CAPACITY BALANCING FOR MULTIUSER MIMO SYSTEMS

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ABSTRACT

This paper investigates the problem of transceiver design with individual rate constraints for multiuser MIMO systems. We focus on linear processing with two design goals: one is to maximize the minimum rate per user under a total power constraint, and the other is to minimize the total transmit power while maintaining certain rate requirements. The optimization is carried out in an alternating manner in both virtual uplink and downlink channels. Each iteration contains the optimization of uplink power allocation, and uplink and downlink MMSE receive filters. The uplink power control to balance the rates or to achieve the rate requirements is taken by optimizing the product of MSEs, which can be formulated as a Geometric Programming (GP) problem.

Additionally, this alternating optimization approach is suitable for the case with successive interference cancellation (SIC) in the uplink and interference pre-compensation (IPC) in the downlink as well. With a fixed precoding ordering, this provides new sub-optimal solutions to the above problems.

Index Terms— multiuser MIMO, capacity region, transceiver optimization, max-min fairness

1. INTRODUCTION

The achievable sum capacity region of multiuser MIMO Gaussian broadcast channels was characterized in an information theoretic context. However, the search for efficient practical schemes to achieve the limit is still ongoing. Existing schemes are usually under the assumption of 'dirty paper precoding', e.g., [1, 2] for sum-capacity optimization and [3, 4, 5] for individual rate optimization. However, dirty paper precoding techniques are difficult for practical implementation. Moreover, the optimal precoding ordering for individual rate optimization is still an open issue [3, 5].

With linear processing, one commonly used strategy is to block-diagonalize the channel, e.g., [6, 7]. However, such a zero-forcing (ZF) approach suffers from noise/power enhancement and has a restriction on the number of transmit and receive antennas. Therefore, ZF is generally not capacity achieving strategy. To achieve the capacity limit, MMSE estimation can be utilized, since it plays an important role in approaching the information-theoretic limits of linear Gaussian channels [8].

In this paper, we first focus on linear processing with MMSE estimation, and later we show how the concepts can be extended to the case that successive interference cancellation (SIC) is applied in the uplink and interference pre-compensation (IPC) in the downlink (also known as 'dirty paper precoding').

We consider a multiuser MIMO system, where K users can perform spatial multiplexing with several data streams (layers). The transmit and receive filters are jointly optimized with respect to two design criteria. One is to balance the rates among users with a total power constraint

$$\max \min_{k} R_k \quad \text{s.t.} \quad P_{\mathrm{sum}} \leq P_{\max}, \tag{1}$$

where R_k is the achieved rate for the kth. P_{sum} and $P_{\max}$ are the required total power and the total power limit, respectively. The other problem is to minimize the total transmit power under individual rate constraints, i.e.,

$$\min P_{\mathrm{sum}} \quad \text{s.t.} \quad R_k \geq \gamma_k, \quad \forall k, \tag{2}$$

where γ_k is the target for the kth user.

For both design goals, the optimization is over the powers, transmit and receive filters, by switching between the virtual uplink and downlink channels. Specifically, we first allocate the uplink powers to all layers by optimizing the product of MSEs, which can be formulated as a Geometric Programming (GP) problem. This differs from the approach of formulating a GP problem with respect to SINR [9], where an approximation $SINR \approx 1 + SINR$ is made. Actually, this approach can not be applied to the user-rate optimization, since each user can have more than one data stream, which means that the rate per user is the sum of rates per layer. Therefore, balancing the product of SINR is not equivalent to balancing the user-rates; furthermore, to achieve a certain user-rate requires the optimal individual layer-SINR targets, however, this is unknown in general. Then, the transmit and receive filters are updated as MMSE filters, which minimize the layer-MSE independently, so that each user achieves its maximum rate. Additionally, MSE duality [10] ensures that the same performance can always be achieved in both virtual links. This leads to a monotonic sequence of rates.

This alternating optimization approach can be applied to the case with SIC/IPC for a fixed precoding/decoding ordering. Note that, in [4] the solution for (2) is found by bi-section over the transmit power obtained by solving (1). Generally, such an approach is not computational efficient. Although our proposed algorithms need additional ordering strategies, we observe that with a simple ordering, where the users are ordered according to the channel norms, the gap to the global optimum [4] is small.

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2. SYSTEM MODEL AND MSE DUALITY

We consider a standard multiuser MIMO downlink model with N_T transmit antennas at the base station and K decentralized receivers, each with N_{R_k} antennas. The channel matrix is $\boldsymbol{H} = [\boldsymbol{H}_1, ..., \boldsymbol{H}_K]$ where $\boldsymbol{H}_k \in \mathbb{C}^{N_T \times N_{R_k}}$ models the channel between the kth user and the base station. Assume that independent unity-power symbols $\boldsymbol{d} = [\boldsymbol{d}_1^T, ..., \boldsymbol{d}_K^T]^T$ with $\mathrm{E}\{\boldsymbol{d}\boldsymbol{d}^H\} = \boldsymbol{I}$, are transmitted, where $\boldsymbol{d}_k \in \mathbb{C}^{M_k \times 1}$ is the data vector to be transmitted to the kth mobile. The total number of the transmit (active) data streams (layers) is $N_d = \sum_{k=1}^K M_k$. Zero-mean white Gaussian noise is denoted by $\boldsymbol{n} = [\boldsymbol{n}_1^T, ..., \boldsymbol{n}_K^T]^T \sim \mathcal{N}(0, \sigma_n^2 \boldsymbol{I})$. The data and the noise are statistically independent.

For convenience, the transmit and receive filters are separated into two parts, a matrix with unity-norm columns and diagonal matrices. In particular, the transmit filter is $\overline{\boldsymbol{U}} = \boldsymbol{U}\boldsymbol{Q}^{1/2}$ and receive filters are $\overline{\boldsymbol{T}}_k^H = \boldsymbol{Q}_k^{-1/2}\boldsymbol{\beta}_k\boldsymbol{T}_k^H, \forall k$, where $\boldsymbol{Q} = \mathrm{diag}\{\boldsymbol{Q}_1, ..., \boldsymbol{Q}_K\}$ contains the transmit powers of all users. Filters $\boldsymbol{U} = [\boldsymbol{U}_1, ..., \boldsymbol{U}_K]$ and $\boldsymbol{T} = \mathrm{diag}\{\boldsymbol{T}_1, ..., \boldsymbol{T}_K\}$ are with normalized columns $\|\boldsymbol{u}_i\|_2 = 1$ and $\|\boldsymbol{t}_i\|_2 = 1, \forall i$, respectively. The matrix $\boldsymbol{\beta} = \mathrm{diag}\{\boldsymbol{\beta}_1, ..., \boldsymbol{\beta}_K\}$ is a diagonal matrix. The system model is given by

$$\hat{d}_i = \bar{\boldsymbol{t}}_i^H \boldsymbol{H}^H \sum_{j=1}^{N_d} \overline{\boldsymbol{u}}_j d_j + \bar{\boldsymbol{t}}_i^H \boldsymbol{n}, \quad \forall i \in \{1, ..., N_d\}.$$

The equivalent uplink channel is obtained by switching the role of the normalized transmit and receive filters. The kth transmit filter is $\overline{\boldsymbol{T}}_k = \boldsymbol{T}_k\boldsymbol{P}_k^{1/2}$ and receive filter is $\overline{\boldsymbol{U}}^H = \boldsymbol{P}^{-1/2}\boldsymbol{\beta}\boldsymbol{U}^H$. The quantities $\boldsymbol{H}, \boldsymbol{U}, \boldsymbol{T}$ and $\boldsymbol{\beta}$ are the same as for the downlink model. The power allocation $\boldsymbol{P} = \mathrm{diag}\{\boldsymbol{p}\} = \mathrm{diag}\{\boldsymbol{P}_1, ..., \boldsymbol{P}_K\}$, however, may differ from the downlink allocation $\boldsymbol{Q} = \mathrm{diag}\{\boldsymbol{q}\}$. It is assumed that both links fulfill the same sum power constraint, i.e., $\|\boldsymbol{p}\|_1 = \|\boldsymbol{q}\|_1 \leq P_{\max}$.

Defining a diagonal matrix $[\boldsymbol{D}]_{ii} = \beta_i^2 \boldsymbol{u}_i^H \boldsymbol{H}\boldsymbol{t}_i\boldsymbol{t}_i^H\boldsymbol{H}^H\boldsymbol{u}_i - 2\beta_i \Re\mathfrak{e}\{\boldsymbol{u}_i^H\boldsymbol{H}\boldsymbol{t}_i\} + 1$ and a matrix

$$[\boldsymbol{\Psi}]_{ij} = \begin{cases} \boldsymbol{u}_i^H \boldsymbol{H}\boldsymbol{t}_j\boldsymbol{t}_j^H\boldsymbol{H}^H\boldsymbol{u}_i, & i \neq j \\ 0, & i = j \end{cases},$$

we obtain the uplink power allocation

$$\boldsymbol{p} = \sigma_n^2(\boldsymbol{\varepsilon} - \boldsymbol{D} - \boldsymbol{\beta}^2\boldsymbol{\Psi})^{-1}\boldsymbol{\beta}^2\mathbf{1}_{N_d}, \tag{3}$$

and downlink power allocation

$$\boldsymbol{q} = \sigma_n^2(\boldsymbol{\varepsilon} - \boldsymbol{D} - \boldsymbol{\beta}^2\boldsymbol{\Psi}^T)^{-1}\boldsymbol{\beta}^2\mathbf{1}_{N_d}, \tag{4}$$

to achieve the same feasible MSE value $\boldsymbol{\varepsilon} = \mathrm{diag}\{[\varepsilon_1, ..., \varepsilon_{N_d}]\}$.

With the same set of $\boldsymbol{T}$, $\boldsymbol{U}$, $\boldsymbol{\beta}$ and a total power limit $P_{\max}$, it has been shown in [10] that the same MSE values $\varepsilon_1, \ldots, \varepsilon_{N_d}$ can be achieved in the uplink if and only if the same values can be achieved in the downlink. Thus, both links have the same achievable MSE region under a sum power constraint.

2.1. Uplink and Downlink MMSE Receive Filters

In the uplink channel for fixed power allocation $\boldsymbol{P}$ and transmit filter $\boldsymbol{T}$, the MSE of each layer can be minimized independently by MMSE receive filters

$$\boldsymbol{U}_k\boldsymbol{\beta}_k\boldsymbol{P}_k^{-1/2} = (\boldsymbol{H}\boldsymbol{T}\boldsymbol{P}\boldsymbol{T}^H\boldsymbol{H}^H + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{H}_k\boldsymbol{T}_k\boldsymbol{P}_k^{1/2}, \forall k.$$

For convenience, we define

$$\widetilde{\boldsymbol{U}}_k = \boldsymbol{U}_k\boldsymbol{\beta}_k = (\boldsymbol{H}\boldsymbol{T}\boldsymbol{P}\boldsymbol{T}^H\boldsymbol{H}^H + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{H}_k\boldsymbol{T}_k\boldsymbol{P}_k, \forall k. \tag{5}$$

The diagonal matrix $\boldsymbol{\beta}_k$ contains the column norms of $\widetilde{\boldsymbol{U}}_k$.

Similarly, for the downlink channel, with fixed downlink power allocation $\boldsymbol{Q}$ and transmit filter $\boldsymbol{U}$, the MMSE receive filters are given as $\widetilde{\boldsymbol{T}}_k\boldsymbol{Q}_k^{-1/2}$, where

$$\widetilde{\boldsymbol{T}}_k = \boldsymbol{T}_k\boldsymbol{\beta}_k = (\boldsymbol{H}_k^H\boldsymbol{U}\boldsymbol{Q}\boldsymbol{U}^H\boldsymbol{H}_k + \sigma_n^2\boldsymbol{I})^{-1}\boldsymbol{H}_k^H\boldsymbol{U}_k\boldsymbol{Q}_k, \forall k. \tag{6}$$

The diagonal matrix $\boldsymbol{\beta}_k$ contains the column norms of $\widetilde{\boldsymbol{T}}_k$.

2.2. Alternating Optimization Framework

If we want to optimize the transmission over a downlink channel $\boldsymbol{H}^H$, then the equivalent uplink channel $\boldsymbol{H}$ only serves for the purpose of optimization, and vice versa. In combination with the MSE duality, this results in an alternating optimization framework:

repeat

1. uplink channel:
   (a) for fixed $\boldsymbol{T}$, $\boldsymbol{U}$ and $\boldsymbol{\beta}$, find the optimal $\boldsymbol{P}$ according to the optimization problem under consideration
   (b) for fixed $\boldsymbol{T}$ and $\boldsymbol{P}$, update $\boldsymbol{U}$ and $\boldsymbol{\beta}$ with (5)
2. downlink channel:
   (a) for fixed $\boldsymbol{T}$, $\boldsymbol{U}$ and $\boldsymbol{\beta}$, find the downlink power allocation $\boldsymbol{Q}$ with (4) achieving the same MSEs as in the uplink
   (b) for fixed $\boldsymbol{Q}$ and $\boldsymbol{U}$, update $\boldsymbol{T}$ and $\boldsymbol{\beta}$ with (6).

until accuracy is reached.

To derive algorithms from this framework, the essential issue is to specify the uplink power allocation (step 1.(a)) according to the optimization problems under consideration. This will be addressed in the next section.

3. UPLINK POWER CONTROL

3.1. Rate Balancing under a Total Power Constraint (1)

Under the common assumption of Gaussian channels, there is a one-to-one monotonic relationship between rate and SINR

$$R_i = \log_2(1 + \mathrm{SINR}_i), \quad \forall i,$$

where $\mathrm{SINR}_i = \frac{|\bar{\boldsymbol{t}}_i^H\boldsymbol{H}^H\overline{\boldsymbol{u}}_i|^2}{\sum_{j=1, j\neq i}^{N_d}|\bar{\boldsymbol{t}}_i^H\boldsymbol{H}^H\overline{\boldsymbol{u}}_i|^2 + \sigma_n^2\bar{\boldsymbol{t}}_i^H\bar{\boldsymbol{t}}_i}, \forall i.$

Moreover, we have the well known relation between the achieved SINR and minimum MSE

$$\mathrm{MMSE} = \frac{1}{\mathrm{SINR} + 1}. \tag{7}$$

Therefore, the rate of the kth user can be expressed as a function of MMSE, i.e., $R_k = -\log_2\left[\prod_{i=1}^{M_k}\mathrm{MMSE}_i\right]$.

With fixed filters $\boldsymbol{T}$ and $\boldsymbol{\beta}\boldsymbol{U}$, we have

$$\prod_{i=1}^{M_k}\varepsilon_i = \prod_{i=1}^{M_k}\left\{p_i^{-1}[(\boldsymbol{D} + \boldsymbol{\beta}^2\boldsymbol{\Psi})\boldsymbol{p} + \sigma_n^2\boldsymbol{\beta}^2\mathbf{1}_{N_d}]_i\right\},$$

which is a posynomial. Hence, the power optimization for (1) can be formulated as a Geometric Programming (GP) problem [11, 9],

$$\begin{aligned} \min_{\boldsymbol{p},t} \quad & t \\ \text{s.t.} \quad & \prod_{i=1}^{M_k} [p_i^{-1}[(\boldsymbol{D}+\boldsymbol{\beta}^2\boldsymbol{\Psi})\boldsymbol{p}+\sigma_n^2\boldsymbol{\beta}^2\mathbf{1}_{N_d}]_i] \leq t, \ \forall k \\ & p_i \geq 0, \ \|\boldsymbol{p}\|_1 \leq P_{\max}, \ t>0, \ \forall i. \end{aligned} \tag{8}$$

The global optimum can be found by existing optimization tools.

3.2. Individual Rate Constrained Power Minimization (2)

For (2), instead of solving (8), the power has to be minimized while achieving the rate requirements, i.e.,

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \|\boldsymbol{p}\|_1 \\ \text{s.t.} \quad & \prod_{i=1}^{M_k} \{p_i^{-1}[(\boldsymbol{D}+\boldsymbol{\beta}^2\boldsymbol{\Psi})\boldsymbol{p}+\sigma_n^2\boldsymbol{\beta}^2\mathbf{1}_{N_d}]_i\} \leq 2^{-\gamma_k}, \ \forall k \\ & p_i \geq 0, \ \forall i. \end{aligned} \tag{9}$$

Optimization (9) is a GP problem as well. Therefore, the global optimum can be found.

4. JOINT TRANSCEIVER OPTIMIZATION AND CONVERGENCE

Joint transceiver optimization for problems (1) and (2) is based on the alternating optimization framework and the uplink power control strategies in the previous section. Joint transceiver optimization algorithms are summarized in Table 1. Superscript $(\cdot)^{(n)}$ denotes the nth iteration step.

Theorem 1. *The total transmit power $P_{\text{sum}}^{(n)}$ obtained by Algorithm 1 is monotonically decreasing in n. (Sketch of the proof is given in the Appendix.)*

Theorem 2. *The $\max\min_k R_k^{(n)}$ obtained by Algorithm 2 is monotonically increasing in n. (Sketch of the proof is given in the Appendix.)*

The above iterations might stop at local optima depending on different initializations of $\boldsymbol{T}_k^{(0)}$, $\forall k$. Therefore, a 'favorable' initialization is desired. How to choose an initialization which ensures the global optimum is an interesting problem for future work. A recommended initialization is the singular vectors of the channel matrix.

5. EXTENSION: OPTIMIZATION WITH IPC

Note that, the relation between the SINR and MMSE (7) holds as well for the case with SIC in the uplink and IPC in the downlink. Assuming that error propagation or precoding loss are neglected, uplink SIC/downlink IPC imposes a triangular structure on the effective channel. Particularly, without loss of generality, we assume a precoding order N_d to 1 in the downlink and decoding order 1 to N_d in the uplink.

In order to find a transceiver strategy that performs well with respect to the design goals (1) and (2) with IPC, we can follow the alternating approach used in the above sections. To this end, we have to first specify the uplink power control. Fortunately, after substituting the matrix $\boldsymbol{\Psi}$ by $\boldsymbol{\Psi}_u = \text{Triu}(\boldsymbol{\Psi})$ (the upper triangular part of a matrix), the optimization problem (8) and (9) are still GP problems. Optimal solutions exist.

On the other hand, the uplink and downlink receive filters have to take the SIC and IPC into account, respectively. Particularly, in the uplink channel, the receive filter $\boldsymbol{U}\boldsymbol{\beta}\boldsymbol{P}^{-1/2}$ is updated as the MMSE-DFE feedforward filter

$$\beta_i p_i^{-1/2}\boldsymbol{u}_i = p_i^{1/2}[\boldsymbol{H}(\sum_{l=i}^{N_d} p_l \boldsymbol{t}_l \boldsymbol{t}_l^H)\boldsymbol{H}^H + \sigma_n^2\boldsymbol{I})]^{-1}\boldsymbol{H}\boldsymbol{t}_i. \tag{10}$$

Similarly, the downlink MMSE receive filter for each layer is given by

$$\beta_i q_i^{-1/2}\boldsymbol{t}_i = q_i^{1/2}[\boldsymbol{H}_k^H(\sum_{l=1}^{i} q_l \boldsymbol{u}_l \boldsymbol{u}_l^H)\boldsymbol{H}_k + \sigma_n^2\boldsymbol{I})]^{-1}\boldsymbol{H}_k\boldsymbol{u}_i. \tag{11}$$

We summarize Algorithm 3 for (1) and Algorithm 4 for (2) with IPC in Table 1. The convergence of the algorithms can be proved by a similar reasoning as for the linear processing case.

Corollary 1. *The total transmit power $P_{\text{sum}}^{(n)}$ returned by Algorithm 3 is monotonically decreasing in n and the rate $\max\min_k R_k^{(n)}$ returned by Algorithm 4 is monotonically increasing in n.*

6. SIMULATION RESULTS

We illustrate the performance of the proposed algorithms for a three-user MIMO system with 5 transmit antennas at base station and 2 receive antennas at each mobile. The performance measures are averaged over 100 randomly chosen channel realizations (flat fading channel is assumed).

Fig. 1 shows the average balanced individual rate level vs. the number of iterations. The total power is limited to 10. It can be observed that the proposed algorithms converge and the scheme with IPC achieves a higher level than that with linear processing, which is expected. The line with circles denotes the case without ordering and the line with dots is the case with a simple ordering strategy, where the users are ordered according to their channel norms. A slight improvement is observed and the gap to the global optimum [4] (plotted by the dashed line) is small.

The convergence behavior of Algorithm 2 and 4 is shown in Fig. 2. The targets are assumed to be $\gamma_1 = 2$, $\gamma_2 = 1$, $\gamma_3 = 3$. We can see that the average total required power converges. With IPC and ordering, the performance is improved as compared with the case with linear processing.

7. CONCLUSIONS

We propose iterative algorithms to jointly optimize the transmit and receive filters in multiuser MIMO systems. The design goals are maximizing the minimum rate under a total power constraint, and minimizing the total transmit power with individual rate requirements. The important steps in the iterations are the optimal power allocation strategies proposed in Section 3. We show that the optimization of user-rates can be carried out by minimizing the product of layer-MSEs together with MMSE filtering. Facilitated by the new power control strategies and based on the MSE duality, the proposed algorithms are proved to be convergent.

8. APPENDIX

A. Sketch of proof of Theorem 1

During each iteration, the total transmit power does not change, i.e., $P_{\text{sum}}^{(step5,n)} = P_{\text{sum}}^{(step6,n)} = P_{\text{sum}}^{(step7,n)}$ whereas the geometric MSE values per user is decreased, or equivalently, $-\log_2\prod_i^{M_k}\varepsilon_i^{(step7,n)} \geq \gamma_k$, $\forall k$. In the next iteration, by optimizing the power allocation to fulfill the rate

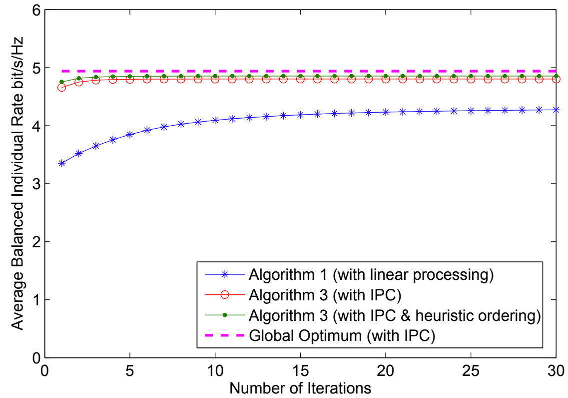

Fig. 1. Average balanced individual rate vs. number of iterations. Parameters: 100 flat fading channel realizations, $P_{\max} = 10$, $K = 3$, $N_T = 5$, $N_{R_1} = N_{R_2} = N_{R_3} = 2, \sigma_n^2 = 1$

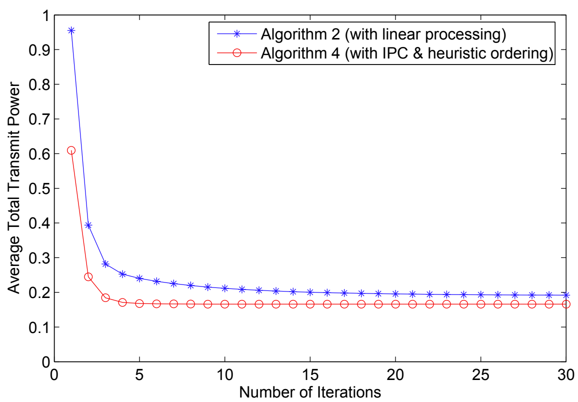

Fig. 2. Average total transmit power vs. number of iterations. Parameters: 100 flat fading channel realizations, individual rate requirement: $r_1 = 2, r_2 = 1, r_3 = 3$, $K = 3$, $N_T = 5$, $N_{R_1} = N_{R_2} = N_{R_3} = 2, \sigma_n^2 = 0.1$

targets γ_k, a lower power level is needed, thus $P_{\text{sum}}^{(step5,n)} \geq P_{\text{sum}}^{(step5,n+1)}$.

B. Sketch of proof of Theorem 2

In iteration n, we have the following (in)equalities

$$\prod_i^{M_k} \varepsilon_i^{(step5.a,n)} \geq \prod_i^{M_k} \varepsilon_i^{(step5.b,n)} = \prod_i^{M_k} \varepsilon_i^{(step6.a,n)}$$
$$\geq \prod_i^{M_k} \varepsilon_i^{(step6.b,n)} = \prod_i^{M_k} \varepsilon_i^{(step7.a,n)} \geq \prod_i^{M_k} \varepsilon_i^{(step7.b,n)}$$
$$\geq \prod_i^{M_k} \varepsilon_i^{(step5.a,n+1)}, \ \forall k.$$

The equalities hold due to the MSE duality and the first three inequalities due to uplink or downlink MMSE filtering. Finding the optimal power allocation by GP, leads to the last inequality. Note that decreasing of the geometric MSE values means increasing of the rates. Thus the individual rate per user is monotonically increasing with each iteration.

Table 1 Algorithm 1, 2, 3, 4: Transceiver Optimization for Rate Balancing (1) and Rate Constrained Power Minimization (2)

1: *initialize:* $[\boldsymbol{Ut}, \boldsymbol{St}, \boldsymbol{V}_k] = \text{svd}(\boldsymbol{H}_k)$, $\boldsymbol{T}_k^{(0)} = \boldsymbol{V}_k$, $[\boldsymbol{P}_k^{(0)}]_{ii} = P_{\max}/K/N_{R_k}, \forall k$, and $n_{\max}$
2: compute $\boldsymbol{U}_k^{(0)}$ and $\boldsymbol{\beta}_k^{(0)}$, $\forall k$, with (5) [**Algorithm 1, 2**] or (10) [**Algorithm 3, 4**]
3: **repeat**
4: $n \leftarrow n + 1$
5: uplink channel:
- a. for given $\boldsymbol{T}_k^{(n-1)}$, $\boldsymbol{U}_k^{(n-1)}$, and $\boldsymbol{\beta}_k^{(n-1)}$, $\forall k$, find optimal power allocation $\boldsymbol{p}$ by solving (9) [**Algorithm 1, 3 (3 with $\boldsymbol{\Psi}_u$)**], or by solving (8) [**Algorithm 2, 4 (4 with $\boldsymbol{\Psi}_u$)**]
- b. update $\boldsymbol{U}_k^{(n)}$ and $\boldsymbol{\beta}_k^{(n)}$, $\forall k$, with (5) [**Algorithm 1, 2**] or with (10) [**Algorithm 3, 4**]

6: downlink channel:
- a. compute $\boldsymbol{Q}^{(n)}$, with (4) [**Algorithm 3, 4 with $\boldsymbol{\Psi}_u$**]
- b. update $\boldsymbol{T}_k^{(n)}$ and $\boldsymbol{\beta}_k^{(n,tmp)}$, $\forall k$, with (6) [**Algorithm 1, 2**] or with (11) [**Algorithm 3, 4**]

7: uplink channel:
- a. compute $\boldsymbol{P}^{(n)}$ with (3) [**Algorithm 3, 4 with $\boldsymbol{\Psi}_u$**]
- b. update $\boldsymbol{U}_k^{(n)}$ and $\boldsymbol{\beta}_k^{(n)}$, $\forall k$, with (5) [**Algorithm 1, 2**] or with (10) [**Algorithm 3, 4**]

8: **until** required accuracy is reached or $n > n_{\max}$

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