal power allocation for overlay and FDMA are shown in Table 1 and 2. For overlay transmission, the second user always put all the power to the eigenmode of the largest eigenvalue, while the first user adapts to put all the power to the orthogonal direction. For FDMA, the optimal frequency allocation is $\alpha_o = 0.48$ when $\sigma \le \left(1 + \frac{P_2}{1-\alpha_o}\right)^{-\frac{1}{2}} = -4.8\text{dB}$, and $\alpha_o = \arg\max_{\alpha \in [0,1]} C_F(\alpha)$ when $\sigma > -4.8\text{dB}$, where

$$C_F(\alpha) = \left\{\alpha \log\left(\frac{P_1}{2\alpha} + 1\right)^2 + \bar{\alpha}\log\left[\frac{\sigma P_2}{2\bar{\alpha}} + \frac{1 + \sigma^2}{2\sigma}\right]^2\right\}$$

So when $\sigma = 0$dB, $\mathbf{S}_{f2} = 0.5\mathbf{I}$, $\alpha_o = 0.5$, $C_F = C = 2$bit. In the neighborhood of 0dB, $\frac{C_F}{C}$ decreases as σ moves away from 0dB. When $\sigma \to \infty$, $\alpha_o \to 0$ and

$$\lim_{\sigma\to\infty} \frac{C_F}{C} = \lim_{\sigma\to\infty} \frac{\log\left[\frac{\sigma}{2} + \frac{\sigma}{2}\left(1 + \frac{1}{\sigma^2}\right)\right]^2}{1 + \log\left(1 + \sigma^2\right)} = 1$$

One user's rate becomes dominant, thus FDMA asymptotically achieves the sum capacity with bandwidth allocation increasingly favoring to the dominant user. However, $\frac{C_F}{C}$ is not a monotone function as in Example 5.

4. CONCLUSION AND EXTENSION

Orthogonal transmission in vector Gaussian MAC was studied in this paper. We derived sufficient and necessary conditions for FDMA to achieve the sum capacity. We note here that the results can be easily generalized for MAC with more than two users. We point out here that parallel results using TDMA can also be obtained.

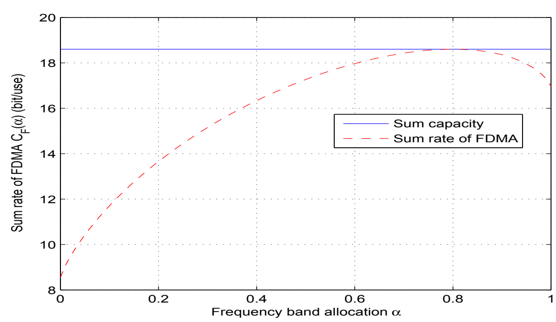

Fig. 1. Sum rate of FDMA versus frequency allocation factor, where $r = t_1 = t_2 = 8$, $P_1 = P_2 = 1$, $\gamma = 2$, $\mathbf{H}$ and $\mathbf{A}$ are randomly chosen.

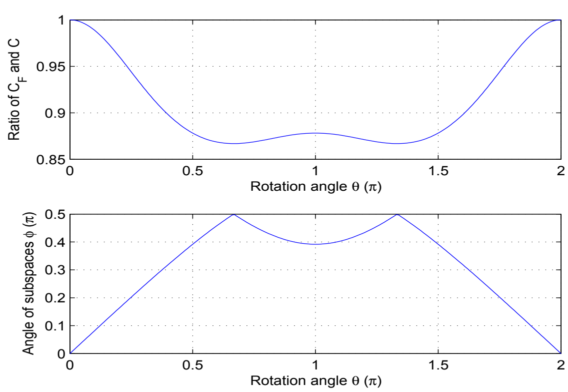

Fig. 2. The top plot is the ratio of C_F and C, and the bottom plot is the angle of subspaces versus the rotation angle.

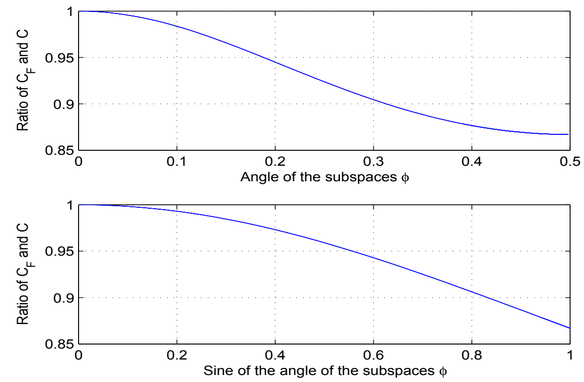

Fig. 3. The ratio of C_F and C versus ϕ and $\sin(\phi)$, the principal angle of the subspaces.

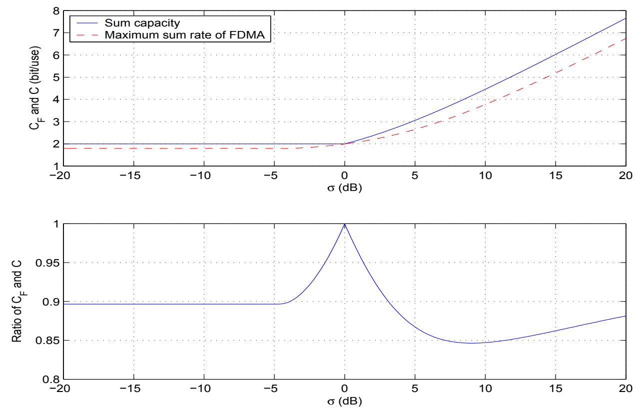

Fig. 4. C, C_F and their ratio versus σ.

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